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A PROGRAM TO DETERMINE ABSOLUTE AND RELATIVE FLUX DENSITIES FROM THE INDUCED ACTIVITY IN FOILS

I R A T A

INTRODUCTION

Foil Activation is a method commonly used to measure the relative distribution and the absolute flux density of high intensity particle beams (order > 10^{10} p cm^{-2}). For proton beams, aluminium is normally activated since Al^{27} produces radionuclides whose half lives are convenient for measurement and the production cross-sections of the radionuclides F^{18}, Na^{24} are well known (ref. [1]).

The usual procedure to find the distribution is to activate two foils and divide them into a series of individual horizontal and vertical strips. These strips are then separately measured with a counter and the counts, corrected for a common time, are plotted on graphs showing the relative horizontal and vertical distributions.

The total flux density follows from the corrected counts after a simple calculation.

THE PROBLEM in the specific case of a Al foil in a proton beam (of a momentum > 8 GeV/c)

When high energy protons pass through a foil of Al^{27} a family of different radionuclides are produced, viz. Na^{24}, Na^{22}, F^{18}, N^{13}, C^{11}, and Be^{8}. (Be^{8} can be ignored as its cross-section as well as its half life is comparatively very small). These radionuclides are unstable and emit electrons and positrons which are counted in the different samples (strips).

If every sample from one foil could be measured simultaneously on identical counters, a relative distribution could be accurately found by plotting these counts.

The samples are measured over an interval of time. So these measurements need to be corrected to some common point in time (e.g.: the end of irradiation, $t = t_0$) in order to create a meaningful distribution. The overall decay, however, is not directly exponential since the total activity consists of a number of different radionuclides decaying. It would be simpler to elucidate the part of the overall count supplied by one particular radionuclide and refer this to $t = t_0$. In fact, though as is shown further on, no particular one radionuclide need be selected, so long as each count is corrected for the time interval.

From these corrected counts the horizontal and vertical graphs are drawn, and also the total number of protons passing calculated.

CALCULATION of the Time Correction Factor and Number of Protons

The basic equation connecting the absolute number of protons passing through a foil and the subsequent electron or positron count (per unit time) of one radionuclide formed at the end of irradiation is given by

$$Np = \frac{N_{\beta 1}(t_0) \cdot \Delta t}{\sigma_1 \, \eta \, (1 - e^{\lambda_1 \Delta t}) \, \xi} \quad \text{with} \quad \lambda = \frac{\log_e 2}{\tau}$$

where

σ_1	is the production cross-section of the radionuclide (1)
τ_1	is the half-life of the radionuclide
η	is the number of atoms cm^{-3}
ξ	is the counting efficiency of the counter used
Δt	is the time of irradiation
Np	is the number of protons
$N_{\beta 1}$	is the number of electrons or positrons from radionuclide (1) at the time t_0 (corrected for background)
t_0	is the time at the end of irradiation

If the half-life τ is sufficiently large compared to Δt then

$$Np = \frac{N_{\beta 1}(t) \cdot e^{\lambda_1 t}}{\sigma_1 \eta \lambda_1 \xi}$$ after a time t from end of irradiation.

From this follows at $t = t_0$

$$Np = \frac{N_{\beta 1}(t_0)}{\sigma_1 \eta \lambda_1 \xi} \qquad (*)$$

Thus $N_{\beta 1}(t) = \xi \cdot Np \cdot \sigma_1 \cdot \eta \cdot \lambda_1 \cdot e^{-\lambda_1 t}$. This holds for each radionuclide produced (if $\tau >> \Delta t$) with different values for σ_1 and λ_1. Thus the total count, at time t is $N_{\beta 1}(t) + N_{\beta 2}(t) + N_{\beta 3}(t) + + N_{\beta 4}(t) + N_{\beta 5}(t)$ ....

Therefore $N_{\beta}(t) = \xi \cdot Np \cdot \eta (\sigma_1 \lambda_1 e^{-\lambda_1 t} + \sigma_2 \lambda_2 e^{-\lambda_2 t} \ldots) =$

$$= \xi \cdot Np \cdot \eta \sum_{i=1}^{n} (\sigma_i \lambda_i e^{-\lambda_i t})$$

to simplify matters call

$$\sum_{i=1}^{n} (\sigma_i \lambda_i e^{-\lambda_i t}) = \Sigma f(t)$$

Thus at a time t the fraction $\frac{N_{\beta 1}(t)}{N_{\beta}(t)}$ is :

$$\frac{\sigma_1 \cdot \lambda_1 \cdot e^{-\lambda_1 t} \cdot \xi \cdot Np \cdot \eta}{\xi \cdot p \cdot \eta \Sigma f(t)}$$

Therefore $N_{\beta 1}(t) = \dfrac{N_{\beta}(t) \cdot \sigma_1 \cdot \lambda_1 \cdot e^{-\lambda_1 t}}{\Sigma f(t)}$

and this $N_{\beta 1}(t)$ count when referred to time t_0 becomes

$$N_{\beta 1}(t_0) = \frac{N_{\beta}(t) \quad \sigma_1 \cdot \lambda_1 \cdot e^{-\lambda_1 t}}{\Sigma f(t)} \cdot e^{\lambda_1 t} = \frac{N_{\beta}(t) \ \sigma_1 \ \lambda_1}{\Sigma f(t)}$$

However, since these counts are relative it is more simple to calculate $\Sigma f(t)$ only and so find

$$\frac{N_{\beta 1}(t_0)}{\sigma_1 \ \lambda_1} = \frac{N_{\beta}(t)}{\Sigma f(t)}$$

This calculation is made for each sample and printed out as "CORR. COUNT".

Going back to equation (*) it can be seen that

$$Np = \frac{N_{\beta 1}(t_0)}{\sigma_1 \ \eta \ \lambda_1 \ \xi} = \frac{N_{\beta}(t)}{\Sigma(t)} \cdot \frac{1}{\eta \cdot \xi} \quad ;$$

In this manner the number of protons passing through each strip could be found and summed to give the total number of protons. In actuality it is more convenient to do a summation of

$$\frac{N_{\beta}(t)}{\Sigma(t)}$$

first, and then the division.

ERRORS

To provide an idea of the statistical accuracy of each count a percentage error is calculated from the count per minute in the form :

$$\text{Statistical error} = \frac{\sqrt{N_\beta(t)_i}}{N_\beta(t)_i} \cdot 100 = x_i \text{ (in \%)} \quad .$$

These errors are then used to weigh the different measurements of one sample when they are averaged for use in the histogram. This is effected by :

$$\text{Weighed count} \quad W_\beta(t_0) = \sum_{i=1}^{M} \left(\frac{N_\beta(t_0)_i}{x_i} \right) \Big/ \sum_{i=1}^{M} \left(\frac{1}{x_i} \right)$$

where M is the number of the measurement of the one sample.

Also at the same time an arithmetic average of errors is made and printed out along side the histogram.

$$Z_i = \frac{\sum_{i=1}^{M} (x_i)}{M} \quad .$$

Finally, an estimate of the error involved in the total number of protons is made by saying :

$$\text{Weighed error} = \frac{\sum_{i=1}^{N} \left(W_\beta(t_0)_i \cdot Z_i \right)}{\sum_{i=1}^{N} \left(W_\beta(t_0)_i \right)}$$

where N = total number of samples.

THE PROGRAM

The method involved in finding the CORR. COUNT has been already explained as depending on a correction function $\Sigma f(t)$. So also has the ERROR. These and an echo of the original input are printed out.

The program then uses the inverse of the error to weigh measurements of one sample taken at different times and sorts these into ascending logical order. With this order the sample numbers, weighed counts, and each count as a percentage of the maximum count are printed out, so producing a histogram of the relative proton distribution. The averaged error as defined above is also printed.

Finally the total number of protons and its error are printed (see above).

The calculation of the correction function $\Sigma f(t)$ has been written as a subroutine (called FACTOR) since this program is applicable to any form of foil activation. For a different type of activation the values σ_i , λ_i , ξ and η are all that would need to be changed.

DATA INPUT

The measurements made on strips all cut from the one same foil, in either the horizontal or vertical plane, are considered by the program separately. To this end each batch of cards, one for each measurement, is headed by a card which contains information about the irradiation itself and the following cards.

Any number of batches may run at one time, so long as the number of cards in one batch is less than 1000.

Format of a batch containing N measurements

CARD 1 The number of sample cards N ; the background at time of counting; the hour when irradiation finished; the minute when it finished; the thickness of the foil in microns.

Format : (I 10, F 10.0, I 10, I 10, F 10.0)

CARD 2 The hour when the sample is measured; the minute when measured; the sample number; the time of counting the count.

Format : (I 6, I 2, I 4, F 8.0, F 10.0)

CARD N + 1

The format of the cards $2 \rightarrow N + 1$ has been chosen as it is the format of the paper supplied in the counting room, except that no provision in the program is made for the DATE. Instead the hour input is I 6 , and this hour is referred to the same zero hour as that of the hour of irradiation.

The SAMPLE must be an integer between 1 and 9999.

The measurement cards (i.e. $2 \rightarrow N + 1$) can be in any order as the histogram will appear in the ascending logical order of the sample numbers.

Values used in the subroutine FACTOR

	Na^{24}	Na^{22}	F^{18}	N^{13}	C^{11}	Ref.
τ hrs	900	2.6 yrs	111	10	20.5	[1]
$\sigma \times 10^{-27}$b	8.6	10.0	6.5	1.1	6.1	[1]

ξ the efficiency = 0.25 (the same efficiency is used for all samples)

η number of atoms cm^{-3} = $6.0 \times 10^{18} \times$ thickness of foil in microns for aluminium.

RESULTS

The histograms of the relative distributions although arranged as percentages of the maximum count always appear with a count $\geqslant$ 1 % , see the EXAMPLE.

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REFERENCE :

[1] J.B. CUMMING
Annual Rev. of Nucl. Sciences 13, 267 (1963)

Distribution (open) :

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EXAMPLE

The following shows the input for a run written on the aforesaid paper.

The samples have been purposely arranged out of any order to display the sorting in the histogram.

Also included is the output for such a run.

N.B.: Programs are readily available upon request.

COUNTER TARGET

DATE	TIME	SAMPLE	TIME OF COUNT	COUNTS	C/M	COINC CORR	BKND	C/M NET	
1	5	10	20	30			40		50
		30	6.0	6			48	45.	
	1218	101	2	6829					
	1220	102	2	2088					
	1223	103	2	380					
	1238	109	2	24					
	1307	99	2	19964					
	1322	92	2	41					
	1320	93	2	64					
	1340	96	2	1178					
	1318	94	2	148					
	1316	95	2	311					
	1314	96	2	1247					
	1311	97	2	5223					
	1309	98	2	13259					
	1305	100	2	16631					
	1225	104	2	174					
	1240	110	2	24					
	1327	102	2	1548					
	1329	101	2	5687					
	1331	100	2	15489					
	1333	99	2	19443					
	1336	98	2	12244					
	1331	100	2	15489					

COUNTER TARGET

DATE	TIME	SAMPLE	TIME OF COUNT	COUNTS	C/M	COINC CORR	BKND	C/M NET	
1	5	10	20	30					
	1227	105	2	77					
	1230	106	2	54					
	1232	107	2	43					
	1333	99	2	19443					
	1336	98	2	12244					
	1338	97	2	4795					
	1325	91	2	32					

SAMPLE	REL. COUNT	NORMALISED DISTRIBUTION	%ERROR
91.	1206.83	X	31.6
92.	1734.[illegible]7	X	26.3
93.	3[illegible]90.48	X	19.6
94.	8[illegible]33.44	X	12.1
95.	17553.38	X	8.2
96.	72795.96	XXXXXX	4.1
97.	300609.86	XXXXXXXXXXXXXXXXXXXXXXXXXX	2.0
98.	762087.55	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	1.3
99.	1180039.88	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	1.0
100.	948063.12	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	1.1
101.	338146.85	XXXXXXXXXXXXXXXXXXXXXXXXXXXXX	1.8
102.	97598.76	XXXXXXXX	3.4
103.	18154.[illegible]4	X	7.4
104.	8[illegible]47.49	X	11.1
105.	3251.36	X	17.5
106.	2122.68	X	21.8
107.	1577.50	X	25.4
108.	1639.53	X	25.0
109.	623.20	X	40.8
110.	627.41	X	40.8

TOTAL REL. COUNT 3757[illegible]03.42

TOTAL NO OF PROTONS 5.581E+13 WEIGHTED ERROR 1.6

TIME	SMPL	CTIME	COUNT	CORR. COUNT	+/-%ERROR
1218	101	2.	6829.	330453.86725	1.71
1220	102	2.	2088.	101344.03111	3.10
1223	103	2.	380.	18154.04326	7.37
1234	108	2.	44.	1639.52752	25.00
1238	109	2.	24.	623.20300	40.82
13 7	99	2.	19964.	1138910.77846	1.00
1322	92	2.	41.	1734.07489	26.26
1320	93	2.	64.	3090.47943	19.61
1340	96	2.	1178.	73557.35696	4.14
1318	94	2.	148.	8033.43626	12.13
1316	95	2.	311.	17553.38190	8.18
1314	96	2.	1247.	72056.13338	4.02
1311	97	2.	5223.	301212.23684	1.96
13 9	98	2.	13259.	760941.68715	1.23
13 5	100	2.	16631.	942680.83950	1.10
1225	104	2.	174.	8047.48692	11.11
1240	110	2.	24.	627.40714	40.82
1327	102	2.	1548.	93244.64140	3.61
1329	101	2.	5687.	346578.42434	1.88
1331	100	2.	15489.	950851.77840	1.14
1333	99	2.	19443.	1200878.30491	1.01
1336	98	2.	12244.	762683.78602	1.28
1331	100	2.	15489.	950851.77840	1.14
1227	105	2.	77.	3251.35924	17.54
1230	106	2.	54.	2122.68404	21.82
1232	107	2.	43.	1577.50499	25.40
1333	99	2.	19443.	1200878.30491	1.01
1336	98	2.	12244.	762683.78602	1.28
1333	97	2.	4795.	299981.11736	2.04
1325	91	2.	32.	1206.83261	31.62