# **EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE**

# **CERN-PS DIVISION**

PS/ AR/ Note 95-06 (MD)

# FIRST ELECTRON COOLING TESTS WITH Pb<sup>53+</sup> IONS (PERFORMED IN DECEMBER 1994)

 $\mathcal{O}(\mathcal{A}^{\mathcal{A}})$  and  $\mathcal{O}(\mathcal{A}^{\mathcal{A}})$  and  $\mathcal{O}(\mathcal{A}^{\mathcal{A}})$  and  $\mathcal{O}(\mathcal{A}^{\mathcal{A}})$ 

S. Baird, J. Bosser, M. Chanel, J. Duran, R. Giannini, P. Lefèvre, R. Ley, R. Maccaferri, S. Maury, I. Meshkov, D. Möhl, G. Molinari, F. Motsch, H. Mulder, U. Oeftiger, J.C. Perrier, E. Roux, G. Tranquille, F. Varenne

# *Contents*

- 1. Aims of the MD<br>2. Linac Beam
- Linac Beam
- 3. Transfer Line and Matching
- 4. LEAR lattices
- 5. Electron Cooler
- 6. Beam Diagnostics<br>7. Injection and Stac
- Injection and Stacking
- 8. Measured Beam Lifetime (With/Without Electron Cooling)<br>9. Measured Electron Cooling Time
- 9. Measured Electron Cooling Time
- 10. Conclusions

#### **1. AIM OF THE MD**

The aim of these tests was the first measurement of the beam lifetime and electron cooling time with  $Pb^{53+}$  as a function of different parameters:

- ion beam (intensity and initial emittances),
- electron cooling device (electron beam intensity, temperature and neutralisation situation),
- LEAR lattice  $(\beta_{x,z}$  and *D* in the electron cooling section).

This MD is the first of a series foreseen for this and coming years. The present configuration of LEAR did not allow these experiments to be performed in the hardware situation described in the project report [1], especially the multiturn injection which could not be tested as the necessary thin septum was not installed.

## **2. THE LINAC BEAM**

The Linac beam had the characteristics of the beam injected into the PSB during the preceding physics run:

Pb<sup>53+</sup> at 4.2 MeV/u  $(B\rho = 1.16$  Tm,  $p/Q = 347.5$  MeV/c/charge),

 $I = 20 \mu A$  *(dN/dt* = 2.3  $\times$  10<sup>6</sup> ions/us)

Pulse duration:  $T \sim 400 \text{ }\mu\text{s}$ 

The Linac pulse length had been optimised for injection into the PSB, but it was far too long for the present purpose (no head or tail clipper were installed). The pulse length required for monotum injection into LEAR, as used in this MD, is 3 µs; therefore, most of the incoming beam was lost in the machine (BHN10), leading to a local pressure bump. This reduced the beam lifetime, especially during the accumulation process where many consecutive pulses are injected. *A way to limit the Linac pulse length has to be studiedfor the next MD.*

## **3. TRANSFER LINE AND MATCHING**

The transfer line was set up with the values scaled from the Linac-LEAR proton transfer for matching the different LEAR lattices (see §4).

The set-up for 347.6 MeV/c/charge put the 106 degree bending magnets (EØ BHNØ2) and 03) close to their saturation limit. As the results obtained were not satisfactory, *a more sophisticated optimisation has to be done* that will take into account the saturation effects and non-linearities of the large bending elements.

The transmission during the MD was estimated to be 50%; *thisfigure has to be verified and significantly improved.*

## **4. LEAR LATTICES**

Three different optical settings had been prepared and tested with protons in previous runs (Fig. <sup>1</sup> and Table 1). "Machine 1" is the familiar optical setting used for antiprotons. "Machine 2" and "Machine 3" are (theoretically) optimised for electron cooling and/or multiturn injection.







Fig. <sup>1</sup> - Dispersion, theoretical curve and measurement points for the three machines tested

Parameters	Machine 1 $(D \# 0, \beta \text{ small})$	Machine 1 $(D = 0, \beta \text{ small})$	Machine 1 $(D = 0, \beta \text{ large})$
$Q_h$	2.315	2.46	1.8
$Q_v$	2.620	2.42	2.42
$1/\gamma_{tr}^2$	$-0.05$	0.125	0.127
$\boldsymbol{D}$ SS1/3	3.51	$\bf{0}$	$\bf{0}$
$\beta_h$ SS1/3	1.9	1.31	10.3
$\beta_{\nu}$ SS1/3	6.36	8.05	6.5
$\boldsymbol{D}$ SS2/4	3.51	9.77	9.6
$\beta_h$ SS2/4	1.9	9.63	6.35
$\beta_{\nu}$ SS2/4	6.36	11.88	14.8

Table <sup>1</sup> - Parameters of the three machines. The lattice functions [in m] are given for the centre of straight sections 1 and 3  $(SS1/3)$ and 2 and 4 (SS2/4). The electron cooler is in SS3.

## **5 . THE ELECTRON COOLER**

The electron current  $I_e$  is adjustable between 0 and 400 mA for grid control voltages varying between 0 and 11 kV.

The main difficulty is an instability which occurs when  $I_e \ge 100$  mA even when the primary ion beam is absent. This phenomenon results from the "natural" neutralisation due to the diameter difference of the vacuum chamber in the gun and interaction regions. This nonuniformity is inherent to the present cooler design. Ways to control the neutralisation (e.g. by a feedback system) are under study but not yet implemented. Therefore, artificial neutralisation has not been used since it would in the present situation worsen the instability conditions. We succeed in reducing the instability by means of a "blower" (which gates off the electron current for a few microseconds once every second, so that the trapped ions can escape) and by reducing the electron cooler magnetic field. With these measures currents up to 400 mA could be stabilised for a few seconds.

CERN/PS has undertaken a collaboration with Russian experts in order to better understand and fully control the neutralisation. Special tests to arrive at stable neutralisation will take place in 1995 and 1996.

## **6. SPECIAL BEAM DIAGNOSTICS AND MEASUREMENT TECHNIQUES**

## **6.1 Diagnostics in the Transfer Line**

Only one SEM grid located in the  $E_0$  loop (between Linac 3 and LEAR) was equipped with the "slow electronics" which permits the measurement of the ion beam. For the next MD, the other SEM grids have to be equipped with the appropriate electronics. It would be extremely useful to have a beam current transformer that works in the transfer line, but this is difficult due to the low current of the ion beam.

#### **6.2 Intensity and Emittance Measurement in LEAR**

Due to the small number of injected ions, it was not possible to use the beam current transformer except when we looked for multi-injection. The Schottky signals however were appropriate to monitor the evolution of the intensity as well as that of the transverse emittances and the momentum distribution.

- The pick-up UCMH40, which combines 12 loop couplers in a travelling wave connection, gives the longitudinal and horizontal Schottky signals.
- The pick-up UCV31, made up of 5 loop couplers connected in the travelling wave fashion, gives the vertical Schottky signals.

All the 50  $\Omega$  matching resistors and the head amplifiers are cooled to low temperature (60 - 80 K).

To measure the beam lifetime we use the longitudinal Schottky signal at harmonic 100 (36.068 MHz) and record by means of a spectrum analyzer set to zero frequency span the Schottky power versus time around the maximum of the power density of the distribution (Fig. 2). As the signal-to-noise ratio is very poor, we have subtracted the noise from the measurement. The Schottky signal is then proportional to  $S_0^2 = V_0^2 - N_0^2$  where  $V_0^2$  is the total signal and  $N_0^2$  the noise power. After a time equal to the lifetime, the Schottky power has decreased by  $1/e \approx 0.37$  and the voltage measured is  $V = \sqrt{0.37S_0^2 + N_0^2}$ . The lifetime with cooling is measured after the beam has reached equilibrium conditions.



Fig. 2 - Lifetime measurement using the longitudinal Schottky signal

To measure the momentum cooling time the longitudinal Schottky signal around harmonic 80 is recorded. The Schottky power density distribution is mixed down to 50 kHz. It is ac quired by a digitizing card mounted in a PC, or by the HP3687A fast Fourier analyzer. With a sampling frequency of 260 kHz, using 2048 points, taking into account the anti-aliasing system, 800 lines are available (bandwith 102.4 kHz) representing about 8 ms of data taking. For each of this time slice we subtract the noise power density and look for the momentum spread for 95% of the particles. For most of the measurements a bandwith of 51.2 kHz with a 200 line resolution and an averaging over two time slices gives one measurement every 8 ms (Fig. 3).

With the electron cooling it is possible to obtain very dense beams. Then coherent waves appear in the Schottky signal masking the true beam momentum distribution. To recover this momentum distribution, the beam transfer function is measured at the same time as the Schottky signal. A computer program has been developed on a PC in order to reconstruct the true momentum distribution from the joint measurement of the beam transfer function and the Schottky signal.



Fig. 3 - An example of a longitudinal cooling time measurement: evolution of  $(\delta p/p)$ 4 $\sigma$  [%o] during the first 180 ms of cooling. At the beginning the cooling time constant observed is 55 ms.

A horizontal Beam Ionisation Profile Monitor (BIPM) has been installed in the vacuum chamber of a bending magnet (BHN20) in LEAR. It observes ions (or electrons) created by the collisions between the beam and the rest gas. They are accelerated to an amplifier system made of Micro-Channel Plates (MCP). The final electrons are collected on a 30 mm wide grid with a resolution of  $\approx$ 1 mm. The system has been roughly calibrated with protons using the neutral hydrogen beam formed in the recombination of the protons with cooling electrons, which gives an "image" of the proton beam. The on-line acquisition software of the BIPM has been continuously improved during the MD. It records the evolution of the beam dimension for typically some seconds with a scan every 30 ms (Fig. 4).



Fig.4 - An example of the evolution of horizontal dimensions of the beam using the Beam Ionisation Profile Monitor. These pictures have been taken with "machine 1" and  $I_e = 47$ mA. A scan is taken every 30 ms. The figure gives the 1st, 5th, 10th, 20th, ..., 60th and 100th scan. The graph gives the evolution of the rms beam width (in mm) during 3000 ms.

Transverse Schottky signals were mainly used to verify the results of the beam profile monitor. The Schottky power density around the centre of a betatron sideband is proportional to the average emittance and can be used to monitor the evolution of the emittance (Figs. 5 and 6).



Fig. 5 - Horizontal cooling time measurement using the horizontal Schottky signal



Fig. 6 - Vertical cooling time measurement using the vertical Schottky signal

# 7. INJECTION AND STACKING

The injection and stacking methods were the same as those used for the oxygen run [2]. The cooled beam is bunched by the rf cavity on harmonic <sup>1</sup> of the revolution frequency. The incoming beam is injected (monotum) on the unstable phase of the rf. After debunching the whole beam is cooled and the process is repeated. We observed:



- The injected intensity is limited by the transmission (§3).
- The stacking is limited by the beam lifetime which is determined by the vacuum (§8). The vacuum is degraded during the stacking process because the loss of about 99% of the long linac pulse (§2) leads to an outgasing in the injection region.

## **8. BEAM LIFETIME MEASUREMENTS**

One can divide these measurements into two parts: those made without and those made with the electron beam. We give below a summary of the measurements. Note that all lifetimes (and later on all cooling time constants) are defined as the time interval during which the initial intensity (and later on the initial horizontal or vertical emittance or the momentum spread) decrease by  $1/e \approx 0.37$ .

## **8.1 Without Electron Cooling**

## *8.1.1. Measurements*

Without electron cooling the lifetime is determined by the charge exchange with the residual gas. With a single injection mode after a sufficiently long recovery period (5-10 min) a beam lifetime of 20 to 23 s was measured. With repeated injection (once per second) the lifetime dropped to 15-17 s.

### *8.1.2. Comparison with theory*

To compare these figures to theoretical values, the average vacuum pressure was worked out using the readings of 17 gauges distributed around the ring and of gas analysers installed in SS1, SS2 and SS4. Unfortunately, the analyser located in the electron cooling section (SS3) was not operational during the MD. From the measurements [3] we conclude that the average pressure was about  $1.1 \times 10^{-11}$  Torr with a gas composition of 95% H<sub>2</sub> and 5% heavier gases like  $N_2$ , CH<sub>4</sub>, H<sub>2</sub>O. From the charge exchange cross sections calculated from Franzke's formula [4], the corresponding beam lifetimes are estimated to be about 40 s (Table 2).

Table 2 - Cross section for charge exchange (Franzke's formula) for Pb<sup>53+</sup> at 4.2 MeV/u. For the calculation of the lifetime the LEAR vacuum is approximated by a gas composition with 95%  $H_2$  and 5%  $N_2$ .



**\* Composite cross section**

With frequent injection a large pressure bump occurred in the downstream end of SS<sup>1</sup> and in the adjacent bending magnet (BHN10) increasing the ring average pressure by almost a factor of 2. Then the estimated beam lifetime drops to 24 s (Table 2). One concludes from Table 2 that the theoretical (Franzke's formula) and the measured beam lifetime agree within a factor of  $\sim$ 2.

It should be mentioned that even with single injection, pressure bumps existed in SS2 where the jet target of the Jetset experiment had operated prior to the run, and in SS2 where a leak (on the valve VVS402) was only provisionally closed by a sealing spray. In addition the gas composition in the cooling region was not known. Thus the above estimate of the LEAR vacuum is subject to considerable uncertainty.

## **8.2 With Electron Cooling**

#### *8.2.1. Measurements*

With cooling a beam lifetime *(1/e)* (Fig. 7) between 12 and 2 s was measured depending on the electron current and -to a lesser extent- on the magnetic field in the cooling section and probably other "hidden" parameters like the stability and the alignment of the electron beam and the degree of neutralisation.

When the electron energy was mismatched (and therefore the ions and electrons did not move at the same velocities) the lifetime improved and approached the value measured without the electron beam. On the other hand, when the apparent longitudinal temperature is increased by modulating the electron energy around the nominal value (by up to 5 V at <sup>1</sup> kHz) the lifetime did not seem to improve by more than a factor of 1.4.



Fig. 7 - Measured lifetime for different electron currents. No strong dependence on the optics (machine 1-3) was observed. Points from the three "machines" are therefore drawn in the diagram. Note the "abnormal" point at 140 mA.

On first sight one concludes from these results that the loss mechanism depends strongly on the energy synchronisation but only weakly on the temperature of the electrons (to the extent that the modulation is equivalent to an increase of the effective temperature).

#### *8.2.2. Comparison with theory*

According to theory the lifetime with cooling is determined by recombination of Pb-ions with cooling electrons. The main capture mechanisms [5-7] known to be of importance here are radiative and dielectronic captures. Other process like multibody collision with electrons [8] or with secondary ions captured in the electron beam seems to be of less importance for the conditions of the experiment.

For Pb53+ no data are available as far as the dielectronic recombination is concerned. For the radiative capture (at an electron temperature  $T_{e\perp} = 0.2$  eV, as extrapolated from measurements with protons) one can estimate a rate coefficient  $<\!\sigma v\!\!> \approx 10^{-8}$  cm<sup>3</sup>/s leading for the conditions of the experiment to a beam lifetime of about 100 s at 400 mA electron current. The curve in Fig. 7 corresponds to a recombination coefficient  $\langle \sigma v \rangle \approx 5 \times 10^{-7}$  cm<sup>3</sup>/s, which is a factor of 50 bigger.

Measurements for various other ions are reported in the litterature [Appendix]. For none of them the dielectronic capture is by more than a factor of 10 faster than the radiative capture. The measurement points in Fig. 7 show a significant spread. These differences may come from parameters which were not investigated during the experiment. Systematic tests have to be pursued in order to obtain reliable results and, if possible, to find the conditions for a smaller recombination rate.

The main goal of our future studies and tests will be oriented towards increasing the lifetime.

#### **9. MEASUREMENTS OF THE COOLING TIME**

Measurements of the cooling time for the three different optics are given in Fig. 8 and in Tables 3,4 and 5.

## **9.1 Measurements on "Machine 1"**

Only measurements with an electron current up to 62 mA were done for this case (Table 3). The time to reach equilibrium (which should not be confused with the *1/e* cooling time constant) and the equilibrium emittance\* are shown in Figs. 8 and 9 as a function of the electron current. Again one notes a large "spread", especially in the equilibrium values.

## **9.2 Measurements with "Machine 2"**

<sup>\*</sup> Throughout in this note we use  $(k\sigma)^2/\beta_{h,v}$  as emittance definition where usually  $k = 1$  ("1 $\sigma$ -emittance").

Machine 1 with $D = 3.6$ m, $\beta_h = 1.9$ m, $\beta_v = 5.3$ m					
I-electron [mA]	Limit $\sigma$ <sub>H</sub> [mm]	Limit of H emittance [mm·mrad]	Time to reach equilibrium $(H$ -plane $)[s]$	<b>Emittance cooling</b> time constant	
24.8	4.65	2.10		$\tau_H \approx \tau_V \approx 0.5$ s	
29	2.51	0.61	2.3		
36.6	4.03	1.58	2.1		
44.7	3.9	1.48	1.9		
53.36	3.47	1.17	1.5		
62.5	3.11	0.94			

Table 3 - Measurements on Machine <sup>1</sup>



Fig. 8 - Time to reach the horizontal equilibrium versus electron current for "Machine 1". The initial cooling time constant *[(l/£)(d£/dt)]-1* is about 1/2 to 1/3 of time to reach equilibrium.



Fig. 9 - Horizontal emittance at equilibrium versus electron current for "Machine 1"

Machine 2 with $D = 0$ , $\beta_h = 1.3$ m, $\beta_v = 6.5$ m				
Electron current [mA]	Limit $\sigma_H$ [mm]	Limit of H emittance [mm·mrad]	Time to reach equilibrium $(H$ -plane $)[s]$	Solenoid field [Gauss]
44.7	3.8	3.0	0.8	
60	2.9	1.8	0.8	630 $(I_s = 400 \text{ A})$
90	$5.6*$	$6.5*$	0.8	
90	4.8	4.8	0.8	
126.5	3.8	3.0	0.8	
151	3.2	2.1	0.9	
204	2.8	1.6	$\mathbf{1}$	
232	2.7	1.5	0.7	
260	2.8	1.7	0.8	
300	2.7	1.6	0.8	420 $(I_s = 270 \text{ A})$
330	2.0	0.8	0.6	
330	2.4	1.2	0.7	
330	4.0	3.4	$\mathbf{1}$	
$330*$	2.1	0.9	0.7	
$410^{#}$	2.4	1.2	0.7	

Table 4 - Measurements with "Machine 2"

**\* electron energy modulated by ±2 × 10<sup>3</sup> at ~1 kHz**

**# blower on (to prevent neutralisation) .**



Fig. 10 - Emittance limit versus electron current for "Machine 2". For the first 3 points the solenoid field was  $630 \text{ G } (I_s = 400 \text{ A})$ , the other was  $420 \text{ G}$  $(I_s = 270 \text{ A}).$ 

Measurements (Table 4 and Fig. 10) were also taken with high electron currents. To maintain the electron beam stable, the solenoid current was reduced from 400 to around 270 A (solenoid field of 630 and 420 G, respectively). This reduction had two consequences:

- 1) The electron energy spread was larger, which explains the better electron beam stability but the cooling forces was also reduced.
- 2) The alignment between the two beams was not optimum even after the scaling of the two dipoles compensating the effect of the toroids.

The time needed to cool the beam dimension to its limits does not depend on the electron current, but the emittance limit and the initial cooling time constant decrease when the electron current is increased. The first two points of Fig. 10, which are taken with the nominal solenoid current, show a better emittance limit indicating that there was an effect of the solenoid current.

## **9.3 Measurements with "Machine 3"**

The change to "Machine 3" was done during the last three hours of the MD time. Thus only very preliminary results could be obtained. Table 5 shows two measurements with low electron current. The emittance limit seems to be very small already with this low electron current. But it appears that the horizontal distribution had a core with a very small dimension ( $\sigma$ )  $\sim$  1 mm) together with an important halo possibly indicating the beams are not perfectly aligned.

Machine 3 with $D = 0$ , $\beta_h = 10$ m, $\beta_v = 6.5$ m				
Electron current [mA]	Limit $\sigma_H$ [mm]	Limit of H emittance [mm·mrad]	Time to reach equilibrium $(H\text{-plane})$ [s]	Comments
45	3.4	1.8	0.9	$\tau_H \approx 0.3$
55	3.03	1.4	0.75	$\tau_H \approx 0.2$

Table 5 - Two measurements made on "Machine 3"

#### **9.4 Some Remarks Common to All Machines**

The cooling time is strongly dependent on the initial beam dimension. For "Machine 1" and 3, most of these measurements start with an unknown beam dimension which is larger than acceptance of the BIPM. In the case of "Machine 2", it seems the injection matching was better (and the Twiss parameter favourable at the BIPM) and we can estimate a profile of  $\sigma$  ~ 10 mm corresponding to an emittance  $\varepsilon(1\sigma) \sim 20$  mm mrad. Then a factor 10 in emittance can be achieved in less than <sup>1</sup> s and the initial horizontal cooling time constant is of the order of 300 ms. But looking to the sum of the charges collected on all the wires of the BIPM, we find that about half of the ions have been lost during that second of cooling. Some of these ions are lost by electron capture but it does not explain entirely the losses.

In all cases, the longitudinal cooling down time was much less than 0.1 s, starting with  $Dp/p \sim 0.001$  and the time constant was of the order of 30 ms to 80 ms, depending on cooler settings.

The beam lifetime (§8.2) did not (or only very weakly) depend on the optics of the ring.

#### **CONCLUSIONS**

The first tests with lead ions in LEAR have demonstrated that a momentum cooling time constant of less than 50 ms and a transverse emittance cooling time constant of less than 300 ms can be already reached with the present state of the cooler. However, the lifetime of the beam decreased to 2 s in the presence of an electron beam current of 400 mA. This effect has to be carefully studied in the next sessions.

The short-term hardware improvements indicated along this note will be prepared for the next MD sessions. Studies will be made with protons in March 1995 especially on the electron cooling stability. Sessions with lead ions will be requested for June, October and for the end of the physics run in December 1995.

The situation will be improved by the following measures:

- An improved transfer line set-up.
- A shorter Linac pulse.
- The use of  $Pb^{52+}$  and  $Pb^{54+}$  in addition to  $Pb^{53+}$ , to investigate the recombination effect.

The first tests of the multi-turn injection using the fast bumpers prepared for December 1994 but not used due to lack of time, will be made in June.

#### **ACKNOWLEDGEMENTS**

We would like to thank our colleagues from the Linac, RF, Kicker and Vacuum Groups, and the LEAR Operation technicians for their special efforts during the MD.

#### **REFERENCES**

- [1] P. Lefèvre and D. Möhl, *A Low Energy Accumulation Ring of Ions for LHC (A Feasibility Study),* CERN/PS 93-62 (DI) and LHC Note 259, 1993.
- [2] S. Baird, J. Bosser, M. Chanel, C. Fernandez Figueroa, C. Hill, P. Lefèvre, R. Ley, D. Manglunki, D. Möhl, G. Molinari, T. Pettersson, P. Tétu, G. Tranquille, and D.J. Williams, *Oxygen Ions in LEAR,* Procs, of EPAC 90, Nice, p. 580, and CERN/PS 90-28 (OP), 1990.
- [3] M. Brouet, *Pression Moyenne LEAR*/16 déc. 1994, private communication.
- [4] B. Franzke, *Vacuum Requirements for Heavy Ion Synchrotrons,* Procs, of the 1981 Part. Acc. Conf., IEEE Trans. Nucl. Sei., Vol. NS-28, No.3, June 1981, p. 2116.
- [5] A. Müller et al., *Recombination of Free Electrons with Ions,* Physica Scripta, Vol. T37, 62-65, 1991.

14

- [6] M. Bell and J.S. Bell, *Capture of Cooling Electrons by Cool Protons,* Particle Accel., 1982, Vol. 12, p. 49.
- [7] D. Möhl and K. Kilian, *Phase-Space Cooling of Ion Beams,* CERN/EP 82-214 and Lecture Notes in Physics, 178, p. 220, 1983.
- [8] H.F. Beyer, O. Guzman and D. Liesen, *On the Total Recombination Between Cooling Electrons and Heavy Ions,* Particle Accel., 1989, Vol. 24, p. 163.

## APPENDIX

Measurements of the recombination rate coefficient for various ions are reported in the literature. They are compiled in Table Al below. Included is the corresponding lifetime for an average density  $\eta n_e = 10^6$  electrons/cm<sup>3</sup>, which corresponds to the LEAR case with  $\eta \approx 0.02$ and  $n_e = 5 \times 10^7$  electrons/cm<sup>3</sup>. Here  $\eta$  is the ratio of the cooling length to the LEAR circumference. The electron beam density in the cooler,  $n_e = I_e / \pi b^2 e \beta c$ , takes the assumed value of  $5 \times 10^7$  for a current  $I_e$  of 440 mA (with  $b = 2.5$  cm beam radius and  $\beta = 0.095$ ).

## Sheet<sup>1</sup>

