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INTRAB, A COMPUTER CODE TO CALCULATE GROWTH RATES, EMITTANCE EVOLUTION AND EQUILIBRIUM WITH INTRABEAM SCATTERING AND COOLING

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ABSTRACT

This note describes a code to estimate the influence of intrabeam scattering (IBS)

The code is composed of three separate programs :

The first, called INTRABT, calculates the growth times for a given set of emittances. It is based on Piwinski's classical theory extended to include the variation of the lattice functions around the ring. Different versions of this program have been used at CERN and elsewhere since about 1974.

Our second program, INTRABS, models the time evolution of the emittances, under the combined action of IBS and phase space cooling (stochastic or electron) starting from given initial emittances.

The third program, INTRABE, searches for the equilibrium emittances with IBS and cooling. Coupling between the horizontal and the vertical planes (e.g. by skew quadrupoles or solenoidal fields) is included in INTRABS and INTRABE by the use of a coupling coefficient as done in the theory of radiation damping in electron machines.

This note is meant to serve as a user guide. It describes the input/output procedures and includes a number of commented sample runs.

Distribution

Geneva, Switzerland

1. INTRODUCTION

In this note we describe a code to estimate the influence of intrabeam scattering (IBS). Originally developed to model growth times, evolution of emittances and equilibrium conditions in the Initial Cooling Experiment (ICE) performed at CERN in 1977-1979, this code was subsequently used in the design and in the interpretation of observations at LEAR as well as in the conceptual study of the Superconducting 12 GeV Antiproton Ring "SuperLEAR"(1).

The code is composed of three separate programs :

- The first program, called INTRABT, calculates the amplitude growth times for a given set of emittances. It is based on Piwinski's classical theory (2), extended to include the variation of the lattice functions around the ring. Different versions of this program have been used at CERN and elsewhere since about 1974. In Appendix 1 we try to sketch a historical review, with the hope to give due credit to the many workers who contributed to the field and to underline the pioneering role which Anton Piwinski has played.
- Our second program, INTRABS, was developed to simulate the beam behaviour in ICE and LEAR. It calculates the time evolution of the emittances, starting from given initial values. Basically, the method is to let the three emittances grow for a fraction (typically 0.1) of the shortest growth time and then recalculate the time constants for the new set of emittances. Phase-space cooling (stochastic or electron) can be included in a rough way by using e-folding with $e^{\tau_{IBS}-\tau_{COOL}}$

  We emphasise that all intrabeam scattering and cooling times used throughout the programs are the e-folding times for the betatron amplitudes a_h, a_v and for the momentum spread $\Delta p/p$. The corresponding time constants for the transverse emittances E_h, E_v (1/2 of the amplitude e-folding times !) are not used for either input or output.

  In the most general case, each of the three cooling times τ_c (horizontal, vertical and longitudinal) is a function of the three emittances. So far, we mostly used constant $\tau_{ch}, \tau_{cv}, \tau_{cs}$.
- The third program, INTRABE, estimates the equilibrium emittances with cooling, specified by the three time constants, as in INTRABS. In fact, INTRABE may be viewed as a version of INTRABS, that searches for conditions where $\left[(1/\tau_{IBS})-(1/\tau_c)\right] \rightarrow 0$ in the three planes.

  Coupling between the horizontal and the vertical plane (e.g. by skew quadrupole or solenoidal fields) can considerably influence the results of IBS as pointed out by G. Parzen (3). This effect is included in a crude way in INTRABS and INTRABE by the use of a coupling coefficient $K \geq 0$, as done in the theory of radiation damping in electron machines. We set :

$$E_{hc} = \frac{E_{hu} + KE_{vu}}{K+1}$$
$$E_{vc} = \frac{E_{vu} + KE_{hu}}{K+1} \qquad (1)$$

where the subscript c stands for *coupled* and the subscript u for *uncoupled*. For $K = 0$ (default value) there is no coupling and for $K = 1$ there is full coupling, which equalizes the two emittances. More details are given in the examples below.

The three programs use the Twiss functions calculated by MAD[4] at a series of points (usually at the entrance and at the exit of elements as used for lattice calculations). Interpolation is then applied to establish the values at a given number (typically 20) of equidistant points in the machine period. Growth rates are calculated for each point and averaged "around the ring".

Gaussian beams are assumed at all steps. In reality (especially in the presence of a cooling system), neither the initial nor the final beams are Gaussian. Thus all results are approximate but hopefully indicative of the trends.

In the following we first describe features common to all three versions of INTRAB and then give details and examples specific to INTRABT, INTRABS and INTRABE.

2. FEATURES COMMON TO THE THREE CODES

2.1 Handling of Input Data

The programs are resident in the CERN VAX CLUSTER[5] and can be run interactively or in Batch. To run the programs in Batch it is necessary to prepare the Batch Command Files INTRABT.COM, INTRABS.COM or INTRABE.COM which contain the Input Data. This is done in an interactive way, as for non-Batch runs.

In the interactive runs as well as during the preparation of the BATCH Command file, the input data have default values. A list of the default values is presented on the screen with a code number for each parameter (see Tables 1 to 3 corresponding to INTRABT, INTRABS and INTRABE, respectively). Note that some of the basic default values (like the storage ring circumference) as used in the sample version are specific to LEAR, whereas others (like the number of points for interpolation) are more general.

We mention also the *internal parameter* CW which is retained for historical reasons. It is set to 1 but can be altered by changing the corresponding statement in the program. For CW = 1 the full theory including the derivatives of the lattice functions is used; for CW = 0 Piwinski's original theory is applied (setting $a_h = 0$ and $D' = 0$).

Table 1 - Default input menu for program INTRABT

```
                INPUT MENU FOR PROGRAM INTRABT
            ENTER CODE NUMBER TO CHANGE A PARAMETER
            "CTRLZ" TO EXIT,"GO" TO START CALCULATION
0   PARTICLES SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
1   TITLE=PROTONS IN LEAR
2   TWISS MAD FILE=octleamad.dat
3   NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4   PRINT SWITCH (IF >0 EXTENDED PRINTOUT)=   -1
5   TRANSVERSE EMITTANCE MULTIPLIER=   1.00000
6   SIGMAP MULTIPLIER =   1.00000
7   SWITCH FOR LINEAR(1)OR GEOMETRIC(2)INCREMENTING OF EMITTANCES=    2
8   NUMBER OF EMITTANCE INCREMENTS=    4
9   FACTOR FOR H-EMITTANCE INCREMENTS=     2.0000
10  NUMBER OF DP/P INCREMENTS=    3
11  FACTOR FOR DP/P INCREMENTS=2.0000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
13  NUMBER OF PARTICLES=          0.1000000E+10
14  RING CIRCUMFERENCE =       78.5400 M
15  MOMENTUM OF THE BEAM=    0.60000     GeV/c
16  STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
18  STARTING DP/P=  0.1000E-03
```

Table 2 -Default input menu for program INTRABS

```
INPUT MENU FOR INTRABS-ENTER CODE NR TO CHANGE ,"CTRLZ" TO EXIT,"GO" TO START
0   PARTICLE SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
1   TITLE=PROTONS IN LEAR
2   TWISS MAD FILE=octleamad.dat
3   NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4   PRINT SWITCH (IF >0 EXTENDED PRINT)=   -1
5   TRANSVERSE EMITTANCES MULTIPLIER=   1.00000
6   SIGMAP MULTIPLIER =   1.00000
7   SWITCH FOR COOLING TIMES VS EMITTANCES (1=CONST,2=TABLE,3=FUNC)=    1
8   H AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
9   V AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
10  DP COOLING TIME CONSTANT=  3600.0     SEC
11  RATIO BETWEEN TIME STEP AND MINIMUM TAU=   0.10000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
13  NUMBER OF PARTICLES          =  0.1000000E+10
14  RING CIRCUMFERENCE =       78.5400 M
15  MOMENTUM OF THE BEAM=    0.60000     GeV/c
16  STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
18  EMITTANCE COUPLING FACTOR=     0.00000
19  STARTING DP/P=  0.1000E-03
20  MAXIMUM NUMBER OF CASES=   40
21  MAXIMUM BEAM STORAGE TIME=   0.360E+05 SEC
```

Table 3 - Default input menu for program INTRABE

```
INPUT MENU FOR INTRABE-ENTER CODE NR TO CHANGE,"CTR&Z" TO EXIT,"GO" TO START
0   PARTICLE SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
1   TITLE=PROTONS IN LEAR
2   TWISS MAD FILE=octleamad.dat
3   NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4   PRINT SWITCH (IF >0 EXTENDED PRINT)=   -1
5   TRANSVERSE EMITTANCES MULTIPLIER=   1.00000
6   SIGMAP MULTIPLIER =   1.00000
7   SWITCH FOR COOLING TIMES VS EMITTANCES (1=CONST,2=TABLE,3=FUNC)=    1
8   H AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
9   V AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
10  DP/P COOLING TIME CONSTANT=  3600.0     SEC
11  RATIO BETWEEN TIME STEP AND MINIMUM TAU=   0.10000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
13  NUMBER OF PARTICLES          =  0.1000000E+10
14  RING CIRCUMFERENCE =       78.5400 M
15  MOMENTUM OF THE BEAM=    0.60000     GeV/c
16  STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
18  EMITTANCE COUPLING FACTOR=     0.00000
19  STARTING DP/P=  0.1000E-03
20  MAXIMUM NUMBER OF CASES=   40
21  TOLERANCE FOR EQUILIBRIUM=   0.200E-01
```

To change an input parameter the user enters the corresponding code number. Then the programme asks for the new value of the parameter and thereafter shows the updated input menu. The process can be repeated ad libitum, i.e. each parameter can be changed several times before the calculation starts. Depending on the options chosen (electrons, protons or ions, bunched or coasting beam, etc.) some lines of the input menu have a slightly different form. This should become clear in the examples given below.

To start the execution the user enters "GO". At the end of the run a file INTRABT.LAS, INTRABS.LAS or INTRABE.LAS is created. This file allows one to start a new run with the last set of input data of the previous run. For batch as well as for interactive runs several input lists can be stored and executed during a run, with the condition that the total CPU time does not exceed "TIMELIM". The variable TIMELIM is set to 4000 sec in the Main Programs and enters the system routine TIMEST.

2.2 Lattice Input

The lattice parameters needed are the betatron functions β_h, β_v, $\alpha_h = -\beta_h'/2$, the horizontal dispersion function $D(s)$ and its derivative $D'(s)$. We only treat plane machines without vertical dispersion and hence α_v and D_v are not needed. The lattice functions have to be specified at a number of points (NMAD, typically 40) in the basic period (superperiod, or half-superperiod in the case of reflection symmetry...). Usually we take the standard MAD

input file to create the table of the lattice functions. The table is written onto the Twiss file issued by MAD(4). The name of this file has to be supplied as an input to INTRAB.

Internally, INTRAB uses the lattice functions at a specified number (NPOINT, default value 20) of equidistant points as explained above. Experience has shown that :

NPOINT < NMAD, typically NPOINT = 0.5*NMAD is a reasonable choice.

2.3 Momentum Spread and Bunch Length Definitions

The momentum distribution ($\Delta p/p$) is assumed to be Gaussian in all cases. Its standard deviation SIGMAP, if desired multiplied by a factor *ml* to comply with the definition preferred by the user, is used as momentum spread:

DP/P=ml*SIGMAP

where *ml* is input, default value = 1.

For the longitudinal density, two cases, namely coasting or bunched beam are allowed. A variable (NBB) to distinguish them is permanently on the input list and can be changed by the user. The further reading of the longitudinal parameters is then controlled by this switch:

i) Coasting beam (NBB = 2): In this case the line density is constant.

$$\lambda = \frac{N}{2\pi R}.$$

where N is the total number of particles and $2\pi R$ the circumference of the ring; N and $2\pi R$ appear on the input list when NBB is 2.

ii) Bunched beam (NBB = 1): In this case the line density of the bunch is Gaussian and given by:

$$\lambda = \frac{N_b}{\sqrt{2\pi}\sigma_s} e^{-s^2/2\sigma_s^2}$$

Here N_b is the number of particles per bunch and σ_s the rms bunch length.

The input parameter for σ_s is the *bunch length* defined by σ_s multiplied by the same factor *ml* as used in the definition of the momentum spread: N_b and $m_l\sigma_s$ appear on the input list when NBB is 1.

2.4 Transverse Emittance Definitions

The programs imply that the transverse distributions are Gaussian. The standard deviations σ_h, σ_v at a location with the beta functions β_h, β_h are specified by the corresponding emittances E_h, Ev defined by :

$$E_{h,v} = \frac{(m\sigma_{h,v})^2}{\beta_{h,v}} \qquad (2)$$

Here m (default 1) is a multiplier permitting the user to work with his preferred emittance definition. We note that for the Gaussian beam, the fraction of particles contained in the emittance (Eq. (2)) is, for each of the two planes:

$$P = 1 - e^{-m^2/2}$$

Values frequently used are given in Table 4.

Table 4 - Relation between transverse multiplier and fraction of particles

m	1	2	2.5
P (%)	39	86	95

Once the values of m and ml are set, the corresponding emittance and momentum spread (and for NBB = 1 also the bunch length) conventions are used for all input and output during the run.

2.5 Particle Species

Electrons (IDP ≤ 0), protons (IDP = 1) and heavier ions (IDP > 1) are treated, depending on the value of the identifier IDP (default 1). For IDP > 1 the charge state number (Q) and the mass number (A) of the ion are requested as input. We use the atomic mass unit with m_0c^2 = 931.50 MeV, hence for protons A = 1.00727, for electrons A = 0.00055 and for C_{12} A = 12.0.

3. SPECIFIC FEATURES OF THE PROGRAM INTRABT

INTRABT calculates a table of intra-beam scattering growth times. The table has a total of NCASE = (SDP+1)*(SEMIT+1) lines, where each line gives the three amplitude growth times for a new set of emittances and dp/p.

Input parameters determining the construction of the table are :

- the switch ICR which controls the mode of incrementing, (linear or geometric progression),
- the starting values of the emittances (HEMITO, VEMITO),
- the number of emittance steps (SEMIT), and
- the increment (DHEMIT) when linear incrementing (ICR = 1) is requested, or the incrementing factor (EMITFAC, default value 2) when geometric incrementing (ICR = 2) is requested. Depending on the value of ICR either DHEMIT or EMITFAC appears on the input list.

- similar parameters (DP0, SDP and DDP or DPFACT) for the relative momentum spread *dp/p*.

The following conventions are used to establish the table:

i) The (only!) parameters varied are the emittances and dp/p.

ii) The emittance ratio E_v/E_h as specified by the starting values is kept the same throughout, i.e. E_v is varied in proportion to E_h.

iii) Beginning with the starting values, *dp/p* is incremented SDP times (this produces the first SDP+1 lines).

iv) Then the emittance HEMIT is incremented once and the loop ii) - iii) is repeated ... etc.

v) For ICR = 1 (linear incrementing) the stepsize DHEMIT and DDP are used.
For ICR = 2 (geometric progression) the values are multiplied with DPFACT or EMITFAC at each step (i.e. doubled when the defaults DPFACT = EMITFAC = 2 are used).

Details should become clear from the examples given below.

3.1 Examples of Runs of INTRABT

3.3.1 Notation

In the following examples an {i} at the beginning of a line is used to indicate the input entered by the user. Comments in the text below are in small case letters. They do not appear during execution (neither does the {i}). All input and output that appear during the run are indicated in capital letters below.

3.3.2 First Sample Run: Coasting Beam of Protons in LEAR

Prior to the run a Twiss file (OCTLEAMAD.DAT in this example) has been prepared. To start execution one enters:

{i} RUN INTRABT

Then the program responds:

WELCOME TO "INTRABT" TO CALCULATE A TABLE OF INTRABEAM SCATTERING GROWTH TIMES.

ENTER

```
 1          TO START FROM THE DEFAULT INPUT MENU
-1          TO START FROM THE LAST INPUT MENU (FILE INTRABT.LAS)
 0          TO PREPARE THE FILE INTRABT.COM FOR BATCH RUNS
CTRL & Z    TO EXIT
```

IF INTRABT.LAS IS USED THE MENU WILL SCROLL DOWN SEVERAL TIMES ON THE SCREEN.DONT WORRY ! THIS IS NORMAL !

{i} 1 Now the default input menu (Table 1) appears on the screen.

Table 5 - Input for example 1 of INTRABT Coasting Beam of Protons in LEAR at p=0.309 GeV/c

```
                INPUT MENU FOR PROGRAM INTRABT
            ENTER CODE NUMBER TO CHANGE A PARAMETER
            "CTR&Z" TO EXIT,"GO" TO START CALCULATION
 0  PARTICLES SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
 1  TITLE="PROTONS IN LEAR"
 2  TWISS MAD FILE=octleaaad.dat
 3  NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
 4  PRINT SWITCH (IF >0 EXTENDED PRINTOUT)=   -1
 5  TRANSVERSE EMITTANCE MULTIPLIER=   1.00000
 6  SIGMAP MULTIPLIER =   1.00000
 7  SWITCH FOR LINEAR(1)OR GEOMETRIC(2)INCREMENTING OF EMITTANCES=    2
 8  NUMBER OF EMITTANCE INCREMENTS=    4
 9  FACTOR FOR H-EMITTANCE INCREMENTS=     2.0000
 10 NUMBER OF DP/P INCREMENTS=    3
 11 FACTOR FOR DP/P INCREMENTS=2.0000
 12 BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
 13 NUMBER OF PARTICLES=          0.1000000E+10
 14 RING CIRCUMFERENCE =       78.5400 M
 15 MOMENTUM OF THE BEAM=    0.30900     GeV/c
 16 STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
 17 STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
 18 STARTING DP/P=  0.1000E-02
```

Suppose that we want to calculate a table of IBS times for a beam momentum (0.309 GeV/c), and a starting value of *dp/p* (0.001) different from the default values. One enters:

{i} 15

The program responds :

ENTER MOMENTUM OF THE BEAM (GEV/C)

After entering

{i} 0.309

the updated input list is displayed

....

....

One continues :

{i} 18

ENTER STARTING *dp/p*

{i} 0.001

Then the new input menu (Table 5) is displayed (as after each change of a parameter).

....

....

Now that all parameters are as desired we start the calculation by :

{i} GO

The program responds :
LAST CHANCE TO CHECK!

TO START CALCULATION WITH THE ABOVE INPUT ENTER "GO" AGAIN.
TO GO BACK TO THE INPUT MENU ENTER ANY OTHER CHARACTER.
{i} GO

Now the program displays :
BE PATIENT. INTRABT IS CALCULATING LINE 1 OF THE TABLE
LAST LINE WILL BE THE nth

and so on for each line. At the end the program displays:

CALCULATION SUCCESSFULLY TERMINATED. ENTER 1 TO COMPUTE ANOTHER CASE, CTRL & Z TO EXIT.

If "1" is entered, the input list is displayed again and the user can change parameters to start a new run. If CTRL & Z is typed, the input data are stored onto the file INTRABT.LAS that can be used in a subsequent run. The output file INTRABT.LST can be read after exit (see below). An extended output (useful in case of trouble or doubt) can be obtained setting the IPR switch to a positive value.

3.1.3 Output file

The results are written onto the file INTRABT.LST that can be viewed and/or printed, e.g. on our printer PS3812 with the instruction

XPRINT INTRABT.LST/PRINTER=PS3812/CC=F

The file INTRABT.LST contains the table of lattice parameters at equidistant points as well as other input parameters on a first page.

The table of the growth times is printed on a new page, which also contains a summary of beam parameters like momentum, intensity, coasting or bunched beam, etc. The output file for the above sample run is reproduced in Table 6.

3.1.4 A second sample run of INTRABT: Coasting beam of Pb^{+53} in LEAR

The input for the case of ions is obtained from the input menu by modifying the switch for the particle species (IDP) to 2. The new input list then includes the charge state number and the mass number of the ion with default values. The input list after the change to Pb^{+53} is reproduced in Table 7.

The second page of the output file INTRABT.LST for this case is shown in Table 8.

Table 6 - Output for Example 1 of INTRABT

```
 9-JUL-1993 13:17:23
CALCULATION OF A TABLE OF INTRABEAM SCATTERING GROWTH TIMES
"PROTONS IN LEAR"
COASTING  BEAM OF PROTONS

RING CIRCUMFERENCE                                                  78.54000 M
NUMBER OF PARTICLES                                                  0.1000000E+10
MOMENTUM                                                            .30900      GeV/c

Emittance Definition E = PI (     1.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =      1.00 SIGMAP
Starting Values of the Table
HORIZONTAL EMITTANCE                                                0.25000E-05 PI M RAD
VERTICAL   EMITTANCE                                                0.25000E-05 PI M RAD
RELATIVE MOMENTUM SPREAD                                            0.10000E-02

PRINT SWITCH=    -1 IF >0 EXTENDED PRINT
REQUESTED NUMBER OF LINES IN TABLE                                  20
obtained by combining      5 different EMITTANCES VALUES with     4 different DP/P VALUES
Emittances and DP/P increase GEOMETRICALLY (ICR=     2)
HORIZONTAL EMITTANCE INCREMENTING FACTOR                            2.000000
Emittances ratio Ev/Eh is constant throughout the Table
DP/P INCREMENTING FACTOR                                            2.0000

7.2/VM  TWISS   88/11/2109:09   GIA126         1       F      26
FILE=2H302V73 LEAR WITH DUMMY SEXTUPOLES 27 MAY 87
NUMBER OF ELEMENTS IN TWISS  FILE octleamad.dat                  Issued by MAD =   26

TABLE OF TWISS PARAMETERS AT    20 EQUIDISTANT POINTS
OBTAINED BY INTERPOLATING THE INPUT FILE
   ------------------------------------------------------
    BH(M)      AH         BV(M)      DH(M)       DHP      L(M)
    1.968      0.000      5.232      3.584       0.000    0.000
    2.258     -0.263      5.341      3.584       0.000    0.517
    2.549     -0.525      5.451      3.584       0.000    1.033
    3.217     -0.788      5.702      3.584       0.000    1.550
    4.395     -1.050      6.145      3.584       0.000    2.067
    5.837     -1.313      6.687      3.584       0.000    2.584
    7.279     -1.575      7.230      3.584       0.000    3.100
    8.722     -1.838      7.772      3.584       0.000    3.617
   10.196     -1.207      8.942      3.507      -0.297    4.134
    8.520      4.053     13.863      2.846      -2.171    4.650
    5.908      1.312     19.975      1.910      -1.488    5.167
    5.760     -0.894     19.054      1.323      -0.835    5.684
    6.800     -1.055     14.915      0.892      -0.835    6.201
    7.917     -1.129     11.167      0.474      -0.787    6.717
    9.065     -1.061      8.022      0.094      -0.681    7.234
   10.103     -0.901      5.437     -0.231      -0.565    7.751
   10.880     -0.637      3.495     -0.483      -0.422    8.267
   11.063     -0.412      2.601     -0.591      -0.274    8.784
   11.245     -0.187      1.707     -0.699      -0.127    9.301
   11.402      0.000      0.934     -0.792       0.000    9.817
```

Table 6 - Continued

```
 ---INTRA-BEAM SCATTERING GROWTH TIMES IN SECONDS.
 FOR CW=0 PIWINSKIS THEORY IS APPLIED AT NPOINT EQUIDISTANT LOCATIONS IN  THE SUPERPERIOD AND 1/TAU AVERAGED.
 FOR CW=1 THE DERIVATIVE OF BETAH AND D IS INCLUDED
 NUMBER OF POINTS IS NPOINT =   20   CW=1.00
COASTING  BEAM OF PROTONS

RING CIRCUMFERENCE                                                    78.54000 M
NUMBER OF PARTICLES                                                    0.1000000E+10
MOMENTUM                                                              .30900      GeV/c

Emittance Definition E = PI (    1.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =     1.00 SIGMAP

TABLE OF INTRABEAM SCATTERING AMPLITUDE GROWTH TIMES
----------------------------------------------------

"PROTONS IN LEAR"

EH(PI*RAD*M) EV(PI*RAD*M) DP/P     TAUH(SEC) TAUV(SEC) TAUE(SEC)


  0.250E-05 0.250E-05 0.100E-02    0.143E+05 0.499E+05 0.395E+06
  0.250E-05 0.250E-05 0.200E-02    0.169E+05 0.460E+05-0.343E+06
  0.250E-05 0.250E-05 0.400E-02    0.235E+05 0.617E+05-0.119E+07
  0.250E-05 0.250E-05 0.800E-02    0.377E+05 0.993E+05-0.636E+07

  0.500E-05 0.500E-05 0.100E-02    0.749E+05 0.401E+06 0.158E+06
  0.500E-05 0.500E-05 0.200E-02    0.821E+05 0.241E+06-0.162E+07
  0.500E-05 0.500E-05 0.400E-02    0.106E+06 0.279E+06-0.318E+07
  0.500E-05 0.500E-05 0.800E-02    0.158E+06 0.416E+06-0.143E+08

  0.100E-04 0.100E-04 0.100E-02    0.403E+06 0.133E+08 0.196E+06
  0.100E-04 0.100E-04 0.200E-02    0.415E+06 0.146E+07 0.115E+08
  0.100E-04 0.100E-04 0.400E-02    0.493E+06 0.134E+07-0.100E+08
  0.100E-04 0.100E-04 0.300E-02    0.686E+06 0.180E+07-0.346E+08

  0.200E-04 0.200E-04 0.100E-02    0.218E+07-0.122E+08 0.319E+06
  0.200E-04 0.200E-04 0.200E-02    0.218E+07 0.118E+08 0.461E+07
  0.200E-04 0.200E-04 0.400E-02    0.240E+07 0.705E+07-0.472E+08
  0.200E-04 0.200E-04 0.800E-02    0.309E+07 0.819E+07-0.930E+08

  0.400E-04 0.400E-04 0.100E-02    0.117E+08-0.284E+08 0.605E+06
  0.400E-04 0.400E-04 0.200E-02    0.118E+08 0.398E+09 0.573E+07
  0.400E-04 0.400E-04 0.400E-02    0.122E+08 0.427E+08 0.337E+09
  0.400E-04 0.400E-04 0.800E-02    0.145E+08 0.395E+08-0.294E+09

THE DATA TO RERUN THE LAST CASE HAVE BEEN  STORED IN File INTRABT.LAS
```

Table 7 - Input file for the 2nd example of INTRABT. Coasting beam of lead 53+ at 4.2 MeV/u in LEAR.

```
                INPUT MENU FOR PROGRAM INTRABT
            ENTER CODE NUMBER TO CHANGE A PARAMETER
            "CTR&Z" TO EXIT,"GO" TO START CALCULATION
0   PARTICLES SPECIES=IONS      (A=   207.000 Q=     53.00)
1   TITLE=Pb 53 times charged in LEAR
2   TWISS MAD FILE=octleamad.dat
3   NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4   PRINT SWITCH (IF >0 EXTENDED PRINTOUT)=   -1
5   TRANSVERSE EMITTANCE MULTIPLIER=   1.00000
6   SIGMAP MULTIPLIER =   1.00000
7   SWITCH FOR LINEAR(1)OR GEOMETRIC(2)INCREMENTING OF EMITTANCES=    2
8   NUMBER OF EMITTANCE INCREMENTS=    3
9   FACTOR FOR H-EMITTANCE INCREMENTS=     2.0000
10  NUMBER OF DP/P INCREMENTS=    3
11  FACTOR FOR DP/P INCREMENTS=2.0000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
13  NUMBER OF PARTICLES=          0.1000000E+10
14  RING CIRCUMFERENCE =       78.5400 M
15  MOMENTUM OF THE BEAM=    0.88556E-01 GeV/c per Nucleon
16  STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.12500E-05 PI*M*RAD
18  STARTING DP/P=  0.5000E-03
```

Table 8 - 2nd page of the output for example 2
of INTRABT

```
 ---INTRA-BEAM SCATTERING GROWTH TIMES IN SECONDS.
 FOR CW=0 PIWINSKIS THEORY IS APPLIED AT NPOINT EQUIDISTANT LOCATIONS IN  THE SUPERPERIOD AND 1/TAU AVERAGED.
 FOR CW=1 THE DERIVATIVE OF BETAH AND D IS INCLUDED
 NUMBER OF POINTS IS NPOINT =   20   CW=1.00
COASTING  BEAM OF                IONS      (A=    207.00 Q=     53.00)

RING CIRCUMFERENCE                                                  78.54000 M
NUMBER OF PARTICLES                                                  0.1000000E+10
MOMENTUM                                                            .88556E-01 GeV/c per Nucleon

Emittance Definition E = PI (    1.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =     1.00 SIGMAP

TABLE OF INTRABEAM SCATTERING AMPLITUDE GROWTH TIMES
----------------------------------------------------

Pb 53 times charged in LEAR

EH(PI*RAD*M) EV(PI*RAD*M) DP/P     TAUH(SEC) TAUV(SEC) TAUE(SEC)


  0.250E-05 0.125E-05 0.500E-03    0.254E+01 0.152E+01 0.132E+01
  0.250E-05 0.125E-05 0.100E-02    0.212E+01 0.129E+01-0.124E+02
  0.250E-05 0.125E-05 0.200E-02    0.227E+01 0.158E+01-0.243E+02
  0.250E-05 0.125E-05 0.400E-02    0.300E+01 0.239E+01-0.106E+03

  0.500E-05 0.250E-05 0.500E-03    0.154E+02 0.113E+02 0.165E+01
  0.500E-05 0.250E-05 0.100E-02    0.119E+02 0.693E+01 0.958E+02
  0.500E-05 0.250E-05 0.200E-02    0.112E+02 0.731E+01-0.754E+02
  0.500E-05 0.250E-05 0.400E-02    0.135E+02 0.101E+02-0.258E+03

  0.100E-04 0.500E-05 0.500E-03    0.924E+02 0.135E+03 0.275E+01
  0.100E-04 0.500E-05 0.100E-02    0.710E+02 0.426E+02 0.369E+02
  0.100E-04 0.500E-05 0.200E-02    0.595E+02 0.362E+02-0.349E+03
  0.100E-04 0.500E-05 0.400E-02    0.639E+02 0.447E+02-0.686E+03

  0.200E-04 0.100E-04 0.500E-03    0.527E+03-0.180E+04 0.531E+01
  0.200E-04 0.100E-04 0.100E-02    0.434E+03 0.321E+03 0.465E+02
  0.200E-04 0.100E-04 0.200E-02    0.337E+03 0.196E+03 0.273E+04
  0.200E-04 0.100E-04 0.400E-02    0.319E+03 0.208E+03-0.214E+04

THE DATA TO RERUN LASt CAICULATION HAVE BEEN  STORED IN File INTRABT.LAS
```

4.1 Specific Features of INTRABS

INTRABS simulates the beam evolution starting from a given set of emittances and momentum spread. The output consists of a table which lists beam parameters as a function of time. The time step corresponds to a fraction (default 1/10) of the fastest of the three e-folding rates. Here the "e-folding rates" for each of the three planes are defined by the difference between the IBS-growth rate and the cooling rate due to stochastic or electron cooling.

$$\frac{1}{\tau} = \frac{1}{\tau_{IBS}} - \frac{1}{\tau_{COOL}}$$

As everywhere in the programme all time constants are amplitude growth and amplitude cooling times.

The input and default values for INTRABS are similar to those of INTRABT except for the parameters controlling the layout of the table and the additional parameters (cooling time constants, emittance coupling) of the present problem.

The cooling system can be characterized by the user specified subroutine GETCOO which, in the most general case, gives the three time constant CTAUH (E_h, E_v, dp), CTAUV (E_h, E_v, dp), CTAUE (E_h, E_v, dp) as a function of the three emittances. So far we have mainly worked with constant cooling times.

4.2 Emittance Coupling

To include coupling in a simple (but crude!) way in INTRABS and INTRABE we use a coefficient $K \geq 0$ as outlined before. The program checks if the ratio R of the starting emittances is compatible with the requested value of K. In fact, Eqs. (1) above imply for the ratio (R_u) of the uncoupled and that of the coupled emittances :

$$R_u = \frac{R_c - K}{1 - KR_c} \tag{3}$$

and therefore

$$K \leq R_c \leq \frac{1}{K} \tag{4}$$

The check of the initial emittance ratio is performed once the user has validated the input list (by typing twice "GO" as in INTRABT). Then

i) If there is no coupling calculation starts.

ii) If there is coupling two possibilities exist

A) The emittance ratio is not compatible with Eq. (4). In that case INTRABS applies Eq. (1) to obtain coupled starting values and the following message appears.

RATIO OF THE STARTING EMITTANCES (E_h= . . . E_v= . . .)
WAS NOT COMPLATIBLE WITH THE COUPLING (K= . . .). COUPLING HAS BEEN APPLIED.NEW STARTING VALUES ARE (E_h= . . . E_v= . . .)

B) The emittance ratio is compatible with Eq, 4). In that case the Program asks :

STARTING EMITTANCES (E_h= . . . , E_v= . . .) ARE COMPATIBLE WITH THE COUPLING (K= . . .) ENTER 1 TO CONTINUE WITH THESE VALUES, ANY OTHER NUMBER TO COUPLE EMITTANCES ONCE MORE PRIOR TO EXECUTION

During the rest of the calculation the coupling is only applied to the emittance *increments i.e* the growth during one step is :

$$\Delta E_{c,h} = \frac{\Delta E_h + K\Delta E_v}{K+1}$$

$$\Delta E_{c,v} = \frac{\Delta E_v + K\Delta E_h}{K+1}$$

Here ΔE_h and ΔE_v are the increments without coupling as calculated from the joint action of IBS and cooling with time constants obtained for the emittances at the beginning of the step. With this simple procedure we usually obtain good results for strong and for zero coupling, but for intermediate values of K the results after a large number of steps are often unsatisfactory.

4.3 SAMPLE RUNS OF INTRABS

4.3.1 INPUT INSTRUCTIONS

Prior to the run a Twiss file (OCTLEAMAD.DAT in the example) has been prepared. To start executions one enters :

{i} RUN INTRABS

The program responds :

WELCOME TO PROGRAM INTRABS, TO SIMULATE THE BEAM EVOLUTION UNDER THE COMBINED ACTION OF INTRABEAM SCATTERING AND COOLING
ENTER

1	TO START FROM DEFAULT INPUT MENU
-1	TO STRAT FROM LAST INPUT MENU (FILE INTRABS.LAS)
0	TO PREPARE THE FILE INTRABS.COM FOR BATCH RUNS

CTRL & Z TO EXIT.

{i} 1

Then the default input menu (table 2) appears on the screen.

To validate the input list one types "GO" twice as in the example for INTRABT above.

After the second "GO", as there is no coupling, the following message appears.

BE PATIENT INTRABS IS CALCULATING LINE 1 OF THE TABLE
TOTAL NUMBER OF LINES WILL NOT EXCEED n
and so on for the other lines.

Once the calculation is finished the file INTRABS.LAS containing the last set of the input data is created and a message is written onto the output file. Then the input menu is displayed again and the user can change parameters to start a new run or exit to read the output. An extended output (useful in case of trouble or doubt) can be obtained setting the IPR-switch to a positive value.

4.3.2 COMMENTS TO SOME INPUT PARAMETERS

One notes that most input parameters are as for INTRABT. To elucidate those specific to INTRABS, we list below the messages which appear when the corresponding code is entered and add some comments.

CODE=7	SWITCH FOR DEPENDENCE OF COOLING TIME-CONSTANT ON EMITTANCE (1=CONST, 2=FUNC, 3=TABLE)

This switch (called ICT, default 1) has the following significance

ICT=1	Constant cooling times (independent of the three emittances) are used
ICT=2 or 3	User Routines are used. To this end the user has to write the cooling time dependence into the FORTRAN Routines READCOOH, READCOOV, READCOOE
CODE=8	ENTER THE HORIZONTAL AMPLITUDE COOLING TIME CONSTANT (sec)
CODE=9	ENTER THE VERTICAL AMPLITUDE COOLING TIME CONSTANT (sec)
CODE=10	ENTER THE MOMENTUM COOLING TIME CONSTANT (sec)

These amplitude and Δp cooling time constants are used throughout when ICT=1 (cooling speed independent of emittances). For ICT=2 or 3 they can serve as input for the user subroutine.

CODE=11	ENTER RATIO BETWEEN THE TIME STEP AND THE MINIMUM TAU. DURING THE TIME STEP EMITTANCES AND DP/P EVOLVE AT CONSTANT RATE.AFTER EACH STEP THE GROWTH TIMES ARE UPDATED.

This parameter (called FRACT, default 0.1) determines the time width *dt* of each step : dt=(FRACT* the shortest of the three e-folding times) as explained above. During the time *dt* the three emittances grow with their proper e-folding rate. Thereafter the time constants are recalculated.

CODE=18 ENTER EMITTANCE COUPLING CONSTANT K (≥0)

This is the coupling constant explained above.

CODE=20 ENTER MAXIMUM NUMBER OF CASES

CODE=21 ENTER MAXIMUM BEAM STORAGE TIME (sec)

Execution stops when either the maximum number of cases (default : 40) is exceeded or when the beam evolution has been followed for a time exceeding the "beam storage time" (default 36000 s).

4.3.3 OUTPUT FILE

The results are written into the File INTRABS.LST that contains the output in a form similar to INTRABT.LST. This file can be viewed or printed (e.g. on our printer PS 3812) by the instruction

XPRINT INTRABS.LST/PRINTER=PS3812/CC=F

4.4 TWO SAMPLE RUNS OF INTRABS FOR BUNCHED BEAMS OF IONS AND PROTONS IN LEAR

The input table and the output for two examples, namely :

i) a bunched proton beam in Lear an

ii) a bunched beam of lead ions in Lear

are reproduced in tables 9 to 12 below.

Table 9 - Input menu for the first sample run of INTRABS : bunched proton beam at 0.6 GeV/c in LEAR with cooling times of 1h.

```
INPUT MENU FOR INTRABS-ENTER CODE NR TO CHANGE ,"CTR&Z" TO EXIT,"GO" TO START
0  PARTICLE SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
1  TITLE="BUNCHED PROTONS IN LEAR"
2  TWISS MAD FILE=octleamad.dat
3  NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4  PRINT SWITCH (IF >0 EXTENDED PRINT)=   -1
5  TRANSVERSE EMITTANCES MULTIPLIER=   2.00000
6  SIGMAP MULTIPLIER =   2.00000
7  SWITCH FOR COOLING TIMES VS EMITTANCES (1=CONST,2=TABLE,3=FUNC)=    1
8  H AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
9  V AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
10 DP COOLING TIME CONSTANT=  3600.0     SEC
11 RATIO BETWEEN TIME STEP AND MINIMUM TAU=   0.10000
12 BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    1
13 NUMBER OF PARTICLES PER BUNCH=  0.1000000E+10
14 BUNCH LENGTH       =       19.6358 M
15 MOMENTUM OF THE BEAM=    0.60000     GeV/c
16 STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17 STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
18 EMITTANCE COUPLING FACTOR=     0.00000
19 STARTING DP/P=  0.1000E-03
20 MAXIMUM NUMBER OF CASES=   40
21 MAXIMUM BEAM STORAGE TIME=   0.179E+05 SEC
```

Table 10 - Output for example 1 of INTRABS

```
 9-JUL-1993 16:58:46
BEAM EVOLUTION UNDER THE COMBINED ACTION OF INTRABEAM SCATTERING AND COOLING
"BUNCHED PROTONS IN LEAR"
BUNCHED   BEAM OF PROTONS

BUNCH LENGTH                                                          19.63500 M
NUMBER OF PARTICLES                                                    0.1000000E+10
MOMENTUM                                                              .60000      GeV/c

Emittance Definition E = PI (     2.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =      2.00 SIGMAP
Starting Values
HORIZONTAL EMITTANCE (UNCOUPLED)                                      0.25000E-05 PI M RAD
VERTICAL   EMITTANCE (UNCOUPLED)                                      0.25000E-05 PI M RAD
RELATIVE MOMENTUM SPREAD                                               .10000E-03

INITIAL COOLING TIME CONSTANTS
HORIZONTAL                                                            3600.000      SEC
VERTICAL                                                              3600.000      SEC
LONGITUDINAL                                                          3600.000      SEC

MAXIMUM TIME THE BEAM CAN STAY IN THE RING                           17900.00      SEC
REQUESTED NUMBER OF LINES                                           40
SWITCH FOR THE DEPENDENCE OF COOLING TIMES ON EMITTANCES =            1    (CONSTANT)
FRACTION OF THE MINIMUM EFFECTIVE TAU DURING WICH THE EMITTANCES EVOLVE  0.10
EMITTANCE COUPLING CONSTANT K =                                      0.00000
Coupled Emittances EH=(EH0+K*EV0)/(1+K)   EV=(EV0+K*EH0)/(1+K)

7.2/VM  TWISS   88/11/2109:09   GIA126        1       F      26
FILE=2H302V73 LEAR WITH DUMMY SEXTUPOLES 27 MAY 87
NUMBER OF ELEMENTS IN TWISS  FILE octleamad.dat                  issued by MAD =   26

TABLE OF TWISS PARAMETERS AT    20 EQUIDISTANT POINTS
OBTAINED BY INTERPOLATING THE INPUT FILE
   ------------------------------------------------------
    BH(M)      AH        BV(M)      DH(M)       DHP      L(M)
    1.968      0.000     5.232      3.584       0.000    0.000
    2.258     -0.263     5.341      3.584       0.000    0.517
    2.549     -0.525     5.451      3.584       0.000    1.033
    3.217     -0.788     5.702      3.584       0.000    1.550
    4.395     -1.050     6.145      3.584       0.000    2.067
    5.837     -1.313     6.687      3.584       0.000    2.584
    7.279     -1.575     7.230      3.584       0.000    3.100
    8.722     -1.838     7.772      3.584       0.000    3.617
   10.196     -1.207     8.942      3.507      -0.297    4.134
    8.520      4.053    13.863      2.846      -2.171    4.650
    5.908      1.312    19.975      1.910      -1.488    5.167
    5.760     -0.894    19.054      1.323      -0.835    5.684
    6.800     -1.055    14.915      0.892      -0.835    6.201
    7.917     -1.129    11.167      0.474      -0.787    6.717
    9.065     -1.061     8.022      0.094      -0.681    7.234
   10.103     -0.901     5.437     -0.231      -0.565    7.751
   10.880     -0.637     3.495     -0.483      -0.422    8.267
   11.063     -0.412     2.601     -0.591      -0.274    8.784
   11.245     -0.187     1.707     -0.699      -0.127    9.301
   11.402      0.000     0.934     -0.792       0.000    9.817
```

Table 10 - Continued

```
 ---INTRA-BEAM SCATTERING GROWTH TIMES IN SECONDS.
 FOR CW=0 PIWINSKIS THEORY IS APPLIED AT NPOINT EQUIDISTANT LOCATIONS IN  THE SUPERPERIOD AND 1/TAU AVERAGED.
 FOR CW=1 THE DERIVATIVE OF BETAH AND D IS INCLUDED
   NB..  ALL TIMES ARE AMPLITUDE GROWTH TIMES
 NUMBER OF POINTS IS NPOINT =    20    CW=1.00
BUNCHED   BEAM OF PROTONS

BUNCH LENGTH                                                          19.63500 M
NUMBER OF PARTICLES                                                    0.1000000E+10
MOMENTUM                                                              .60000      GeV/c

EMITTANCE COUPLING CONSTANT K =                                    0.00000
Coupled Emittances EH=(EH0+K*EV0)/(1+K)    EV=(EV0+K*EH0)/(1+K)

Emittance Definition E = PI (     2.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =      2.00 SIGMAP

BEAM EVOLUTION UNDER THE COMBINED ACTION OF INTRABEAM SCATTERING AND COOLING
--------------------------------------------------------------------------------
"BUNCHED PROTONS IN LEAR"
                                                          AMPLITUDE GROWTH TIMES                AMPLITUDE COOLING TIMES
TIME(SEC)      EH(PI*RAD*M)    EV(PI*RAD*M)    DP/P      TAUH(SEC)   TAUV(SEC)   TAUE(SEC)   CTAUH(SEC) CTAUV(SEC)  CTAUE(SEC)

   0.00000E+00   0.250E-05    0.250E-05    0.100E-03    0.221E+04   -0.365E+04    27.8        3600.0     3600.0      3600.0
    2.7987       0.250E-05    0.249E-05    0.111E-03    0.225E+04   -0.382E+04    35.0        3600.0     3600.0      3600.0
    6.3306       0.251E-05    0.248E-05    0.122E-03    0.230E+04   -0.402E+04    44.2        3600.0     3600.0      3600.0
   10.804        0.251E-05    0.247E-05    0.135E-03    0.234E+04   -0.426E+04    56.0        3600.0     3600.0      3600.0
   16.494        0.251E-05    0.246E-05    0.149E-03    0.239E+04   -0.455E+04    71.3        3600.0     3600.0      3600.0
   23.766        0.252E-05    0.244E-05    0.165E-03    0.245E+04   -0.491E+04    91.1        3600.0     3600.0      3600.0
   33.110        0.252E-05    0.242E-05    0.182E-03    0.250E+04   -0.535E+04    117.        3600.0     3600.0      3600.0
   45.189        0.253E-05    0.239E-05    0.201E-03    0.256E+04   -0.592E+04    151.        3600.0     3600.0      3600.0
   60.916        0.254E-05    0.236E-05    0.223E-03    0.263E+04   -0.668E+04    195.        3600.0     3600.0      3600.0
   81.563        0.255E-05    0.232E-05    0.246E-03    0.270E+04   -0.776E+04    254.        3600.0     3600.0      3600.0
  108.94         0.256E-05    0.226E-05    0.272E-03    0.277E+04   -0.939E+04    333.        3600.0     3600.0      3600.0
  145.69         0.258E-05    0.220E-05    0.300E-03    0.285E+04   -0.122E+05    440.        3600.0     3600.0      3600.0
  195.80         0.260E-05    0.212E-05    0.332E-03    0.294E+04   -0.185E+05    584.        3600.0     3600.0      3600.0
  265.57         0.262E-05    0.203E-05    0.367E-03    0.303E+04   -0.468E+05    783.        3600.0     3600.0      3600.0
  365.60         0.265E-05    0.191E-05    0.406E-03    0.313E+04    0.533E+05   0.106E+04   3600.0     3600.0      3600.0
  515.62         0.268E-05    0.177E-05    0.448E-03    0.325E+04   -0.140E+05   0.145E+04   3600.0     3600.0      3600.0
  757.71         0.272E-05    0.160E-05    0.495E-03    0.338E+04    0.693E+04   0.202E+04   3600.0     3600.0      3600.0
 1132.5          0.276E-05    0.145E-05    0.537E-03    0.351E+04    0.438E+04   0.271E+04   3600.0     3600.0      3600.0
 2145.6          0.280E-05    0.131E-05    0.589E-03    0.365E+04    0.301E+04   0.393E+04   3600.0     3600.0      3600.0
 3072.6          0.278E-05    0.145E-05    0.576E-03    0.357E+04    0.410E+04   0.360E+04   3600.0     3600.0      3600.0
 4539.4          0.280E-05    0.131E-05    0.576E-03    0.365E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
 5549.0          0.277E-05    0.145E-05    0.577E-03    0.356E+04    0.410E+04   0.361E+04   3600.0     3600.0      3600.0
 7021.7          0.280E-05    0.131E-05    0.576E-03    0.366E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
 8032.8          0.277E-05    0.145E-05    0.577E-03    0.356E+04    0.410E+04   0.361E+04   3600.0     3600.0      3600.0
 9504.5          0.280E-05    0.131E-05    0.576E-03    0.366E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
10515.           0.277E-05    0.145E-05    0.577E-03    0.356E+04    0.410E+04   0.361E+04   3600.0     3600.0      3600.0
11986.           0.280E-05    0.131E-05    0.576E-03    0.366E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
12996.           0.277E-05    0.145E-05    0.577E-03    0.356E+04    0.410E+04   0.361E+04   3600.0     3600.0      3600.0
14467.           0.280E-05    0.131E-05    0.576E-03    0.367E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
15477.           0.277E-05    0.145E-05    0.577E-03    0.356E+04    0.410E+04   0.361E+04   3600.0     3600.0      3600.0
16948.           0.280E-05    0.131E-05    0.576E-03    0.367E+04    0.306E+04   0.359E+04   3600.0     3600.0      3600.0
17958.           0.277E-05    0.145E-05     .577E-03    0.356E+04    0.410E+04   0.36  04    3600.0     3600.0      3600.0

THE DATA TO RERUN THE LAST CASE HAVE BEEN  STORED IN File INTRABS.LAS
```

Table 11 : Input Menu for second example of INTRABS : bunched beam of Pb 53 at 4.2 MeV/u in LEAR with electron cooling time constants of 0.1 sec.

```
INPUT MENU FOR INTRABS-ENTER CODE NR TO CHANGE ,"CTR&Z" TO EXIT,"GO" TO START
0   PARTICLE SPECIES=IONS       (A= 207.00000 Q=     53.00)
1   TITLE="Bunched Beam of Pb 207 in LEAR "
2   TWISS MAD FILE=octleamad.dat
3   NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
4   PRINT SWITCH (IF >0 EXTENDED PRINT)=   -1
5   TRANSVERSE EMITTANCES MULTIPLIER=   2.00000
6   SIGMAP MULTIPLIER =   2.00000
7   SWITCH FOR COOLING TIMES VS EMITTANCES (1=CONST,2=TABLE,3=FUNC)=    1
8   H AMPLITUDE COOLING TIME CONSTANT= 0.10000     SEC
9   V AMPLITUDE COOLING TIME CONSTANT= 0.10000     SEC
10  DP COOLING TIME CONSTANT= 0.10000     SEC
11  RATIO BETWEEN TIME STEP AND MINIMUM TAU=   0.10000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    1
13  NUMBER OF PARTICLES PER BUNCH=  0.1000000E+10
14  BUNCH LENGTH        =       19.6350 M
15  MOMENTUM OF THE BEAM=    0.88556E-01 GeV/c per Nucleon
16  STARTING HORIZONTAL EMITTANCE=    0.10000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.50000E-06 PI*M*RAD
18  EMITTANCE COUPLING FACTOR=     0.00000
19  STARTING DP/P=  0.2000E-03
20  MAXIMUM NUMBER OF CASES=   40
21  MAXIMUM BEAM STORAGE TIME=   0.700     SEC
```

Table 12 - 2nd page of the output of example 2
of INTRABS

```
 ---INTRA-BEAM SCATTERING GROWTH TIMES IN SECONDS.
 FOR CW=0 PIWINSKIS THEORY IS APPLIED AT NPOINT EQUIDISTANT LOCATIONS IN  THE SUPERPERIOD AND 1/TAU AVERAGED.
 FOR CW=1 THE DERIVATIVE OF BETAH AND D IS INCLUDED
   NB..  ALL TIMES ARE AMPLITUDE GROWTH TIMES
 NUMBER OF POINTS IS NPOINT =   20   CW=1.00
BUNCHED   BEAM OF                IONS      (A=    207.00 Q=     53.00)

BUNCH LENGTH                                                          19.63500 M
NUMBER OF PARTICLES                                                    0.1000000E+10
MOMENTUM                                                              .88556E-01 GeV/c per Nucleon

EMITTANCE COUPLING CONSTANT K =                                      0.00000
Coupled Emittances EH=(EH0+K*EV0)/(1+K)   EV=(EV0+K*EH0)/(1+K)

Emittance Definition E = PI (    2.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =     2.00 SIGMAP

BEAM EVOLUTION UNDER THE COMBINED ACTION OF INTRABEAM SCATTERING AND COOLING
--------------------------------------------------------------------------
"Bunched Beam of Pb 207 in LEAR "
                                                    AMPLITUDE GROWTH TIMES          AMPLITUDE COOLING TIMES
TIME(SEC)     EH(PI*RAD*M)   EV(PI*RAD*M)   DP/P      TAUH(SEC)  TAUV(SEC)  TAUE(SEC)    CTAUH(SEC) CTAUV(SEC)  CTAUE(SEC)

  0.00000E+00   0.100E-05   0.500E-06   0.200E-03   0.113E-01  0.930E-02  0.157E-02    0.10000    0.10000    0.10000
  0.15917E-03   0.103E-05   0.516E-06   0.221E-03   0.115E-01  0.875E-02  0.226E-02    0.10000    0.10000    0.10000
  0.39013E-03   0.106E-05   0.541E-06   0.244E-03   0.120E-01  0.888E-02  0.332E-02    0.10000    0.10000    0.10000
  0.73377E-03   0.112E-05   0.581E-06   0.270E-03   0.130E-01  0.977E-02  0.500E-02    0.10000    0.10000    0.10000
  0.12596E-02   0.120E-05   0.640E-06   0.298E-03   0.149E-01  0.117E-01  0.766E-02    0.10000    0.10000    0.10000
  0.19236E-02   0.129E-05   0.707E-06   0.323E-03   0.174E-01  0.145E-01  0.109E-01    0.10000    0.10000    0.10000
  0.27716E-02   0.140E-05   0.782E-06   0.346E-03   0.207E-01  0.181E-01  0.150E-01    0.10000    0.10000    0.10000
  0.38790E-02   0.153E-05   0.864E-06   0.369E-03   0.251E-01  0.228E-01  0.200E-01    0.10000    0.10000    0.10000
  0.53551E-02   0.167E-05   0.955E-06   0.391E-03   0.307E-01  0.287E-01  0.263E-01    0.10000    0.10000    0.10000
  0.73721E-02   0.182E-05   0.106E-05   0.414E-03   0.379E-01  0.363E-01  0.343E-01    0.10000    0.10000    0.10000
  0.10224E-01   0.200E-05   0.117E-05   0.437E-03   0.472E-01  0.460E-01  0.443E-01    0.10000    0.10000    0.10000
  0.14475E-01   0.220E-05   0.129E-05   0.461E-03   0.590E-01  0.582E-01  0.570E-01    0.10000    0.10000    0.10000
  0.21434E-01   0.243E-05   0.142E-05   0.486E-03   0.741E-01  0.737E-01  0.731E-01    0.10000    0.10000    0.10000
  0.35475E-01   0.268E-05   0.157E-05   0.512E-03   0.935E-01  0.935E-01  0.934E-01    0.10000    0.10000    0.10000
  0.10737       0.296E-05   0.174E-05   0.539E-03   0.119      0.119      0.119        0.10000    0.10000    0.10000
  0.13923       0.268E-05   0.157E-05   0.512E-03   0.935E-01  0.935E-01  0.934E-01    0.10000    0.10000    0.10000
  0.21111       0.296E-05   0.174E-05   0.539E-03   0.119      0.119      0.119        0.10000    0.10000    0.10000
  0.24296       0.268E-05   0.157E-05   0.512E-03   0.935E-01  0.935E-01  0.934E-01    0.10000    0.10000    0.10000
  0.31479       0.296E-05   0.174E-05   0.539E-03   0.119      0.119      0.119        0.10000    0.10000    0.10000
  0.34662       0.268E-05   0.157E-05   0.512E-03   0.935E-01  0.935E-01  0.935E-01    0.10000    0.10000    0.10000
  0.41833       0.296E-05   0.174E-05   0.538E-03   0.118      0.119      0.119        0.10000    0.10000    0.10000
  0.45008       0.268E-05   0.157E-05   0.512E-03   0.936E-01  0.935E-01  0.936E-01    0.10000    0.10000    0.10000
  0.52146       0.295E-05   0.174E-05   0.538E-03   0.118      0.119      0.118        0.10000    0.10000    0.10000
  0.55299       0.268E-05   0.157E-05   0.512E-03   0.940E-01  0.934E-01  0.938E-01    0.10000    0.10000    0.10000
  0.62338       0.294E-05   0.174E-05   0.537E-03   0.116      0.119      0.117        0.10000    0.10000    0.10000
  0.65433       0.269E-05   0.157E-05   0.513E-03   0.948E-01  0.931E-01  0.946E-01    0.10000    0.10000    0.10000
  0.72217       0.290E-05   0.174E-05   0.534E-03   0.113      0.120      0.114        0.10000    0.10000    0.10000

THE DATA TO RERUN THE LAST CASE HAVE BEEN  STORED IN File INTRABS.LAS
```

5.1 FEATURES SPECIFIC TO INTRABE

INTRABE searches for the equilibrium between IBS and cooling. The procedure is similar to that of INTRABT. Starting from a set of emittances, the evolution is followed until the emittance growth during the time step becomes very small.

We have found good convergence in cases where the starting values are "not too far" from equilibrium. To find starting values, we run INTRABT and/or INTRABS first.

Default and input parameters are the same as for INTRABS except for the criterion of the "maximum beam storage time" which is replaced by TOL (the relative tolerance for equilibrium). INTRABS stops when one of the following two conditions is satisfied.

A) The maximum number of cases NCASE (default 40) is reached, or

B) $\sqrt{\left(\frac{dE_h}{E_h}\right)^2+\left(\frac{dE_v}{E_v}\right)^2+\left[\frac{d(DP/P}{DP/P}\right]^2} < TOL$,i.e. if the r.m.s emittance change during a step is less than the specified tolerance (default 0.02).

The relative error in the difference between IBS and cooling rate

$$\frac{1}{TM}=\frac{\sqrt{\left(\frac{1}{\tau_{hcool}}-\frac{1}{\tau_h}\right)^2+\left(\frac{1}{\tau_{vcool}}-\frac{1}{\tau_h}\right)^2+\left(\frac{1}{\tau_{ecool}}-\frac{1}{\tau_e}\right)^2}}{\sqrt{\frac{1}{\tau_h^2}+\frac{1}{\tau_v^2}+\frac{1}{\tau_e^2}}} \qquad (5)$$

is also printed for the purpose of checking. Ideally 1/TM → 0 at equilibrium. Emittance coupling is treated in the same way as in INTRABS (see section 4.2 above).

5.2 SAMPLE RUNS OF INTRABE

5.2.1 Input instructions

It is again assumed that before execution a Twiss file (OCTLEAMAD.DAT in the example) has been prepared. To start executions one enters

RUN INTRABE

Then the program responds :

WELCOME TO PROGRAM INTRABE TO SEARCH FOR THE EQUILIBRIUM

BETWEN INTRABEAM SCATTERING AND COOLING

ENTER

1 TO START FROM DEFAULT INPUT MENU

-1 TO START FROM LAST INPUT MENU (FILE INTRABE.LAS)

0 TO PREPARE THE FILE INTRABE.COM FOR BATCH RUNS

CTRL & Z TO EXIT

{i}1

Then the input menu (table 3 above) is displayed :

5.2.2 Output file

The results are written into the File INTRABE.LST that can be viewed or printed, on our system e.g. via the instruction

XPRINT INTRABE.LST/PRINTER=PS3812/CC=F

The output file INTRABE.LST is similar to that of INTRABS. The last column gives the difference between IBS and cooling rate as defined by Eq. (5) above.

5.2.3 A sample run of INTRABE for LEAR

The input list and the output file for a coasting beam of protons at 0.6 GeV/c in LEAR are reproduced on the next pages (tables 13 and 14).

Table 13 - Input for example 1 of INTRABE :
coasting beam of protons at 0.6 GeV/c

```
INPUT MENU FOR INTRABE-ENTER CODE NR TO CHANGE,"CTR&Z" TO EXIT,"60" TO START
 1  PARTICLE SPECIES=PROTONS   (A=   1.00728 Q=      1.00)
 .  TITLE="PROTONS IN LEAR"
 2  TWISS MAD FILE=octleamad.dat
 3  NUMBER OF POINTS FOR TWISS FUNCTION INTERPOLATION=   20
 4  PRINT SWITCH (IF >0 EXTENDED PRINT)=   -1
 5  TRANSVERSE EMITTANCES MULTIPLIER=   2.00000
 6  SIGMAP MULTIPLIER =   2.00000
 7  SWITCH FOR COOLING TIMES VS EMITTANCES (1=CONST,2=TABLE,3=FUNC)=    1
 8  H AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
 9  V AMPLITUDE COOLING TIME CONSTANT=  3600.0     SEC
10  DP/P COOLING TIME CONSTANT=  3600.0     SEC
11  RATIO BETWEEN TIME STEP AND MINIMUM TAU=   0.10000
12  BUNCH SHAPE(1=BUNCHED,NOT 1 =COAS)=    2
13  NUMBER OF PARTICLES          =  0.1000000E+10
14  RING CIRCUMFERENCE =       78.5400 M
15  MOMENTUM OF THE BEAM=    0.60000     GeV/c
16  STARTING HORIZONTAL EMITTANCE=    0.25000E-05 PI*M*RAD
17  STARTING VERTICAL EMITTANCE=    0.25000E-05 PI*M*RAD
18  EMITTANCE COUPLING FACTOR=     0.00000
19  STARTING DP/P=  0.7000E-03
20  MAXIMUM NUMBER OF CASES=   29
21  TOLERANCE FOR EQUILIBRIUM=   0.139E-02
```

Table 14 - Output of example 1 of INTRABE

```
12-JUL-1993 16:13:18
SEARCH FOR THE EQUILIBRIUM BETWEEN INTRABEAM SCATTERING AND COOLING
"PROTONS IN LEAR"
COASTING  BEAM OF PROTONS

RING CIRCUMFERENCE                                                          78.54000 M
NUMBER OF PARTICLES                                                          0.1000000E+10
MOMENTUM                                                                    .60000     GeV/c

Emittance Definition E = PI (    2.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =     2.00 SIGMAP
Starting Values
HORIZONTAL EMITTANCE (UNCOUPLED)                                           0.25000E-05 PI M RAD
VERTICAL   EMITTANCE (UNCOUPLED)                                           0.25000E-05 PI M RAD
RELATIVE MOMENTUM SPREAD                                                    .70000E-03

INITIAL COOLING TIME CONSTANTS
HORIZONTAL                                                                   3600.000     SEC
VERTICAL                                                                     3600.000     SEC
LONGITUDINAL                                                                 3600.000     SEC

TOLERANCE FOR EQUILIBRIUM                                                    0.1390000E-02
MAXIMUM NUMBER OF STEPS FOR SEARCH                                           29
SWITCH FOR THE DEPENDENCE OF COOLING TIMES ON EMITTANCES =                   1    (CONSTANT)
FRACTION OF THE MINIMUM EFFECTIVE TAU DURING WICH THE EMITTANCES EVOLVE(STARTING VALUE)  0.100
EMITTANCE COUPLING CONSTANT K =                                             0.00000
Coupled Emittances EH=(EH0+K*EV0)/(1+K)   EV=(EV0+K*EH0)/(1+K)

7.2/VM  TWISS   88/11/2109:09   GIA126         1       F      26
FILE=2H302V73 LEAR WITH DUMMY SEXTUPOLES 27 MAY 87
NUMBER OF ELEMENTS IN TWISS  FILE octleamad.dat                  issued by MAD =   26

TABLE OF TWISS PARAMETERS AT    20 EQUIDISTANT POINTS
OBTAINED BY INTERPOLATING THE INPUT FILE
   ------------------------------------------------------
    BH(M)      AH        BV(M)      DH(M)       DHP      L(M)
    1.968      0.000     5.232      3.584       0.000    0.000
    2.258     -0.263     5.341      3.584       0.000    0.517
    2.549     -0.525     5.451      3.584       0.000    1.033
    3.217     -0.788     5.702      3.584       0.000    1.550
    4.395     -1.050     6.145      3.584       0.000    2.067
    5.837     -1.313     6.687      3.584       0.000    2.584
    7.279     -1.575     7.230      3.584       0.000    3.100
    8.722     -1.838     7.772      3.584       0.000    3.617
   10.196     -1.207     8.942      3.507      -0.297    4.134
    8.520      4.053    13.863      2.846      -2.171    4.650
    5.908      1.312    19.975      1.910      -1.488    5.167
    5.760     -0.894    19.054      1.323      -0.835    5.684
    6.800     -1.055    14.915      0.892      -0.835    6.201
    7.917     -1.129    11.167      0.474      -0.787    6.717
    9.065     -1.061     8.022      0.094      -0.681    7.234
   10.103     -0.901     5.437     -0.231      -0.565    7.751
   10.880     -0.637     3.495     -0.483      -0.422    8.267
   11.063     -0.412     2.601     -0.591      -0.274    8.784
   11.245     -0.187     1.707     -0.699      -0.127    9.301
   11.402      0.000     0.934     -0.792       0.000    9.817
```

Table 14 - Continued

```
 ---INTRA-BEAM SCATTERING GROWTH TIMES IN SECONDS.
 FOR CW=0 PIWINSKIS THEORY IS APPLIED AT NPOINT EQUIDISTANT LOCATIONS IN  THE SUPERPERIOD AND 1/TAU AVERAGED.
 FOR CW=1 THE DERIVATIVE OF BETAH AND D IS INCLUDED
   NB..  ALL TIMES ARE AMPLITUDE GROWTH TIMES
 NUMBER OF POINTS IS NPOINT =    20    CW=1.00
"PROTONS IN LEAR"
COASTING  BEAM OF PROTONS

RING CIRCUMFERENCE                                                          78.54000 M
NUMBER OF PARTICLES                                                          0.1000000E+10
MOMENTUM                                                                    .60000      GeV/c

EMITTANCE COUPLING CONSTANT K =                                             0.00000
Coupled Emittances EH=(EH0+K*EV0)/(1+K)    EV=(EV0+K*EH0)/(1+K)

Emittance Definition E = PI (     2.00 SIGMA)**2/BETA
Momentum Spread Definition : DP/P =      2.00 SIGMAP

SEARCH FOR THE EQUILIBRIUM BETWEEN INTRABEAM SCATTERING AND COOLING
------------------------------------------------------------------
"PROTONS IN LEAR"
                                                    AMPLITUDE GROWTH TIMES              AMPLITUDE COOLING TIMES
TIME(SEC)   EH(PI*RAD*M) EV(PI*RAD*M)     DP/P      TAUH(SEC)   TAUV(SEC)   TAUE(SEC)   CTAUH(SEC) CTAUV(SEC) CTAUE(SEC)   ER(1/TAU)

 .00000E+00   0.250E-05   0.250E-05   0.700E-03   0.342E+04   0.333E+05   0.445E+04   3600.      3600.      3600.      0.527
 201.82       0.252E-05   0.226E-05   0.693E-03   0.338E+04   0.200E+05   0.425E+04   3600.      3600.      3600.      0.483
 421.34       0.253E-05   0.205E-05   0.686E-03   0.338E+04   0.131E+05   0.410E+04   3600.      3600.      3600.      0.427
 669.37       0.256E-05   0.185E-05   0.680E-03   0.339E+04   0.909E+04   0.398E+04   3600.      3600.      3600.      0.355
 967.34       0.258E-05   0.168E-05   0.675E-03   0.343E+04   0.654E+04   0.388E+04   3600.      3600.      3600.      0.264
 1368.0       0.261E-05   0.152E-05   0.670E-03   0.349E+04   0.483E+04   0.380E+04   3600.      3600.      3600.      0.151
 2075.2       0.265E-05   0.137E-05   0.663E-03   0.358E+04   0.364E+04   0.369E+04   3600.      3600.      3600.      0.159E-01
 16926.       0.278E-05   0.124E-05   0.600E-03   0.407E+04   0.291E+04   0.241E+04   3600.      3600.      3600.      0.323
 17657.       0.265E-05   0.137E-05   0.664E-03   0.360E+04   0.361E+04   0.371E+04   3600.      3600.      3600.      0.177E-01
 29450.       0.266E-05   0.135E-05   0.600E-03   0.363E+04   0.370E+04   0.238E+04   3600.      3600.      3600.      0.298
 30149.       0.265E-05   0.134E-05   0.664E-03   0.359E+04   0.339E+04   0.373E+04   3600.      3600.      3600.      0.401E-01
 33126.       0.266E-05   0.148E-05   0.645E-03   0.363E+04   0.458E+04   0.324E+04   3600.      3600.      3600.      0.140
 33969.       0.265E-05   0.134E-05   0.662E-03   0.359E+04   0.340E+04   0.369E+04   3600.      3600.      3600.      0.373E-01
 36998.       0.267E-05   0.148E-05   0.648E-03   0.365E+04   0.456E+04   0.331E+04   3600.      3600.      3600.      0.132
 37853.       0.265E-05   0.134E-05   0.662E-03   0.358E+04   0.340E+04   0.368E+04   3600.      3600.      3600.      0.366E-01
 38767.       0.266E-05   0.138E-05   0.658E-03   0.361E+04   0.370E+04   0.357E+04   3600.      3600.      3600.      0.168E-01
 40712.       0.265E-05   0.134E-05   0.661E-03   0.359E+04   0.340E+04   0.366E+04   3600.      3600.      3600.      0.352E-01
 41635.       0.265E-05   0.138E-05   0.658E-03   0.360E+04   0.370E+04   0.357E+04   3600.      3600.      3600.      0.166E-01
 43584.       0.265E-05   0.134E-05   0.661E-03   0.359E+04   0.340E+04   0.366E+04   3600.      3600.      3600.      0.351E-01
 44508.       0.265E-05   0.138E-05   0.658E-03   0.360E+04   0.370E+04   0.357E+04   3600.      3600.      3600.      0.166E-01
 45158.       0.265E-05   0.136E-05   0.659E-03   0.360E+04   0.360E+04   0.360E+04   3600.      3600.      3600.      0.259E-03
 91918.       0.266E-05   0.138E-05   0.657E-03   0.361E+04   0.371E+04   0.355E+04   3600.      3600.      3600.      0.186E-01
 92552.       0.265E-05   0.136E-05   0.659E-03   0.360E+04   0.360E+04   0.360E+04   3600.      3600.      3600.      0.801E-03
 .11946E+06   0.264E-05   0.137E-05   0.666E-03   0.356E+04   0.361E+04   0.376E+04   3600.      3600.      3600.      0.256E-01
 .12030E+06   0.266E-05   0.137E-05   0.659E-03   0.361E+04   0.361E+04   0.359E+04   3600.      3600.      3600.      0.241E-02
 .12526E+06   0.264E-05   0.135E-05   0.660E-03   0.357E+04   0.351E+04   0.363E+04   3600.      3600.      3600.      0.161E-01
 .12598E+06   0.265E-05   0.137E-05   0.659E-03   0.360E+04   0.361E+04   0.360E+04   3600.      3600.      3600.      0.204E-02
 .13110E+06   0.265E-05   0.135E-05   0.659E-03   0.360E+04   0.351E+04   0.361E+04   3600.      3600.      3600.      0.143E-01

EQUILIBRIUM HAS   BEEN REACHED WITH THE SPECIFIED TOLERANCE(  0.14E-02).THE ACTUAL RELATIVE EMITTANCE ERROR IS   0.25427E-03
THE 1/TAU RELATIVE ERROR IS  0.1425997E-01

THE DATA TO RERUN THE LAST CASE HAVE BEEN  STORED IN File INTRABE.LAS
```

6.1 Final Remark

Please, send your user comments

to : GIANNINI @ CERNVM.CERN.CH

or : MOEHL @ CERNVM.CERN.CH

APPENDIX 1: Historical Notes

There were earlier publications by J. Le Duff, H. Bruck, C. Pellegrini, J. Augustin, E. Keil, K. Hübner, H.G. Hereward as cited by A. Piwinski in his classical 1974 paper[2], but it is the article by Piwinski which gave, for the first time, a fairly complete theory.

Apart from the assumption of Gaussian beams, which was (to our knowledge) retained by all subsequent authors, other simplifications are made in this paper, which were later removed by Piwinski himself and by other authors.

i) the lattice functions (β_h, β_v, D) were assumed constant, i.e. $\beta'_h = \beta'_v = D' = 0$

ii) average values were taken.

The latter of these simplifications is easily removed by calculating the growth rates at a sufficiently large number of locations around the ring, and then taking the average. The other restriction is more fundamental. Generalising Piwinski's approach one ends up with three rather than with one single "form function" f(a,b,c), each of them given by a triple integral. This increases the amount of calculations needed, which is however not too serious since computer programmes have to be used anyway to deal with the form function.

An important result of the general theory, after removal of the restriction i) above, is the fact that the three emittances can grow simultaneously, below as well as above transition energy. The simultaneous growth in all three degrees of freedom can be explained by the transfer of energy from the common longitudinal motion into transverse and longitudinal energy spread. There are two mechanisms to achieve this : the negative mass effect (i.e. the longer revolution time for particles with too high momentum) and the "friction" due to the change of the lattice functions. The first mechanism is only present above transition whereas the second one can be effective at all energies.

A computer program based on Piwinski's original theory was developed in 1974/75 by H. Hereward and K. Hübner[6] to estimate IBS for the ISR. All later program used at CERN are based on this code. One of us (D.M.) got a copy and all the necessary information from K. Hübner in about 1976. Together with Frank Sacherer we included the averaging around the ring (point ii above). We also started to remove the simplification i) and Frank managed to produce a complete version just prior to his untimely death in 1978. The generalisation of the theory was independently accomplished by A.Piwinski. His letters [7] to W. Hardt and one of us (D.M.) in early 1979, where he explained this work, were enlightening and most helpful.

In the period 1977-1982 the programmes were applied to several machines at CERN. We collaborated with H. Herr[8] to examine the situation in ICE. It was in this context that the versions to calculate the beam evolution and the equilibrium with cooling were developed. Later, these programmes were used in the design and development of LEAR.

Frank Sacherer's interest was oriented towards the antiproton accumulator. He collaborated with Simon Van der Meer who introduced Michel Martini into this field. Michel[9] was the first to bring the basic program (corresponding to INTRABT as described above) into a clean and well documented form. He also gave a thorough description of the underlying equations. Prior to this, only "working versions" existed. L. Evans and B. Zotter[10] used such a working version to find out the importance of IBS in the SPS Collider. In passing they showed that one of the three integrations in Piwinski's form function can be done analytically.

In 1979 we gave the copy of a working version to A. Ruggiero who took it to Fermilab. A. Ruggiero's copy was used up to 1983, when better documented versions became available.

In 1983 J. D. Bjorken and S. Mtingwa[11] published their general theory of IBS, based on the classical two-body scattering rules (the "golden rule") used in Quantum Mechanics. Results fully agree with Piwinski's generalised theory but the approach is complementary.

Since then, the theory has been applied to a large number of machines. The work of G. Parzen, A. Ruggiero and J. Wei[12][13] for the heavy ion ring RHIC should be mentioned.

There, owing to the $(Q^2/A)^2$ scaling, IBS is a major limitation. A. Ruggiero and collaborators have also worked on equilibrium conditions with cooling using Fokker-Planck techniques [12][13] which are more precise than our "global approach" outlined above.

Concerning summary articles there are excellent contributions in the proceeding of the CERN Accelerator School (CAS 1986 by A. Sorensen[14] and CAS 1991 by A. Piwinski[15].)

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