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# ENERGY LEVEL INEQUALITIES AND "WRINKLED" QUARKONIUM POTENTIAL

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*Abstract*

The "concave" downward property of the standard static $q\bar{q}$ potentials leads to the energy level inequalities: $E_{n+2} - E_{n+1} < E_{n+1} - E_n$ in the quarkonium mass spectrum. However, this inequality is experimentally observed to be reversed for n=2 in charmonium and n=3 in bottomonium, a fact that is inexplicable in terms of any known concave downward potential. We attempt to explain this by allowing for the violation of the concavity condition in a small interval, i.e., a "wrinkle" in some recently proposed quarkonium potentials.

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## I *Introduction:*

Most of the $q\bar{q}$ potentials given in the literature[1] are concave downwards i.e. for a given potential V(R) ,

$$\frac{\partial V}{\partial R} > 0 \quad ; \quad \frac{\partial^2 V}{\partial R^2} < 0 \qquad (1)$$

Bachas [2] has demonstrated this property on the lattice for potentials derived from gauge theories. However, the absence of a viable theory of strong interactions in the non-perturbative region and the lack of proof of confinement allows us to consider phenomenological potentials which violate this condition, at least in the region where there still exist theoretical uncertainties. Furthermore, it has been shown by Lichtenberg[3] in a recent communication that the data on the energy level spacings in heavy quarkonia show that one cannot obtain complete agreement with the experimentally observed[4] levels in the charmonium and bottomonium through the use of a concave downward potential.

For a concave downward potential the adjacent energy levels follow the rule

$$E_{n+2} - E_{n+1} < E_{n+1} - E_n \qquad (2)$$

while the reverse is true for a convex downward potential.Charmonium and Bottomonium data[4] show that this inequality is violated for n=2 in Charmonium and n=3 in bottomonium , i.e, for the J/ψ family :

$$E_4 - E_3 > E_3 - E_2 \ , \qquad (3a)$$

and for the ϒ family:

$$E_5 - E_4 > E_4 - E_3 \ . \qquad (3b)$$

This trend has led Lichtenberg[3] to postulate that for a description of quarkonia based on a completely static potential (with no openings for decay channels)one must have a potential that violates the concavity condition in a small interval, i.e., a potential with a "wrinkle".

## II *The Wrinkled Potentials*

Recently, we have proposed [5,6] two new potentials which violate the concavity condition (eqn.1) in a small region. We present, in this paper, an analysis of our potentials to test Lichtenberg's claim. For a better appreciation of the structure and role of the wrinkle, we give here the essential details of the potentials in question and elaborate their dependence on the various parameters used in them.

The string inspired large distance potential in conjunction with the short distance QCD inspired "Coulomb" potential is our zeroth-order potential:

$$V_0(R) = -4\alpha_s/3R \qquad R<R_c$$
$$= K\sqrt{R^2 - R_c^2} \qquad R>R_c \qquad (4)$$

Where $\alpha_s(R) = 12\pi/[(33-2n_f)\ln(\Lambda^2R^2)]$ is the scaled strong interaction coupling constant to first order in perturbation theory and $R_c = \sqrt{\pi/6K}$ .

As can be seen from the form of the potential (fig. 1), the long distance form is unphysical below $R= R_c$ , the short distance form is that due to perturbation theory and is dependent on the QCD cutoff $\Lambda$. By choosing $\Lambda$ to be well within experimental limits of $100 < \Lambda < 200$ Mev , the maximum of the coulombic part can be made to coincide with $R = R_c$. For numerical calculations we have chosen throughout $K=0.16\,GeV^2$ and $\Lambda=0.11$ GeV. This however leads to a discontinuity as shown in fig 1. Smoothing out the discontinuity at $R= R_c$ leads us to the following wrinkled potential:

$$V_{BDKS}(R)=V_1(R) + V_2(R) \qquad (5)$$

Where[7]

$$V_1(R) = -\frac{4}{3}\alpha_s(R)\frac{[\ln(\Lambda R)]^2}{R\{\ln(\Lambda R)\}^2 + A\{\tanh\frac{R-R_0}{b} + \tanh\frac{R_0}{b}\}} \qquad (6)$$

and

$$V_2(R) = K\tanh\left(\frac{R}{R_c}\right)\sqrt{R^2-\left(R_c\tanh\frac{R}{R_c}\right)^2} \qquad (7)$$

Here $R_0$ , A and b are in units of $GeV^{-1}$. The original BDKS[7] potential of reference 5 which is closest to the roughened potential $V_0$ is plotted in fig 2. The parameters used in this are $A=15\,GeV^{-1}$ ; $b = R_c/60$ and $R_0= R_c-b = x R_c$ with $x=0.95$.

A few remarks about the rather complicated form of this potential are in order here.

1. The short distance part $V_1(R)$ seeks to approximate $V_0(R)$ for $R < R_c$ while for $R > R_c$ it is negligible. $V_2(R)$ does the same for $R>R_c$ . The forms of $V_1(R)$, $V_2(R)$ and $V_{BDKS}(R)$ are shown in figure 2.Thus $V_{BDKS}(R)$ is a continuous potential that retains all the features of $V_0(R)$.
2. The parameters A and b are related to $R_c$ and are chosen to ensure that $V_{BDKS}(R)$ is equal to $V_0(R)$ in as large a region as possible and lead to a "wrinkle" in the potential. These parameters A, b and $R_0$ determine the depth, width and position of the "wrinkle" respectively.
3. The parameter A is essential to avoid the infinity in $V_0(R)$ at $R=1/\Lambda$. The variation of the potential with A is shown in figure (3).
4. The dependence of the potential on $R_0$ , the parameter that controls the position of the wrinkle is shown in figure (4).The combination of A and $R_0$ determines the shape of the potential $V_{BDKS}(R)$ at the upper end of the wrinkle and its values were initially chosen to get the shape as close to the bare $V_0(R)$ as possible.
5. The parameter 'b' determines the width of the wrinkle. A smaller b keeps the potential $V_{BDKS}(R)$ close to $V_0(R)$, while a large enough b will get rid of the wrinkle, in particular a wrinkle free smooth monotonic concave downward potential is obtained for large A and large b, e.g., with $b\cong R_C/4$ and $A\sim25\,GeV^{-1}$ gives curve-3 in fig.5. [Hereafter referred to as $V^{WF}(R)$]. The interpolation is truely logarithmic only for this wrinkle-free case.

It should be noted that a large number of potentials, with less than three parameters, could have been selected for the intermediary region. However, for freedom to choose the width, depth and position of the wrinkle, we select this particular one.

## III *Results and Discussions*

In this section we examine Lichtenberg's comment on the effect of the wrinkle on the energy-level inequalities (eqns. 2 and 3). As we have seen in the earlier section, the adjustments of the parameters, A and b , allow us control over the wrinkle . For the values of the parameters ($A= 15\,GeV^{-1}$; $b= 0.032\,GeV^{-1}$, $R_0 \cong R_c=1.86\,GeV^{-1}$) chosen in references 5 and 6 these inequalities are not satisfied (table-1, cols.2 and 3). This is because the energy-levels under consideration fell above the wrinkle and were governed completely by the concave part of the potential. However, if we adjust the parameters such that the energy levels under consideration are in the region of transition then the presence of the wrinkle would alter the character of the level spacings in the desired manner.

One must observe that the potential governing the wrinkle can be visualized as a superposition of two parts ( see fig.6), i.e.,

$$V_{BDKS}(R) = V^{WF}(R) + V_{AG}(R) \quad (8)$$

Where $V_{AG}$ is an asymmetric Gaussian-type of function (fig.6, curve-3) obtained by subtracting the unwrinkled potential $V^{WF}(R)$ (fig.6 curve-1) from the wrinkled $V_{BDKS}$ (R) (fig. (6) curve-2). Thus we see that for the potential $V_{BDKS}(R)$ of refs. 5 and 6 involves an asymmetric Gaussian. The `wrinkle' in the potential is confined to a very narrow region. The theoretical uncertainty in the potential is only over a distance $.95R_c$ to $1.05R_c$ which, for K=0.14 $GeV^2$ ,corresponds to 1.7 $GeV^{-1}$ to 1.9 $GeV^{-1}$ (0.36 to 0.38 Fermi).

However, if we relax this stringent condition and allow the intermedeary region between the `Coulombic' and the `Roughened' string potential to span a distance $R_c$ to $2R_c$, then the energy levels under consideration ($\psi(2S)$, $\psi(3S)$, $\psi(4S)$, Y(3S), Y(4S), Y(5S)) fall in or around the region of the wrinkle. This happens for values of the parameters A =0.36 $GeV^{-1}$, $R_c$ =1.809$GeV^{-1}$ $\cong R_0$, $\Lambda$=.11GeV and K = 0.16$GeV^2$ (fig.7, curve-2). Then the energy level spacings are in reasonable agreement with experimental values and are given in table-1 column-4. We see that qualitatively ,the potential with a wide, shallow wrinkle, within the framework of the BDKS model gives us the desired inversion effect required by equations 2 and 3.

In order to obtain better quantitative agreement, we can take a purely phenomenological approach, by supposing the wrinkle to be caused by a *symmetric* Gaussian of the form

$$V(R)=\mathcal{V}_0\exp[-\{(R-3R_c)/R_c\}^2], \quad (9)$$

which is subtracted out of the unwrinkled potential.VWF(R) This gives a fairly good fit to data as shown in table 1 column 5. Here $R_c$=1.809$GeV^{-1}$ , $\mathcal{V}_0$ = 0.5 GeV. It may be emphasized that our purpose in this communication has not been to fit the experimental data, but to demonstrate in some details the structure of the BDKS wrinkle, and its qualitative impact on the energy level differences of quarkonia.

The Gaussian wrinkle is a purely phenomenological potential for which we can claim no theoretical justification in a purely static model for heavy quarkonia. However, we can make a conjecture that if we find a way of including dynamical processes such as quark pair production[8] in the non-perturbative regime into an effective interquark potential then we may be able to produce a wrinkle. It has been shown by Tornquist[9] that coupled channel effects may be able to produce the inversion of energy levels in the case of the Y system. However, he has also shown that these effects cannot be absorbed into the parameters of any static quark model. We have shown that the qualitative behaviour of experimentally observed mass-differences can be explained by means of a potential with two different types of wrinkles. It would be interesting to see if a systematic calculation of coupled channel effects can result in an effective potential which when added to a purely concave static potential can give rise to these wrinkles. This is currently under investigation and shall be reported in the near future.

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(b) The ref. 5(a) has a few misprints : (i) The euqation (10) on page 1770 should read :

$$V_1(R) = -\frac{4}{3}\,\alpha_s(R)\frac{[\ln(\Lambda R)]^2}{R\{\ln(\Lambda R)\}^2 + A\{\tanh\frac{R-R_0}{b} + \tanh\frac{R_0}{b}\}}$$

$$V_2(R) = K\,\tanh(\frac{R}{R_c})\sqrt{R^2-(R_c\tanh\frac{R}{R_c})^2}$$

(ii) The value of A quoted on page 1771 (paragraph just below Fig.2) is a misprint. It should read as $1/A = 0.06\ GeV^{-1}$

6 (a) R.Kaur and B.A.Bambah, Phys. Rev. **D47**, 5079 (1993); Erratum, *ibid*, submitted (1995).

(b) The ref.6(a) has a few misprints : (i) Equation (5) on page 5079 should read :

$$V_1(R) = -\frac{4}{3}\,\alpha_s(R)\frac{[\ln(\Lambda R)]^2}{R\{\ln(\Lambda R)\}^2 + A\{\tanh\frac{R-R_0}{b} + \tanh\frac{R_0}{b}\}}$$

$$V_2(R) = K\,\tanh(\frac{R}{R_c})\sqrt{R^2-(R_c\tanh\frac{R}{R_c})^2}$$

(ii) The value of A quoted on page 5080 should read as $1/A = 0.06\ GeV^{-1}$.

7 *A note regarding the notation* : In all future communications we shall follow the notations of the present paper. It may be noted that $V_{BDKS}(R)$ of the present paper [eq.5] is the same as $V_2(R)$ of Ref.5 and the Vr(R) of Ref.6

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## *Figure Captions :*

1. The "bare" potential $V_0(R)$ of equation 4, with $\Lambda$=0.11GeV and K=0.16 $GeV^2$; $n_f$=4.
2. The detailed structure of BDKS potential[5] [$V_{BDKS}(R)$]. ooooooooo is the "bare" potential $V_0(R)$ (equation 4) . V (R) (eqn. 6) and V (R) (eqn. 7) constituting $V_{BDKS}(R)$ are shown separately for qualitative comparison. The parameters are A=15$GeV^{-1}$ $\Lambda$=0.11GeV , K=0.16 $GeV^2$ and b=Rc/60
3. Variation of the BDKS-potential.$V_{BDKS}(R)$ with the parameter A. Curves 1,2and 3 correspond to A=25, 100 and 1(in $GeV^{-1}$) respectively. The other parameters are the same as in fig. 2. ooooooooo is the "bare" potential $V_0(R)$ of fig. 1.
4. Variation of the BDKS-potential $V_{BDKS}(R)$ with the parameter $R_0$.The curves 1,2 and 3 correspond to x=0.8,0.95and 1.15 respectively. A=25 $GeV^{-1}$ and ooooooooo is the "bare" potential $V_0(R)$ of fig. 1.
5. Variation of the BDKS-potential $V_{BDKS}(R)$ with parameter the b. The curves 1,2 and 3 correspond to b=$R_c$/60,$R_c$/15 and $R_c$/4 respectively A=15 $GeV^{-1}$ and ΔΔΔΔΔΔΔ is the "bare" potential $V_0(R)$ of fig. 1.The curve 3 is the wrinkle -free $V^{WF}(R)$.
6. The asymmetric Gaussian ($V_{AG}$) (curve 3) obtained by subtracting the wrinkled-BDKS potential $V_{BDKS}(R)$ (curve 2)from the unwrinkled one $V^{WF}(R)$ (curve 1).
7. The wrinkled-BDKS potential (curve 2) which produces the required energy level inversion. The parameters used are given in the text. The curve-1 is the original BDKS wrinkled potential of ref.5 with the parameters same as in fig. 2.
8. The unwrinkled potential $V^{WF}(R)$ (curve 1) with the symmetric Gaussian (curve2) of equation 9 .

## *Table Caption:*

I. Energy-level differences (in GeV) of charmonium and Bottomonium

**Table I : Energy level differences ( GeV ) of Charmonium and Bottomonium**

| | | 1<br>Experimental<br>(Ref. 4) | 2<br>$V_{BDKS}$<br>(Ref. 5) | 3<br>$V^{wr}$ | 4<br>$V_{BDKS}$<br>(present work) | 5<br>$V^{wr}+V$<br>(V as in eq.9) |
|---|---|---|---|---|---|---|
| CHARM | 2S - 1S | 0.589 | 0.569 | 0.481 | 0.541 | 0.457 |
| | 3S - 2S | 0.354 | 0.388 | 0.368 | 0.435 | 0.333 |
| | 4S - 3S | 0.375 | 0.325 | 0.318 | 0.439 | 0.372 |
| | M (Ψ 1S) | 3.097 | 3.044 | 3.074 | 3.134 | 3.074 |
| BOTTOM | 2S - 1S | 0.563 | 0.603 | 0.419 | 0.412 | 0.420 |
| | 3S - 2S | 0.332 | 0.275 | 0.278 | 0.300 | 0.269 |
| | 4S - 3S | 0.225 | 0.232 | 0.229 | 0.210 | 0.171 |
| | 5S - 4S | 0.285 | 0.207 | 0.202 | 0.243 | 0.214 |
| | 6S - 5S | 0.154 | 0.189 | 0.185 | 0.209 | ** |
| | M (Υ 1S) | 9.460 | 9.421 | 9.438 | 9.431 | 9.439 |

** There seems to be a spurious inversion here. This needs to be investigated more carefully, theoretically as well as experimentally.

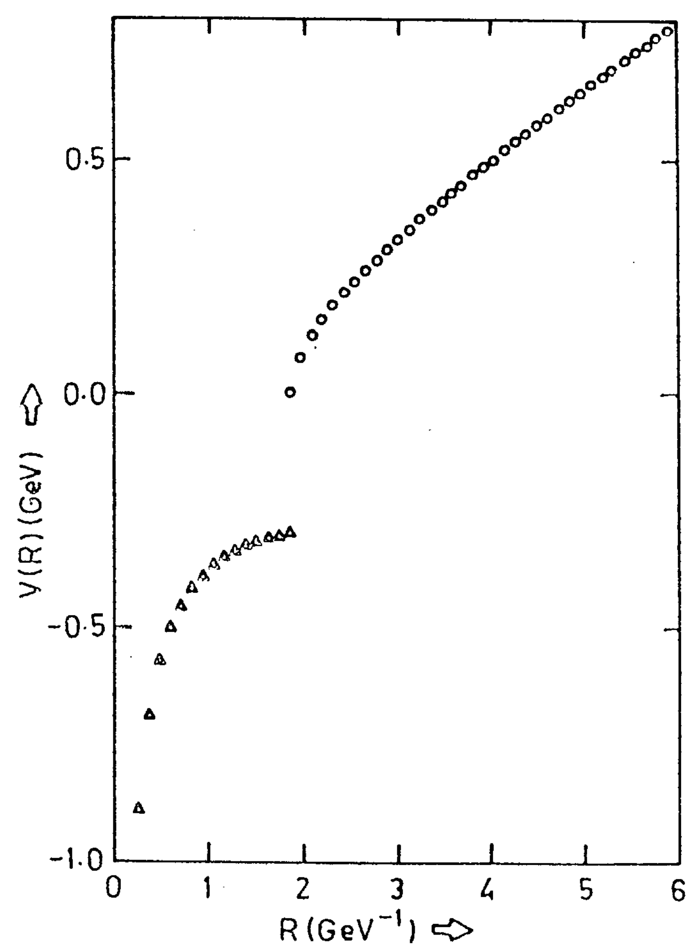


Fig.1 →

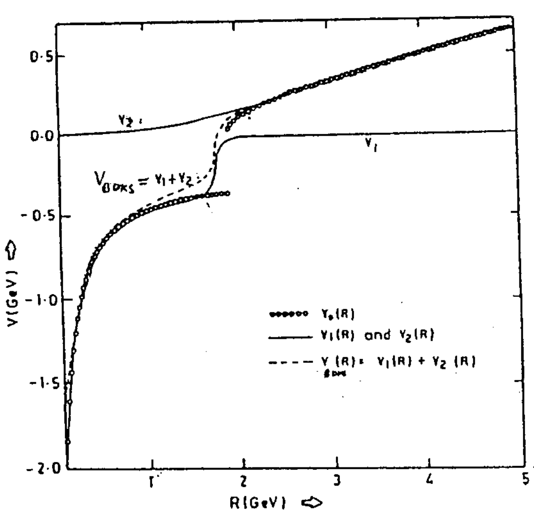


Fig.2

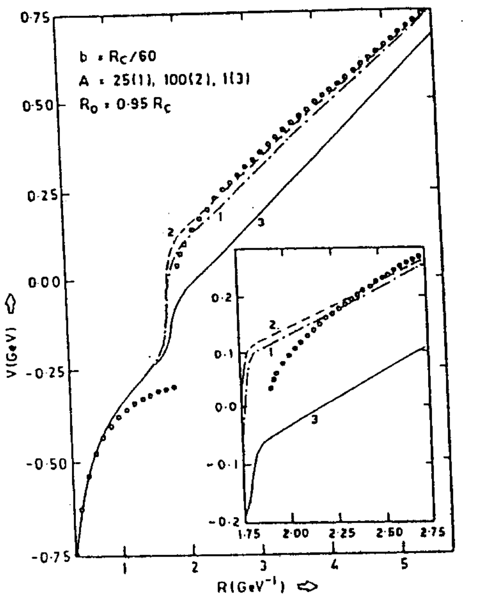


Fig.3

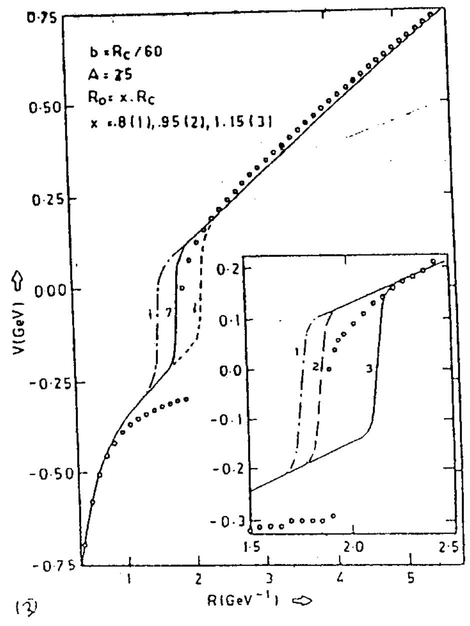


Fig.4

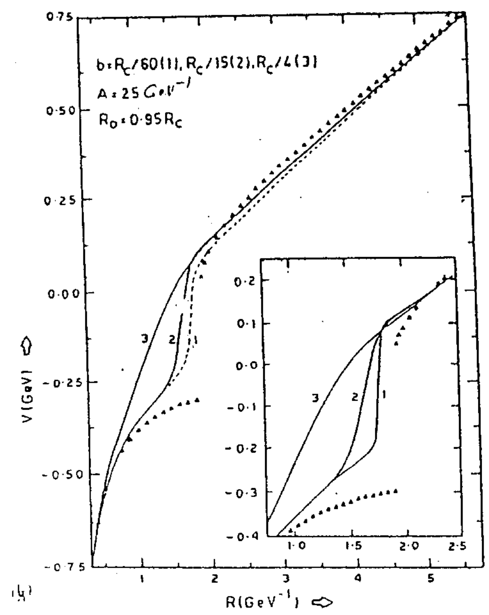


Fig.5

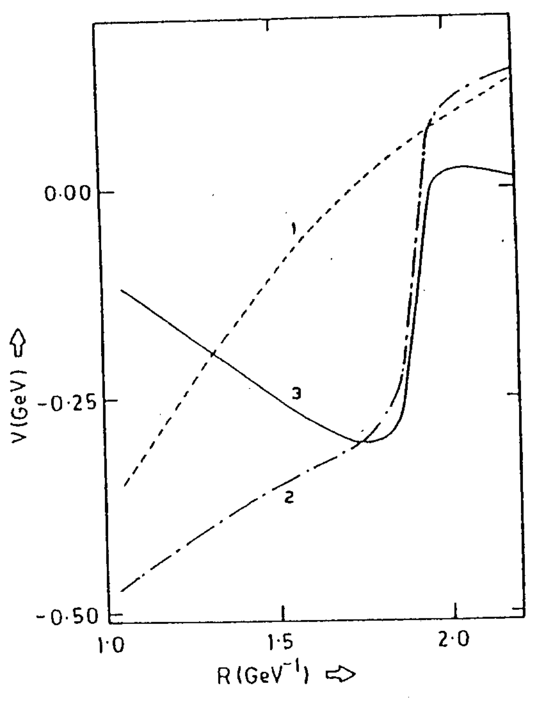


Fig. 6

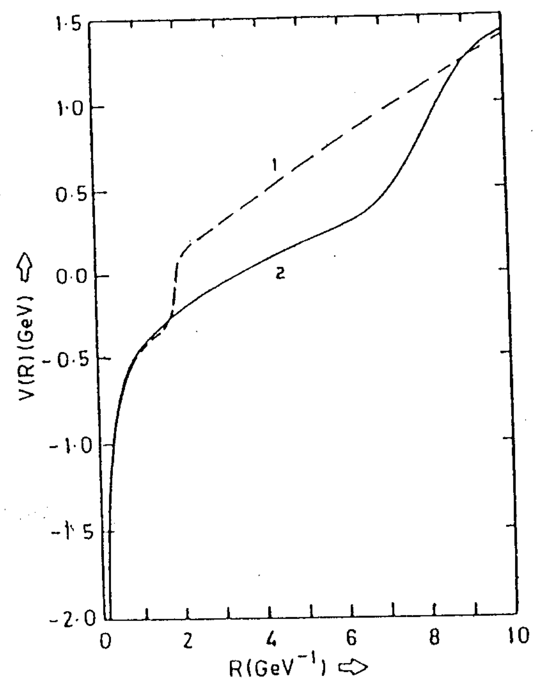


Fig. 7

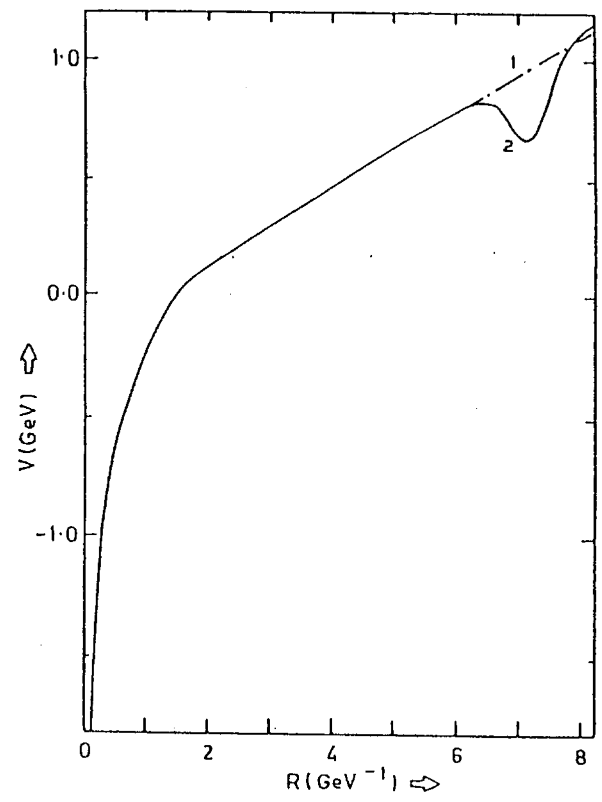


Fig. 8.