MPS/AE/Note 74-6
16.4.1974

# SIGNAL TRANSMISSION THEORY OF THE LOW ENERGY UHF WIDE BAND PICK-UP STATION FOR THE NEW LINAC

G.C. Schneider

Table of Contents

## 1. INTRODUCTION

The resistive UHF pick-up station proposed in Ref. 1 requires a special screening for low energy beams ($\beta \ll 1$). One solution was discussed in Ref. 2. A simpler configuration shown in Fig. 1 will be investigated in the following.

The gap (impedance $Z_2$) between 2 vacuum pipes contains 4 resistors R placed at 90°, 180°, 270° and 360° positions across the gap. 4 radial strip lines (impedance $Z_1/2$) sense the field of the beam. Four cables of impedance Z = R transmit the signals to a summing point.

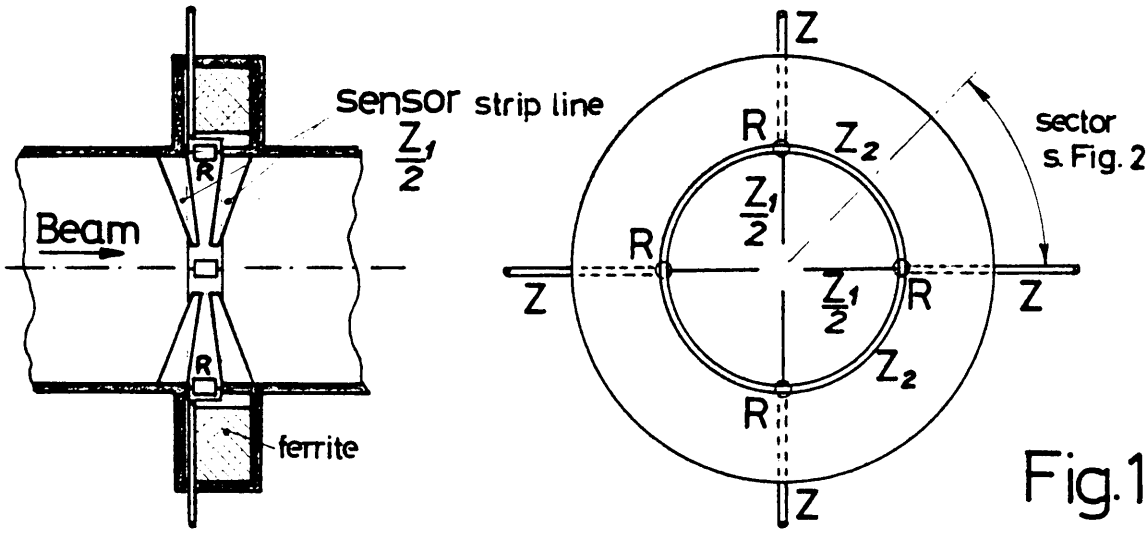



Fig.1

## 2. SIGNALS OF A CENTRED BEAM

In this case only a sector of 45° must be considered due to symmetry. The equivalent circuit of such a sector is given in Fig. 2.

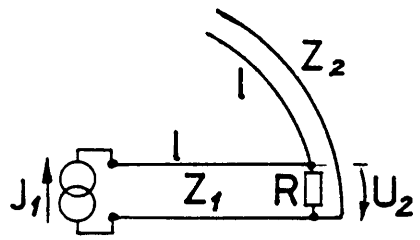



Fig. 2

The beam is represented by a current source $J_1$ at the free end of the radial strip line ($Z_1$). The open line $Z_2$ corresponds to 1/8 of the circular gap. The length of both lines is $\ell$. The output signal $U_2$ appears across the resistor R. The transfer function $U_2/J_1$ will be evaluated in the following.

The circuit of Fig. 2 can be simplified (Fig. 3) when the input admittance of the open line $Z_2$

$$Y = \frac{j \ tg \ \beta\ell}{Z_2} \qquad (1)$$

($\beta$ = phase constant of the line)

is added to the conductance

$$G = 1/R \qquad (2)$$

e.g.

$$Y_p = G + Y \qquad (3)$$

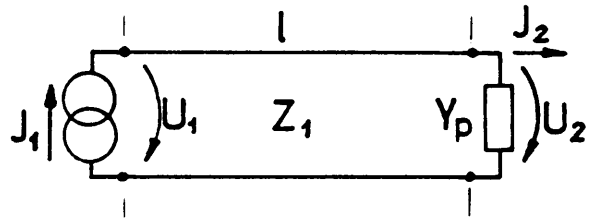



Fig. 3

The signal transfer (in Fig. 3) can be described with the chain matrix product

$$\begin{pmatrix} U_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} \cos \beta\ell; & j \, Z_1 \sin \beta\ell \\ \frac{j}{Z_1} \sin \beta\ell; & \cos \beta\ell \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y_p & 1 \end{pmatrix} \begin{pmatrix} U_2 \\ J_2 \end{pmatrix} \qquad (4)$$

where the first expression between brackets on the right is the transmission line matrix of a line without losses and the second is the matrix of the admittance $Y_p$.

The desired ratio $U_2/J_1$ can be found from equ. (4) with $J_2 = 0$.

$$\frac{U_2}{J_1} = \frac{1}{\frac{j}{Z_1}\sin\beta\ell + Y_p\cos\beta\ell} \tag{5}$$

and with equ. (3) and (1)

$$\frac{U_2}{J_1} = \frac{R}{\cos\beta\ell + j\,R\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\beta\ell}\ . \tag{6}$$

If the co-factor of $j\sin\beta\ell$

$$R\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right) = 1 \tag{7}$$

equation (6) simplifies with Euler's formula

$$\frac{U_2}{J_1} = R\cdot e^{-j\beta\ell} = R\cdot e^{-j\frac{\omega\ell}{c}} \tag{8}$$

with the phase constant

$$\beta = \frac{\omega}{c} \tag{9}$$

($\omega$ = radian frequency; c = velocity of light).

The transfer function $\frac{U_2}{J_1}$ (see equ. (8)) has a constant amplitude R and a linear phase.

The group delay

$$\tau_g = \frac{d\phi}{d\omega} = \frac{\ell}{c} \tag{10}$$

is a constant versus frequency, i.e. the transmission from $J_1$ to $U_2$ occurs theoretically for all frequencies without distortions.

Equation (8) pretends that even for the $\lambda/4$ frequency, where the open ended line ($Z_2$) represents a short-circuit at the input (see Fig.2), the transfer function $U_2/J_1$ has a constant amplitude R. The explanation is that the current flowing into the resistor R has a simple pole at the $\lambda/4$ frequency where the line ($Z_2$) represents a short-circuit, and hence the singularity is removed if equ. (7) is satisfied.

Equation (7) can be rewritten

$$Z_2 = \frac{1}{\left(\frac{1}{R} - \frac{1}{Z_1}\right)} \tag{11}$$

This function is plotted in Fig.A for 3 parameters of R (50, 75, 125 Ω).

The ideal transmission property in the time domain can be explained by the reflection theory. This is done in the following for the more general case shown in Fig. 4.

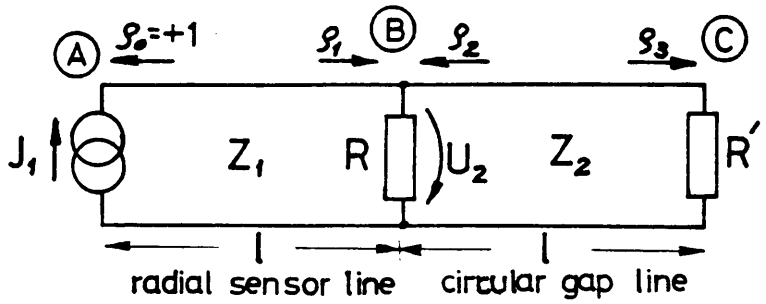



Fig. 4

The difference with respect to the circuit of Fig. 2 is the arbitrary termination with R' at the end of line $Z_2$ which causes there a reflection $\rho_3 = (R' - Z_2)/(R' + Z_2)$.

The signal coming from the source $J_1$ is partially reflected at point B (reflection coefficient $\rho_1$). The transmitted part $(1 + \rho_1)$ travels down the line from B to C and is partially reflected at point C (reflection coefficient $\rho_3$). The condition for undistorted transmission is that the reflections coming back from both ends (A and C) have the same magnitude but opposite polarity and cancel then at point B.

This is mathematically described by the following equation

$$\rho_1\rho_0(1 + \rho_1) = -(1 + \rho_1)\rho_3(1 + \rho_2) \tag{12}$$

and simplified with $\rho_0 = +1$

$$\rho_1 = -(1 + \rho_2)\rho_3 . \tag{13}$$

Introducing for the reflection coefficients

$$\rho_1 = \frac{\frac{1}{Z_1} - \left(\frac{1}{Z_2} + \frac{1}{R}\right)}{\frac{1}{Z_1} + \left(\frac{1}{Z_2} + \frac{1}{R}\right)} \tag{14}$$

and

$$\rho_2 = \frac{\frac{1}{Z_2} - \left(\frac{1}{Z_1} + \frac{1}{R}\right)}{\frac{1}{Z_2} + \left(\frac{1}{Z_1} + \frac{1}{R}\right)} \tag{15}$$

in equ. (14) yields

$$Z_2 = \frac{(2\rho_3 - 1)}{\frac{1}{R} - \frac{1}{Z_1}} \tag{16}$$

This equation is plotted in Fig. B ($Z_2$ versus $Z_1$) for R = 75 Ω and $\rho_3$ as parameter. It contains of course the special case of Fig. A for $\rho_3$ = +1 and R = 75 Ω. Figures A and B allow to determine quickly the impedance combinations $Z_1$, $Z_2$ for an ideal signal transmission under the given conditions of R and $\rho_3$.

## 3. SIGNALS OF AN OFF-CENTRE BEAM

A position independent signal requires a summation of the four signals coming from the four resistors R. Fig. 5 shows the pick-up with a resistive summing network. The beam touches in an extreme beam position the end of one radial strip line.

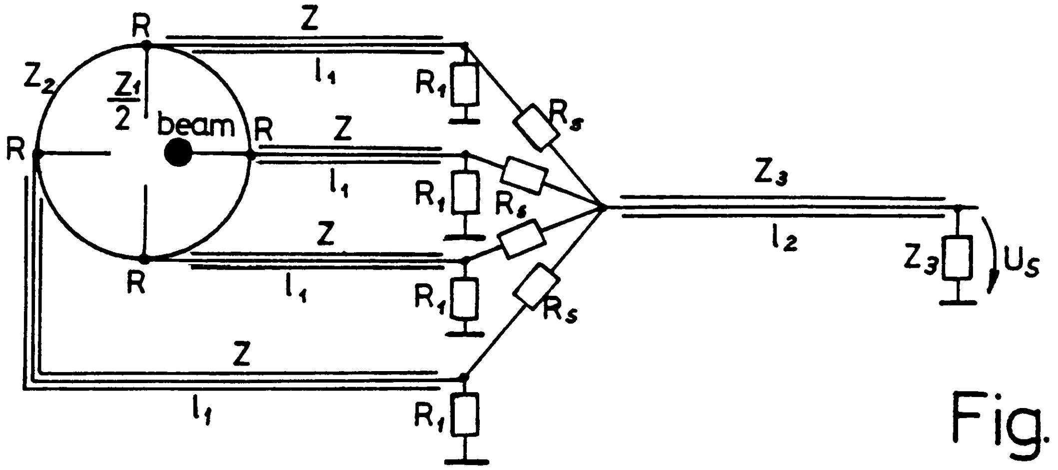



Fig. 5

This case can be regarded as the superposition of the four cases I + II + III + IV shown in Fig. 6. The cases II, III and IV have current sources of the same magnitude but opposite polarity.

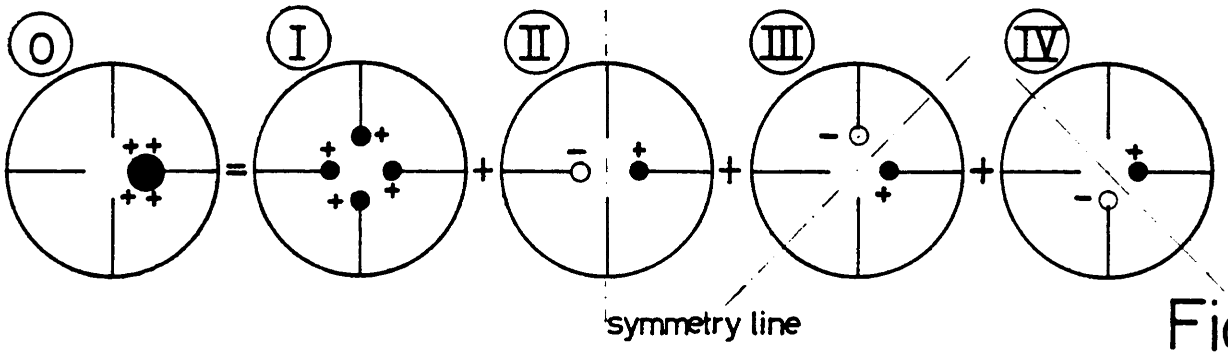



Fig. 6

The signals in the cases II, III and IV cancel at the summing point due to symmetry. (The summing point lies on the symmetry line). The sum signal of case I is therefore the same as the one of the original configuration (case 0). Hence the sum signal in Fig. 5 is an undistorted wide band signal with the same amplitude as for a centred beam, provided that no reflections occur at the summing network and that equ. (11) is satisfied.

## 4. SIGNALS OF A CENTRED DOUBLE BEAM

An extreme double beam is shown in Fig. 7a. This case can be regarded as the sum of the cases I + II + III.

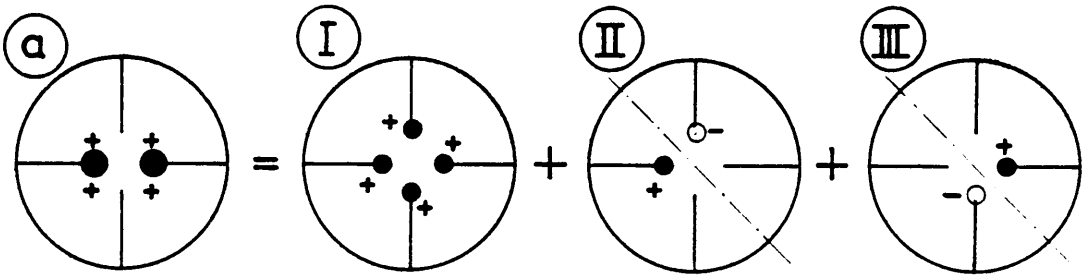



Fig. 7

The cases II and III in Fig. 7 do not contribute to the sum signal due to symmetry. The same reflections as in §3 lead also for a double beam to an undistorted sum signal of the same amplitude as for a centred circular beam.

## 5. CHOICE OF THE PARAMETERS

The gap impedance $Z_2$ depends on the gap width d, the thickness $\delta$ of the tube, the diameter of the tube D and on the dimensions of the outer elements. A gap impedance $Z_2 = 100\ \Omega$ can be realized with d = 2 mm, $\delta = 0.2$ mm and a tube diameter D = 50 mm (determined experimentally with a model 5 : 1, including the ferrite ring and the external short-circuit). The corresponding impedance $Z_1$ becomes for R = Z = 75 Ω

$$Z_1 = 300\ \Omega \quad \text{(see Fig A).}$$

This means the sensor-strip line impedance equals

$$\frac{Z_1}{2} = 150\ \Omega \quad \text{(see Fig. 1).}$$

The equivalent circuit for the sum signal is given in Fig. 8.

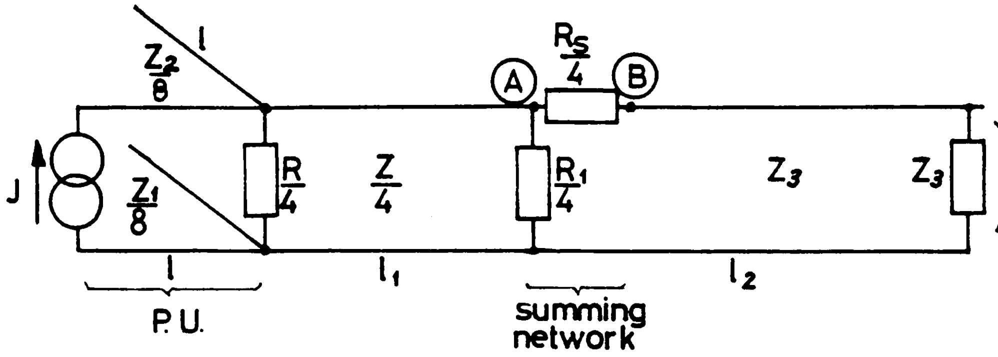



Fig. 8

The matching condition at points A and B gives for a minimum loss pad with the parameters Z = 75, $Z_3$ = 50 Ω (see Ref. 3))

$$R_1 = \frac{Z}{\sqrt{1 - Z/4\,Z_3}} = \frac{75}{\sqrt{1 - \frac{75}{200}}} = 94.87\ \Omega$$

and

$$R_S = 4\,Z_3 \cdot \sqrt{1 - Z/4\,Z_3} = 200\sqrt{1 - \frac{75}{200}} = 158.11\ \Omega.$$

The following list is a summary of the determined parameters corresponding to the circuit of Fig. 5 (in Ohm) :

$R = 75$

$Z_2 = 100$

$\frac{Z_1}{2} = 150$

$Z = 75$

$R_1 = 94.87$

$R_S = 158.11$

$Z_3 = 50.$

## 6. CONCLUSION

In the preceding paragraphs it could be shown that not only centred but even excentric beams give an undistorted wide band sum signal under the condition that all the field emerging from the beam charge goes to the ends of the four sensor strip lines. This is not a hundred per cent true. Therefore signal distortions of a few per cent and a rise time limitation must be expected.

Furthermore, the frequency dependent attenuation of the transmission cables limit the band width. The following table gives the attenuations at 3 frequencies.

| Impedance | Cable | Length | Attenuation at (db) | | |
|---|---|---|---|---|---|
| | | | f = 1 GHz | 2 GHz | 3 GHz |
| Z = 75 Ω | Suhner G 3233 | $l_1$ = 1 m | 0.55 | 0.83 | 1.06 |
| $Z_3$ = 50 Ω | Flexwell 7/8" | $l_2$ = 20 m | 0.76 | 1.40 | 1.80 |
| Total cable attenuation (db) | | | 1.31 | 2.23 | 2.86 |

The real band width of the complete pick-up system and a low energy proton beam must be finally determined experimentally.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge H.H. Umstätter for interesting and useful discussions. I should also like to thank D. Bloess and A. Krusche for their interest and support in the development of this pick-up station.

REFERENCES

1) G.C. Schneider: Proposal for a Resistive, Non-Destructive UHF Wide Band Pick-Up Station with Beam Position Measurement, MPS/SR/Note 73-32.

2) Addendum to 1), MPS/SR/Note 73-32 Add.

3) Howard W. Sams & Co., ITT: Reference data for radio engineers, p. 10-5, 1972.

Distribution open

/bb

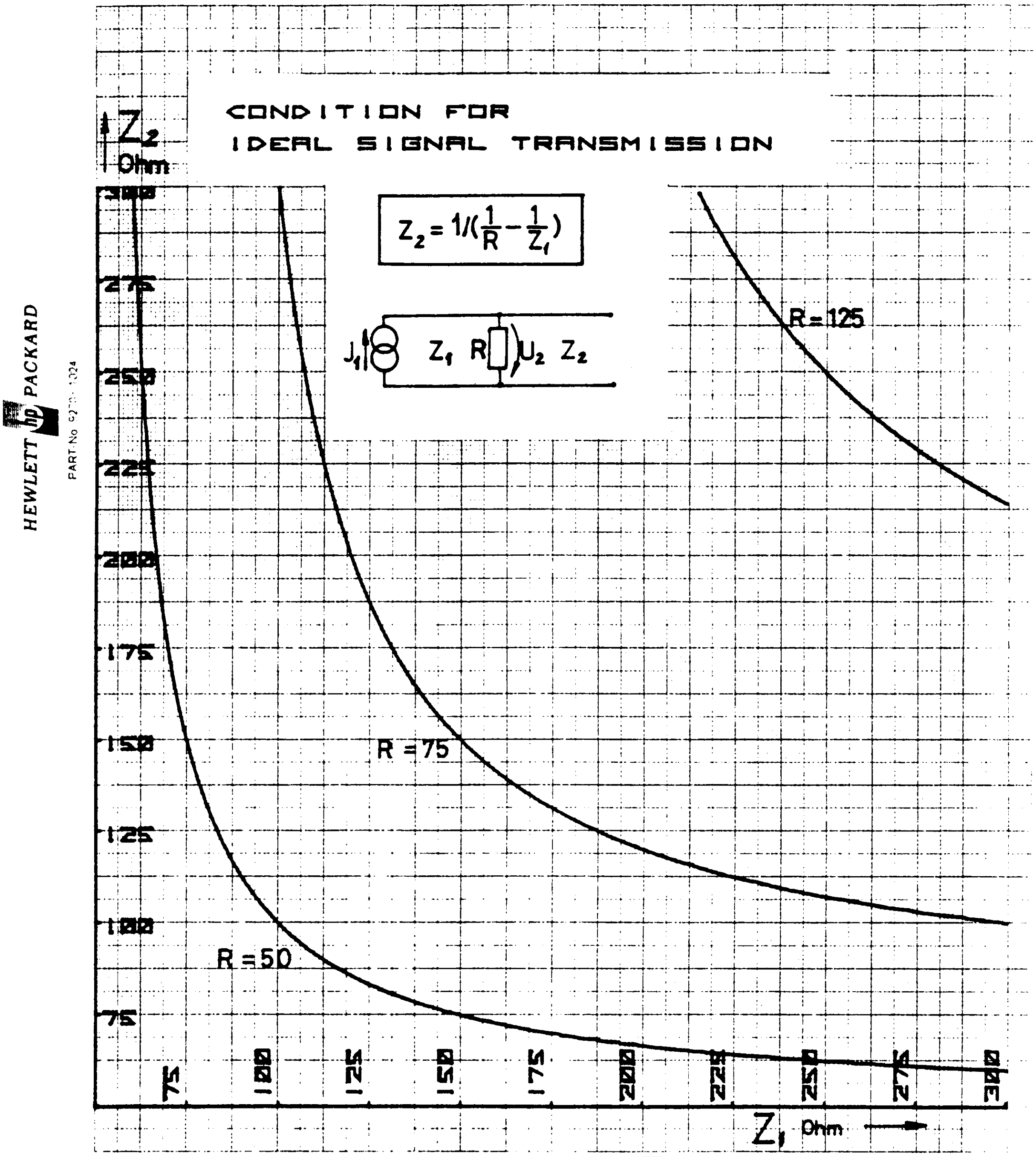
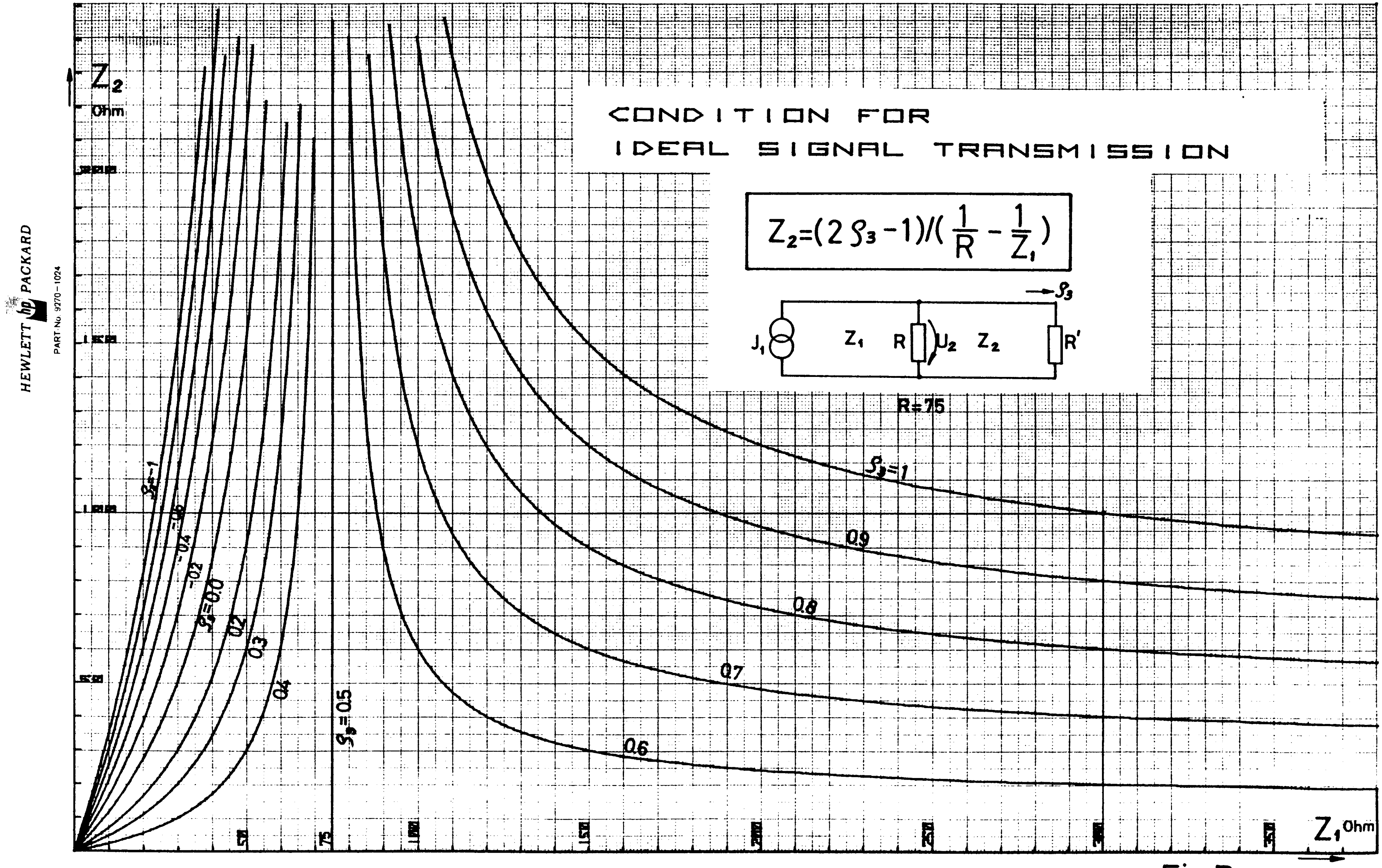

CONDITION FOR
IDEAL SIGNAL TRANSMISSION
$Z_2 = 1/(\frac{1}{R} - \frac{1}{Z_1})$
$J_1$ $Z_1$ R $U_2$ $Z_2$
$Z_2$ Ohm
300
275
250
225
200
175
150
125
100
75
R = 125
R = 75
R = 50
75
100
125
150
175
200
225
250
275
300
$Z_1$ Ohm
HEWLETT PACKARD

Fig.A



CONDITION FOR
IDEAL SIGNAL TRANSMISSION
$Z_2=(2\rho_3-1)/(\frac{1}{R}-\frac{1}{Z_1})$
$J_1$ $Z_1$ R $U_2$ $Z_2$ R' $\rho_3$
R=75
$Z_2$
Ohm
$Z_1$ Ohm
75
$\rho_3=1$
0.9
0.8
0.7
0.6
$\rho_3=0.5$
0.4
0.3
0.2
$\rho_3=0.0$
-0.2
-0.4
$\rho_3=-1$
HEWLETT PACKARD
PART No. 9270-1024


Fig.B