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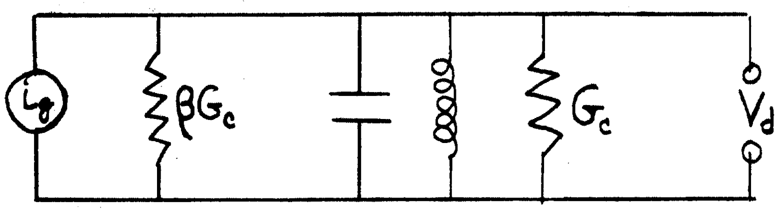
STABILITY IN SUPERCONDUCTING SEPARATOR SYSTEMS

Notes on a talk given 14th May, 1969, at the seminar on the application of superconductivity to RF particle separators.

I. EQUIVALENT CIRCUIT

A superconducting resonant cavity can be represented by the following equivalent circuit (see, for example, refs. 1 and 3). All quantities are referred to the cavity side of the coupling network.



The following definitions are introduced :

$$G_c = \frac{2}{rL}\,, \quad i_g = \sqrt{8\,P_0\,\beta\,G_c}$$

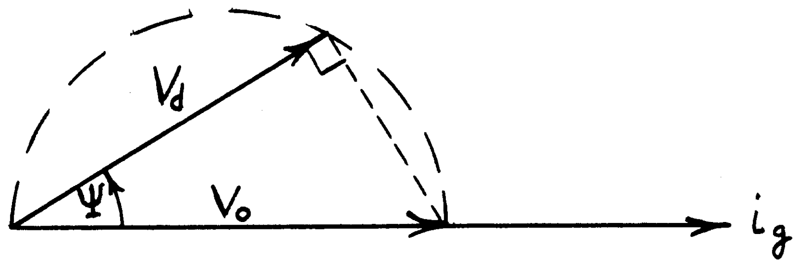
$$\mathrm{Tan}\,\Psi = -2Q_L\left(\frac{\omega - \omega_0}{\omega_0}\right)$$

$$Q_L = Q_0/(1+\beta) \qquad\qquad Q_e = Q_0/\beta$$

Here r is the shunt impedance per unit length, P_0 is the available power for the generator (= klystron power), β is the coupling coefficent, and Ψ is the tuning angle. The transverse deflection voltage can be written, using the above circuit, as

$$V_d = \sqrt{P_o rL}\ \frac{2\sqrt{\beta}}{1+\beta}\ \cos\Psi = V_o \cos\Psi \quad . \tag{1}$$

As the tuning angle Ψ varies, the tip of the vector V_d traces out a semi-circle as shown in the following figure :



II. OVERCOUPLED CASE

It will normally be desirable to operate a superconducting separator cavity in the overcoupled condition, in order to reduce the value of the loaded Q . If $\beta >> 1$, then Eq. (1) becomes

$$V_d \sim 2\sqrt{P_o rL/\beta} = 2\sqrt{P_o LQ_e(r/Q)} \ .$$

Since $Q_e \approx Q_L$ for the overcoupled case, we have

$$Q_L \approx \frac{E_d^2}{4(P_o/L)(r/Q)} \ ,$$

where $E_d = V_d/L$ is the average deflecting gradient. We see that the loaded Q is determined by the gradient, the klystron power per unit length, and the factor r/Q, but is independent of the unloaded Q actually obtained in a particular structure.

For typical standing-wave structures, we can make the approximation that $(r/Q) \approx 100/\lambda$. Then

$$Q_L \approx \frac{\lambda E_d{}^2}{400\ (P_o/L)} \tag{2}$$

As an example of the application of Eq. (2), let $\lambda = 0.1$ m, $E_d = 6$ MV/m, and $P_o/L = 1$ kW/m. Then $Q_L \simeq 10^7$, and the filling time is

$$T_F = 2Q_L/\omega \approx 10^{-3} \text{ sec.}$$

We will take these values of Q_L and T_F as being typical for superconducting separator cavities in the calculations to follow.

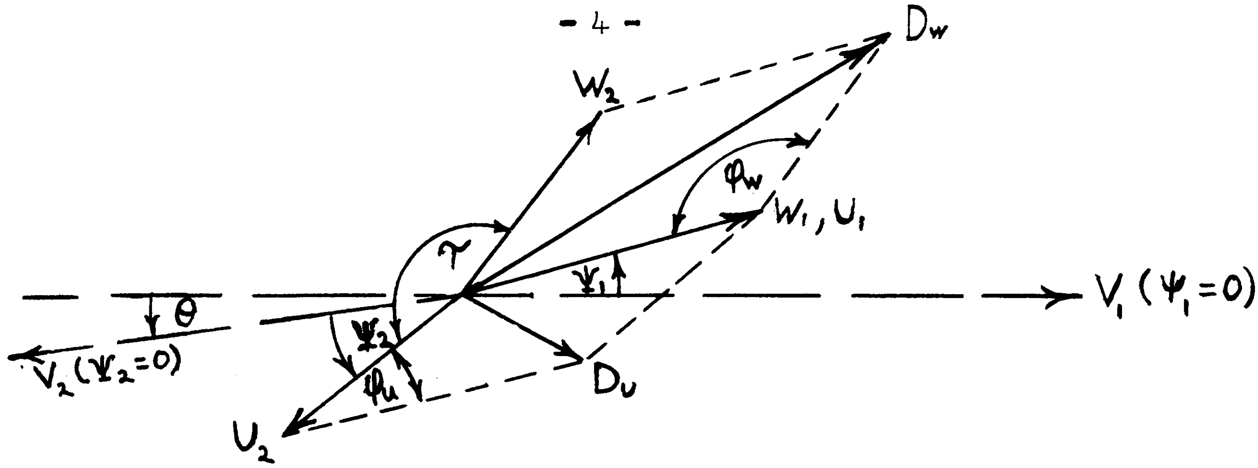
If the unloaded Q of the structure is $Q_o = 10^9$, then $\beta \simeq Q_o/Q_L = 100$ and

$$\frac{P_s}{P_o} = \frac{4\beta}{(1+\beta)^2} \simeq \frac{4}{\beta} = .04$$

where P_s is the power actually dissipated in the structure. Thus if $P_o = 1$ kW/m, $P_s = 40$ W/m. This appears to be rather inefficient operation , but klystron power is cheap and it is a small price to pay to lower Q_L by a factor of 50 below the Q_L obtained at critical coupling. As we will see, lower Q_L implies greater stability of the separator system againt the many effects which can cause changes in the resonant frequency of a superconducting structure.

III. VECTOR DIAGRAM FOR THE NET DEFLECTION IN A SEPARATOR SYSTEM

The following figure shows a vector diagram for calculating the net deflection, D_u , given to the unwanted particles and the net deflection, D_w , given to the wanted particles in a two-cavity separator system.



The angles φ_u and φ_w are given by

$$\varphi_u = \Theta + \Psi_2 - \Psi_1$$

$$\varphi_w = \tau - (\Theta + \Psi_2 - \Psi_1).$$

In the diagram the vectors W_1, U_1 and W_2, U_2 refer to the net deflection given to the wanted and unwanted particles in the first and secnd cavities respectively. The reference phase has been chosen as the phase of the field in the first cavity at resonance ($\Psi_1 = 0$). The relative phasing Θ between the two cavities is taken to be zero when the deflection given to the unwanted particles in the second cavity (at resonance) is exactly 180° out of phase with respect to the deflection imparted in the first cavity. The angle τ is the phase slip between the wanted and unwanted particles (for example, see Eq. (7), Ref. 4). The magnitudes of the various vectors are

$$W_1 = U_1 = V_{01} \cos \Psi_1$$

$$W_2 = U_2 = V_{02} \cos \Psi_2$$

where

$$V_{01} = (P_{01} rL)^{\frac{1}{2}} \cdot \frac{2\sqrt{\beta_1}}{1+\beta_1}$$

$$V_{02} = (P_{02} rL_2)^{\frac{1}{2}} \cdot \frac{2\sqrt{\beta_2}}{1+\beta_2}$$

Using the law of cosines, we can readily calculate D_u from the preceding vector diagram,

$$D_u^2 = V_{01}^2 \cos^2\Psi_1 + V_{02}^2 \cos^2\Psi_2 - 2V_{01} V_{02} \cos\Psi_1 \cos\Psi_2 \cos(\Theta+\Psi_2-\Psi_1)$$

If $\psi_1 = \Psi_2 = \Theta = 0$, then

$$D_u = V_{01} - V_{02} .$$

If $\Theta, \Psi_1, \Psi_2 << 1$, then

$$D_u^2 \approx (V_{01}-V_{02})^2+(V_{01}-V_{02})(V_{02}\Psi_2^2-V_{01}\Psi_1^2)+V_{01}V_{02}(\Theta+\Psi_2-\Psi_1)^2$$

If $V_{01} = V_{02} = V_0$ and if $\tau \approx \pi$, then

$$D_u \approx V_0 (\Theta+\Psi_2-\Psi_1)$$

$$D_w \approx 2 V_0 .$$

The preceding two equations give

$$\frac{D_u}{D_w} \approx \frac{1}{2} (\Theta + \Psi_2 - \Psi_1) . \qquad (3)$$

IV. STABILITY REQUIRED IN A SEPARATOR SYSTEM

The permissible fluctuation in the amplitude of D_u will depend on the type of experiment being conducted. The purity of a beam for counter experiments, for example, may not have to be as good as the purity required for a bubble chamber beam. Let us choose rather arbitrarily that we want to keep the ratio D_u/D_w less than 3 %. For some experiments a ratio of 10 % or even higher may be adequate. If this is the case, all the calculations to follow can readily be scaled to take a larger ratio into account. However, assuming a ratio of 3 %, and assuming that the errors in Θ, φ, and Ψ_2 are of the same order, we have from Eq. (3)

that

$$\Theta,\ \Psi_1,\ \Psi_2 \approx \frac{(2)(.03)}{\sqrt{3}} \approx .035 \approx 2^\circ \quad .$$

The stabilization of the relative phase angle Θ is essentially a room temperature problem. It has been shown that this phase can be held constant to the order of $\pm 2°$ (see Ref. 4). Let us now see what the stability requirement on the tuning angle implies for the frequency stability of a superconducting cavity. From the definition of tuning angle we have, for $\Psi << 1$, that $\Psi \approx 2\ Q_L\ (\Delta\omega/\omega)$. Thus for a loaded Q of 10^7, as discussed in Section II, we have

$$\frac{\Delta\omega}{\omega} \approx \frac{\Psi}{2Q_L} \approx \frac{(.035)}{(2\times10^7)} \approx 2\times10^{-9} \quad . \qquad (4)$$

It is easy to achieve this order of stability using any one of a number of commercially available frequency sources. But even if the source frequency has an acceptable stability, the resonant frequency of the superconducting cavity can change as the result of a variety of thermal, mechanical, and electronic effects. Some of these effects are discussed in the following section.

V. SOME EFFECTS WHICH CAN CHANGE THE RESONANT FREQUENCY OF A SUPERCONDUCTING STRUCTURE

V.1. THERMAL EFFECTS

a) VARIATION OF THE PENETRATION DEPTH WITH TEMPERATURE

From the BCS theory, for temperatures which are low compared to the transition temperature, the penetration depth can be written

$$\delta(T) = \frac{\delta_0}{[1-2\ \exp\ (-\epsilon_0/kT)]^{\frac{1}{3}}} \quad .$$

Here δ_0 is the penetration depth at $T = 0°K$ and $2\epsilon_0$ is the energy gap. Differentiating,

$$\frac{d\delta}{dT} = \frac{2\delta_0\epsilon_0}{3kT^2} \exp(-\epsilon_0/kT) \quad .$$

For lead at 1.85°K, using $\epsilon_0/k = 14.8$°K and $\delta_0 = 4 \times 10^{-6}$ cm, we calculate that $d\delta/dT = 4 \times 10^{-9}$ cm/°K. The diameter of a typical separator structure given roughly be $D \approx 1.15\ \lambda$. Therefore,

$$\frac{d\omega}{\omega} \approx \frac{2(d\delta)}{D} \approx -1.75 \frac{d\delta}{\lambda} \quad . \tag{5}$$

And,

$$\frac{\Delta\omega}{\omega} \approx \left(\frac{-1.75}{\lambda}\right) \left(\frac{d\delta}{dT}\right) \Delta T \quad .$$

If we assume that the temperature of the liquid helium bath can be controlled to 10 millidegrees, we calculate from the preceding expression (using $\lambda = 10.5$ cm) that $\Delta\omega/\omega \approx 7 \times 10^{-12}$. This is far below the allowable variation of 2×10^{-9} derived earlier (see Eq. 4), and this effect should not, therefore, be troublesome. If the calculation is repeated for niobium, the frequency shift obtained is lower by a factor of 3 .

b) <u>THERMAL EXPANSION</u>

The coefficient of thermal expansion, for both normal and superconducting metals, vanishes rapidly as the temperature approaches absolute zero. For copper at 1.85°K, this coefficient is $\alpha = 6 \times 10^{-10}$/°K (see ref. 5). The frequency shift due to this effect can be written

$$\frac{\Delta\omega}{\omega} \approx \alpha(\Delta T) \quad .$$

Again taking $\Delta T = 10^{-2}$ °K, we calculate that $(\Delta\omega/\omega) \approx 6 \times 10^{-12}$. The conclusion here also is that this effect will not cause a detrimental frequency shift for reasonable fluctuations in temperature.

c) TEMPERATURE RISE IN THE HELIUM BATH DURING PULSED OPERATION

The specific heat of liquid helium at 1.85°K is about 450 J/°K/liter. If we assume that there are 50 liters of liquid per meter of structure, then the total heat capacity per unit length is about 2.2×10^4 J/m/°K. Assume now a power dissipation of 100 Watts during a 1/2 second pulse in a structure 3 meters in length. The heat pulse is then about 16 J/m. The temperature rise is

$$\Delta T \approx (16)/(2.2 \times 10^4) = 7 \times 10^{-4} \text{ °K} .$$

The superfluid helium bath is seen to provide a thermal reservoir with a very large heat capacity. For any reasonable power dissipation and pulse length, temperature variations are effectively damped.

d) TEMPERATURE RISE DUE TO THERMAL IMPEDANCE

Since the diameter of a separator structure is about $D \approx 1.14\ \lambda$, the area per unit length available for heat transfer is about $\pi D = 3.6\ \lambda$. If we approximate the structure by a cylinder of thickness t, and if K is the thermal conductivity, the temperature rise at the superconducting surface due to the finite thermal impedance is

$$\Delta T \approx \left(\frac{0.3}{\lambda}\right)\left(\frac{P_s}{L}\right)\left[\frac{t}{K} + R_1 + R_2\right] .$$

Here P_s/L is the power dissipated in the structure per unit length, and R_1 and R_2 are the Kapitza resistances for the transfer of heat across the boundaries between the superconducting layer and the structure, and between the structure and the helium bath. For a solid niobium structure there is only one Kapitza resistance, which is small compared to t/K (note that $R_2 \approx 10/T^3$ for a Cu-He boundary). Using $K = .005$ W/cm-°K at 1.85° (see Ref. 6), and $\lambda = 10.5$ cm, we calculate that if $t = 0.5$ cm and $P_s/L = 30$ watts/m, then $\Delta T \approx 1$°K. The temperature rise in the disks might be even higher because of the longer path for heat conduction. A temperature rise of this order would cause a serious drop in Q and a frequency shift which is about equal to the limit specified by Eq. (4).

V.2. MECHANICAL EFFECTS

Any variation in the mechanical forces acting on the structure can cause dimensional changes which may be large enough to cause a noticeable shift in resonant frequency. The pressure of the liquid helium bath is such a force. For the original bi-periodic accelerating structure built at Stanford, the effect of a pressure change on frequency was measured to be about

$$\frac{\Delta\omega}{\omega} \approx 10^{-8} \quad \Delta P \,, \tag{6}$$

where ΔP is in Torr. If the frequency is to be stabilized to $\Delta\omega/\omega = 2 \times 10^{-9}$, then ΔP must be held to 0.2 Torr. A pressure change of this order corresponds to a change of 4 millidegrees in the temperature of a liquid helium bath in equilibrium with a saturated vapor. These tolerances may be strict for a large refrigerator system, but probably not impossible. It would be better for a structure to be several times more rigid with respect to the effect of pressure changes on frequency. The coefficient in Eq. (6) is most easily obtained from measurement using a model superconducting structure.

V.3. DEPENDENCE OF SURFACE REACTANCE ON RF AMPLITUDE

According to J. Halbritter[7], the penetration depth varies with the strength of the magnetic field at the superconducting surface according to

$$\delta_H = \delta_0 \left[1 + \alpha \, (H/H_c)^2\right] \tag{7}$$

where H_c is the thermodynamic critical field.

Combining Eq. (5) and (7), the corresponding change in frequency is

$$\frac{\Delta\omega}{\omega} \approx -1.75\ \delta_0\ \alpha\ (H/H_c)^2/\lambda$$

According to Halbritter, $\alpha = 0.08$ for niobium. Using also $\delta_0 = 4 \times 10^{-6}$ cm and $\lambda = 10.5$ cm, we calculate that $\Delta\omega/\omega = 2 \times 10^{-9}$ if $H/H_c = 0.2$. This corresponds to a field of about 400 G for niobium. In a superconducting separator the peak field reached at the outer diameter of the structure is expected to be of this order. Thus this effect might or might not be noticeable. It should be pointed out that the Ginzberg-Landau theory, from which Eq. 7 is derived, is in principle valid only close to the transition temperature. The calculation should be considered only as an indication of a potential source of frequency instability.

V.4. ELECTRONIC LOADING

Loading due to electrons from field emission or other sources can cause both a Q degradation and a frequency shift in a superconducting cavity. The loading can be represented by an admittance

$$Y_L = G_L + jB_L = G_L\ (1+j\gamma)\ , \tag{8}$$

where γ is the ratio of the imaginary to the real part. An exact calculation of Y_L seems to be impossible for the case of a high-field superconducting cavity when the loading current is emitted from unknown locations within the cavity. It can only be stated that G_L and B_L will probably be proportional to the total emitted dc current.

An analysis of the effect of electron loading can be carried a bit further without a detailed model. The cavity impedance without loading can be written, using the equivalent circuit in Sec. 1, as

$$Y_c = G_c(1+\beta) + j2G_cQ_0(\Delta\omega/\omega)\ . \tag{9}$$

By combining Eqs. (8) and (9) it is easy to show that the new unloaded Q with electron loading present, denoted by Q_o' , and the shift in resonant frequency, are given by

$$Q_o' = Q_o G_c / (G_c + G_L) \tag{10a}$$

$$2\, Q_o(\Delta\omega/\omega) = -\gamma G_L / G_c \quad . \tag{10b}$$

For example, suppose $G_L = G_c$ and $\gamma = 1$. The Q then decreases by a factor of two due to loading and the frequency shifts by one-half a bandwidth. The exact numbers are not of much importance. The point is that, if the Q decreases due to electron loading, the frequency can also shift, in principle either upward or downward.

By eliminating G_c and G_L between Eqs. (10a) and (10b) we obtain

$$2\, Q_o(\Delta\omega/\omega) = -\gamma\, [(Q_o/Q_o') - 1] \; . \tag{11}$$

The measurements in Table 5, Ref. 8, agree fairly well with the predictions of Eq. (11), where γ is about 2.8 for one set of measurements and 1.8 for the second.

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VI. REFERENCES

In Refs. 2 and 3, a general survey of the problem of stability in superconducting cavities is given. In Ref. 5, thermal aspects of the stability problem are treated in some detail. Copies of each of these references are available in a file of Stanford and SLAC reports concerning superconducting cavities and accelerators, located in Bldg. 18, Room 2-040.

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