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DECELERATION AND COLLIDER MODE FOR THE FERMILAB ANTIPROTON ACCUMULATOR

S. Maury and M. Church

The lattice of the Accumulator [1] has been designed to accept the injection of antiprotons every few seconds at a kinetic energy of 8 GeV. These are then momentum stacked and stochastically cooled. The antiprotons are accumulated over a period of several hours to get a dense core of antiprotons prior to extracting a high intensity beam towards the Main Ring and Tevatron (Fig. 1). The parameters of the Accumulator ring on the central orbit are shown in Table 1. The lattice parameters for one sector of the ring are shown in Table 6.

Table 1

Momentum	8.816 GeV/c
γ	9.449
β	0.9944
Circumference	474.062 meters
Revolution frequency	0.628840 MHz
Kinetic energy	7.9275 GeV
γ_t, transition energy	5.43
$\eta = (1/\gamma_t^2)-(1/\gamma^2)$	0.023

In addition to the standard operation of mode described above, we study the possiblity of decelerating the beam in the Accumulator down to the lowest momentum technically acceptable. We also study the collider mode, proton-antiproton, in the accumulator.

I. DECELERATION

The Accumulator has been operated successfully as low as 3.6 GeV/c. The passage through transition is accomplished with a γ_t jump of 1.2 units (Fig. 2). The deceleration is accomplished in several steps, to allow for stochastic cooling in all three dimensions to counteract transverse and longitudinal emittance blow-up. A deceleration efficiency to

3.6 GeV/c of 90% has been achieved routinely. From the current vs. momentum curves of previous deceleration, we can extrapolate to 2 GeV/c:

Table 2

Bus	Current (A)
A:IB	254.3
A:QDF	53.6
A.LQ	287.0
A:QT	53.3

The deceleration process has been done with an rf system on harmonic 2 with about 2.5 kV. The Accumulator parameters on the central orbit at 2 GeV/c are listed in Table 3.

Table 3

Momentum	2.00 GeV/c
γ	2.354
β	0.9053
Circumference	474.062 meters
Revolution frequency	0.572520 MHz
Kinetic energy	1.2709 GeV
γ_t, transition energy	4.43
$\eta = (1/\gamma_t^2)-(1/\gamma^2)$	-0.129

Where we have distorted the lattice (1 unit in γ_t,) to give a more favourable η for rf manipulations. At 2 GeV/c several issues may be of concern:

Stochastic Cooling: At 3.6 GeV/c the primary limitation to stochastic cooling has been found to be due to beam instabilities caused by trapped ions and longitudinal impendance (Keil-Schnell criterion). We expect this situation to be similar at 2.0 GeV/c. In cooling systems that are not gain-limited and not noise-limited (Accumulator core systems) the cooling rate is determined by the machine parameter η. At 2 GeV/c this parameter will be 4 times larger that at 3.6 GeV/c (in favour of better cooling). Adversely, the bad mixing (mixing from pick-up to kicker) will be somewhat worse. The momentum band will be limited to approximately $\delta p/p = 0.75 \times 10^{-3}$ (95% full width) for effective transverse cooling in the 4-8 GHz band.

Power Supplies: Power supplies will need to be tested for stability and regulation at these low currents. At this time, there is no reason to believe they will not well regulated.

rf: In the future there will be 5 kV of H = 2 rf available for bunching the beam for deceleration above 4 GeV/c. At 2 GeV/c only about half this will be available due to the fact that the cavities will be off resonance. This is adequate to bunch the beam provided that dc beam can be cooled to $\delta p/p = 2.1 \times 10^{-3}$ (95% full width). In operation, typical beam widths at 3.6 GeV/c have been $\delta p/p = 0.6 \times 10^{-3}$. Variable tuning capacitors could be installed on the cavities if required to increase the rf voltage at low energies.

II. COLLIDER MODE

In this mode, 2×10^{11} antiprotons are stacked and cooled in about 4 hours. Then, 2×10^{11} protons are injected in the opposite direction using the current antiproton extraction line (AP3). We choose 2×10^{11} because this is the number of protons that can be injected into the Accumulator in a single shot, and we choose to use equal amounts of protons and antiprotons. The main parameters to know in this option are the tune shift and luminosity. To get an upper limit on the luminosity, we assume head-on collisions of the proton and antiproton bunches. We consider a round Gaussian beam [2,5] with n particles per unit length and with a density distribution

$$\rho(r) = \frac{ne}{2\pi\sigma^2} e^{-r^2/2\sigma^2} .$$

The Lorentz force on one particle at a radius r is:

$$\vec{F} = e\left(\vec{E} + \vec{v} \wedge \vec{B}\right)$$

$$\vec{F}_r = e\left(E_r \pm \beta c B_\phi\right)\vec{r}$$

The positive sign corresponds to a particle in the other beam and the negative sign to a particle in the same bunch. The radial electric field E_r and the magnetic induction B_ϕ can be obtained from Gauss' theorem and Ampere's law, respectively:

$$2\pi r E_r = \frac{1}{\varepsilon_0}\int_0^r 2\pi r' \rho(r') dr'$$

$$2\pi r B_\phi = \mu_0 \int_0^r 2\pi r' \beta c \rho(r') dr' .$$

Then,

$$F_r = \frac{ne^2}{2\pi r \varepsilon_0}\left(1 \pm \beta^2\right)\left(1 - e^{-r^2/2\sigma^2}\right)$$

The intrabeam scattering, Coulomb collisions between particles in the same bunch (negative sign), is counteracted by the stochastic cooling. For particles in the other beam, the effect of electric and magnetic fields is additive. We define an equivalent magnetic field B_{eq} which gives the same force:

$$B_{eq} = \frac{F_r}{e\beta c} = \frac{E_r}{\beta c} + B_\phi$$

$$= \frac{ne}{2\pi r \varepsilon_0 \beta c}\left(1+\beta^2\right)\left(1-e^{-r^2/2\sigma^2}\right).$$

The linear tune shift $\Delta\nu$ is given by:

$$\Delta\nu = \frac{1}{4\pi} K \beta^* \frac{l_B}{2}$$

where

$$K = \frac{1}{B_{eq}\rho}\frac{\partial B_{eq}}{\partial r}$$

where β^* is the beta function at the interaction point and $l_B/2$ is the effective length of the interaction which is taken to be equal to half the bunch length. Putting

$$B_{eq}\rho = \frac{mc\beta\gamma}{e} \text{ and } r_p = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 1.535\times10^{-18}\text{ m}$$

gives

$$\Delta\nu = \frac{N}{2B}\frac{r_p}{4\pi\sigma^2}\frac{\left(1+\beta^2\right)}{\beta^2\gamma}\beta^*$$

where N is the total number of particles in the beam, B is the number of bunches. We can express the tune shift in terms of the measured emittances containing 95% of the beam. Putting $\beta^* \varepsilon_{95\%} = 6\pi\sigma^2$, gives:

$$\Delta\nu = \frac{3}{2}\frac{N}{B}\frac{r_p}{\varepsilon_{95\%}}\frac{\left(1+\beta^2\right)}{2\beta^2\gamma}.$$

The luminosity per bunch [3] is defined as the product of the number of particles per bunch (N_B) of the two beams per cross-sectional area:

$$L_B = \frac{N_B^2}{4\pi\sigma^2} = \frac{3}{2}\frac{N_B^2}{\beta^*\varepsilon_{95\%}}.$$

The average luminosity is the luminosity per bunch times the collision frequency:

$$L = \frac{3}{2}\frac{N^2 f_{rev}}{B\beta^*\varepsilon_{95\%}}$$

We can write the average luminosity as a function of the tune shift:

$$L = N\frac{f_{rev}}{r_p}\frac{2\beta^2\gamma}{\left(1+\beta^2\right)}\frac{\Delta\nu}{\beta^*}.$$

The parameters of the Accumulator including the tune shift and the luminosity are listed in Table 4.

Table 4

	8.8 GeV/c	2.0 GeV/c
N (total number of particles)	2×10^{11}	2×10^{11}
B (number of bunches)	84	84
$\varepsilon_{95\%}$ (transverse emittance)	$0.2 \times 10^{-6}\ \pi$	$0.2 \times 10^{-6}\ \pi$
β	0.9944	0.9053
γ	9.449	2.355
f_{rev} (revolution frequency)	0.628840 MHz	0.572520 MHz
β^* (effective beta function)	8.0 m	8.0 m
$\Delta\nu$ (tune shift)	0.00093	0.0042
L (average luminosity)	0.9×10^{28} cm^{-2}s^{-1}	0.8×10^{28} cm^{-2}s^{-1}

Measured emittances versus stack size are shown in Figs. 3 and 4.

III. RF SYSTEMS

The bucket area per bunch as a function of rf voltage is [4]:

$$A = 16\frac{E}{H\omega_{rev}}\left(\frac{\beta^2}{H\eta}\frac{eV_{peak}}{2\pi E}\right)^{1/2}.$$

We also know that for debunched beam the phase-space area is:

$$A = \beta^2\frac{E}{f_{rev}}\frac{\delta p_{95\%}}{p}.$$

So, the voltage needed for a full bucket (*bf* = 1) is:

$$eV_{peak} = \frac{\pi^3}{32}H\eta B^2\left(\frac{\delta p_{95\%}}{p}\right)^2 E.$$

The voltage required for a bunch factor less than 1 is obtained from numerical integration of the longitudinal difference equations for particle motion in an rf bucket. The Accumulator parameters needed to bunch the beams are listed in Table 5. For 8.816 GeV/c $\delta p_{95\%}/p$ is extrapolated from data on σ_p from Fig. 5. For 2.0 GeV/c $\delta p_{95\%}/p$ is estimated from measurements at 3.6 GeV/c.

Table 5

		8.816 GeV/c	2.0 GeV/c
H	(harmonic number	84	84
$\delta p_{95\%}/p$		0.0008	0.0006
β		0.9944	0.9053
γ		9.449	2.355
E	(beam energy)	8.866 GeV	2.210
η		0.023	-0.129
A	(phase-space area)	11.1 eV·sec	1.9 eV·sec
eV_{peak}	(rf voltage for $bf = 1$)	10.5 kV	6.9 kV
eV_{peak}	(rf voltage for $bf = 1/2$)	56 kV	43 kV

IV. CONCLUSIONS

In the collider mode, new core stochastic cooling systems will be required to cool the proton beam. Space is available at A20 for the momentum pick-up and space is available at A50 for a momentum kicker. For transverse cooling, one option is the following: move the Schottky pick-ups at A10 and install two new transverse pick-up tanks; remove two stack-tail kicker tanks (thereby compromising the stacking rate somewhat) and replace them with two transverse kicker tanks at A30. The cost of these new systems will be about .75 M$.

Two new H=84 rf cavities would need to be installed in the 50 sector of the Accumulator for rf manipulations of the protons. Space is presently available. We estimate the cost of this system at 1M$. The present system is capable of running dc at about 30 kV. It is not practical to reach much higher than this without installing additional cavities - for which there is not space in the lattice. Hence, it is probably not practical to run in collider mode with bunch factors any smaller than about 1/2.

In calculating the tune shift, we have assumed the beams only cross at one point (A50). This implies the use of electrostatic separators to separate the beam at the other zero dispersion regions (A10 and A30). We have not investigated such a system in detail, but estimate the cost at 1.5 M$. At the normal Accumulator operating point (tunes at 0.609, 0.607) a tune shift of about 0.005 could be tolerated.

Luminosity could probably be increased (perhaps a factor of 5?) by the addition of a low beta insert at A50, although we have not considered this option in detail. However, the relatively long bunch length spreads the interaction region away from the β_{min} point and makes the low beta insertion much less effective than it would be with a short bunch length. The luminosity in the detector should also be precised, especially in the case where the length of the detector is shorter than the bunch length.

REFERENCES

1. *Design Report, Tevatron I Project,* Sept. 1994.

2 L.R. Evans and J. Gareyte, *Beam-beam Effects,* in CERN 87-03 (1987), p. 159.

3. D.A. Edwards and M.J. Syphers, *An Introduction to the Physics of Particle Accelerators,* in "Physics of Particle Accelerators", p. 13.

4. G Dome, *Theory of RF Acceleration,* in CERN 87-03 (1987), p. 110.

5. D. Siergiej, private communication.

Table 6

Accumulator Reference. Central Orbit.
*** CYC -3 // R/6

BETATRON FUNCTIONS THRU R/6

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POS		S(M)	NUX	NUY	BETAX(M)	BETAY(M)	XEQ(M)	YEQ(M)	ZEQ (M)	ALPHAX	ALPHAY	DXEQ	DYEQ
0		0.0000	0.00000	0.00000	7.36998	5.92350	-0.00740	0.00000	0.0000	0.00000	0.00000	0.00000	0.00000
1	A10	0.0000	0.00000	0.00000	7.36998	5.92350	-0.00740	0.00000	0.0000	0.00000	0.00000	0.00000	0.00000
2	BP1	7.7065	0.12855	0.14570	15.42844	15.94978	-0.00740	0.00000	0.0000	-1.04567	-1.30101	0.00000	0.00000
3	LS	7.9465	0.13099	0.14805	15.93809	16.58389	-0.00740	0.00000	0.0000	-1.07822	-1.34152	0.00000	0.00000
4	Q1	8.2665	0.13415	0.15101	16.04701	18.09777	-0.00727	0.00000	0.0000	0.74206	-3.44654	0.00084	0.00000
5	Q1	8.5866	0.13742	0.15363	15.01122	21.10482	-0.00686	0.00000	0.0000	2.45467	-6.06407	0.00166	0.00000
6	01	9.0990	0.14334	0.15700	12.61849	27.78938	-0.00602	0.00000	0.0000	2.21485	-6.98118	0.00166	0.00000
7	Q2	9.7548	0.15217	0.16037	11.66526	32.69482	-0.00537	0.00000	0.0000	-0.68741	-0.11009	0.00034	0.00000
8	Q2	10.4106	0.16038	0.16373	14.61299	28.04939	-0.00556	0.00000	0.0000	-4.03597	6.82548	-0.00093	0.00000
9	BP2	11.2618	0.16788	0.16982	22.34020	17.65981	-0.00636	0.00000	0.0000	-5.04294	5.38152	-0.00093	0.00000
10	02	11.3712	0.16864	0.17084	23.45858	16.50181	-0.00646	0.00000	0.0000	-5.17247	5.19578	-0.00093	0.00000
11	Q3	11.7218	0.17088	0.17458	26.10405	13.70064	-0.00664	0.00000	0.0000	-2.26366	2.91268	-0.00011	0.00000
12	Q3	12.0723	0.17298	0.17891	26.54002	12.29700	-0.00653	0.00000	0.0000	1.03821	1.15040	0.00072	0.00000
13	03	12.9765	0.17860	0.19167	24.72654	10.37110	-0.00588	0.00000	0.0000	0.96741	0.97956	0.00072	0.00000
14	B3	14.5005	0.18900	0.21884	21.96298	7.75674	-0.06168	0.00000	0.0015	0.84825	0.72937	0.08805	0.00000
15	BP3	20.8227	0.24692	0.39536	14.36669	6.42861	0.61833	0.00000	0.0015	0.35327	-0.51930	0.08805	0.00000
16	OB3	20.9241	0.24804	0.39785	14.29585	6.53696	0.62725	0.00000	0.0015	0.34533	-0.53932	0.08806	0.00000
17	Q4	21.1527	0.25062	0.40329	13.90126	6.90790	0.64198	0.00000	0.0015	1.37096	-1.09696	0.04061	0.00000
18	Q4	21.3813	0.25331	0.40834	13.05646	7.55047	0.64577	0.00000	0.0015	2.30355	-1.72990	-0.00753	0.00000
19	BP3*	21.4827	0.25457	0.41043	12.59426	7.90673	0.64500	0.00000	0.0015	2.25457	-1.78352	-0.00753	0.00000
20	04	24.6423	0.33793	0.44691	3.16904	24.45600	0.62122	0.00000	0.0015	0.72847	-3.45426	-0.00753	0.00000
21	Q5	25.0564	0.36029	0.44950	2.80939	25.94525	0.63568	0.00000	0.0015	0.15649	-0.07490	0.07769	0.00000
22	Q5	25.4704	0.38368	0.45209	2.90003	24.57543	0.68616	0.00000	0.0015	-0.37952	3.32105	0.16730	0.00000
23	HT5	32.3087	0.52594	0.65971	26.53787	2.04436	1.83023	0.00000	0.0015	-3.07717	-0.02620	0.16730	0.00000
24	BP4	32.7087	0.52823	0.69030	29.06273	2.14364	1.89715	0.00000	0.0015	-3.23497	-0.22200	0.16730	0.00000
25	05	32.8181	0.52883	0.69833	29.77578	2.19811	1.91547	0.00000	0.0015	-3.27816	-0.27559	0.16730	0.00000
26	Q6	33.1687	0.53065	0.72216	30.88203	2.54684	1.93542	0.00000	0.0015	0.16447	-0.73258	-0.05382	0.00000
27	Q6	33.5192	0.53249	0.74172	29.55131	3.25294	1.87799	0.00000	0.0015	3.58105	-1.30871	-0.27279	0.00000
28	06	37.7063	0.57706	0.81288	7.76397	28.83321	0.73576	0.00000	0.0015	1.62232	-4.80051	-0.27279	0.00000
29	Q7	38.0569	0.58470	0.81473	6.97214	31.05965	0.65441	0.00000	0.0015	0.66705	-1.46547	-0.19290	0.00000
30	Q7	38.4074	0.59286	0.81652	6.80329	30.83299	0.59962	0.00000	0.0015	-0.17888	2.10337	-0.12082	0.00000
31	07	41.2550	0.65196	0.83468	9.05210	20.28039	0.25557	0.00000	0.0015	-0.61084	1.60242	-0.12082	0.00000
32	S7	41.5598	0.65721	0.83713	9.43856	19.31990	0.21874	0.00000	0.0015	-0.65707	1.54880	-0.12082	0.00000
33	OS7	42.8528	0.67711	0.84900	11.39143	15.60872	0.06252	0.00000	0.0015	-0.85322	1.32132	-0.12082	0.00000
34	B7	45.9008	0.71111	0.89078	17.96127	8.83824	-0.03856	0.00000	-0.0042	-1.31323	0.86604	0.05416	0.00000
35	OB7	46.4088	0.71545	0.90039	19.33466	8.00944	-0.01105	0.00000	-0.0042	-1.39029	0.76546	0.05416	0.00000
36	Q8	46.6374	0.71731	0.90501	19.64291	7.80334	0.00139	0.00000	-0.0042	0.04953	0.14124	0.05452	0.00000
37	Q8	46.8660	0.71917	0.90966	19.28988	7.87882	0.01380	0.00000	-0.0042	1.48598	-0.47328	0.05395	0.00000
38	BP5	46.9374	0.71976	0.91110	19.07853	7.94719	0.01765	0.00000	-0.0042	1.47411	-0.48437	0.05395	0.00000
39	08	48.0852	0.73026	0.93245	15.91368	9.26379	0.07958	0.00000	-0.0042	1.28321	-0.66269	0.05395	0.00000
40	B8	52.6572	0.79715	0.99050	7.71130	17.73433	0.91849	0.00000	0.1009	0.53149	-1.12616	0.31726	0.00000
41	OB8	53.1652	0.80799	0.99492	7.21422	18.91151	1.07966	0.00000	0.1009	0.44700	-1.19113	0.31726	0.00000
42	Q9	53.3938	0.81308	0.99682	7.14101	19.13320	1.16169	0.00000	0.1009	-0.12488	0.22695	0.40143	0.00000
43	Q9	53.6224	0.81812	0.99874	.32973	18.70637	1.26372	0.00000	.1009	-0.70540	1.62950	0.49252	0.00000
44	09	53.9272	0.82455	1.00141	.77872	17.73118	1.41384	0.00000	.1009	-0.78768	1.56994	0.49252	0.00000

Table 7

BETATRON FUNCTIONS THRU R/6

POS		S(M)	NUX	NUY	BETAX(M)	BETAY(M)	XEQ(M)	YEQ(M)	ZEQ (M)	ALPHAX	ALPHAY	DXEQ	DYEQ
45	S9	54.2320	0.83060	1.00422	8.26568	16.79229	1.56396	0.00000	0.1009	-0.82995	1.51038	0.49252	0.00000
46	OS9	54.8416	0.84164	1.01033	9.35348	15.02344	1.86420	0.00000	0.1009	-0.95450	1.39127	0.49252	0.00000
47	B9	59.4136	0.89247	1.09174	22.15630	5.80695	4.68542	0.00000	0.9369	-1.87801	0.55611	0.75582	0.00000
48	OB9	59.9216	0.89597	1.10633	24.11708	5.30109	5.06938	0.00000	0.9369	-1.98180	0.44067	0.75582	0.00000
49	Q10	60.1502	0.89746	1.11330	24.85497	5.14898	5.22358	0.00000	0.9369	-1.23825	0.22635	0.59246	0.00000
50	Q10	60.3788	0.89891	1.12042	25.24387	5.09311	5.33993	0.00000	0.9369	-0.45886	0.01862	0.42481	0.00000
51	BP6	60.4002	0.89904	1.12109	25.26353	5.09241	5.34902	0.00000	0.9369	-0.45989	0.01441	0.42481	0.00000
52	O10	60.8868	0.90208	1.13627	25.72244	5.12489	5.55573	0.00000	0.9369	-0.48322	-0.08116	0.42481	0.00000
53	B10	65.4588	0.92764	1.24927	31.07041	9.59989	8.07093	0.00000	2.7046	-0.69997	-0.86389	0.68812	0.00000
54	OB10	65.7507	0.92912	1.25398	31.48314	10.11972	8.27179	0.00000	2.7046	-0.71397	-0.91699	0.68812	0.00000
55	S10	66.0555	0.93065	1.25865	31.92283	10.69562	8.48153	0.00000	2.7046	-0.72859	-0.97243	0.68812	0.00000
56	OS10	66.3474	0.93210	1.26288	32.35226	11.27882	8.68239	0.00000	2.7046	-0.74258	-1.02553	0.68812	0.00000
57	Q11	66.7843	0.93427	1.26870	31.14717	12.91135	8.72986	0.00000	2.7046	3.44751	-2.78312	-0.47184	0.00000
58	Q11	67.2212	0.93666	1.27353	26.55764	16.33239	8.27408	0.00000	2.7046	6.85411	-5.19812	-1.60459	0.00000
59	BP7	67.2943	0.93711	1.27423	25.56495	17.10174	8.15676	0.00000	2.7046	6.72201	-5.32357	-1.60459	0.00000
60	O11	67.7422	0.94027	1.27788	19.90580	22.21478	7.43806	0.00000	2.7046	5.91284	-6.09201	-1.60459	0.00000
61	Q12	68.1283	0.94370	1.28042	16.38860	26.04034	6.98274	0.00000	2.7046	3.33369	-3.66633	-0.76286	0.00000
62	Q12	68.5144	0.94770	1.28269	14.60056	27.70782	6.84453	0.00000	2.7046	1.36699	-0.58711	0.04424	0.00000
63	O12	68.7322	0.95012	1.28394	14.01441	27.96586	6.85416	0.00000	2.7046	1.32420	-0.59768	0.04424	0.00000
64	S12	69.0370	0.95369	1.28566	13.22543	28.33471	6.86765	0.00000	2.7046	1.26431	-0.61247	0.04424	0.00000
65	OS12	69.2548	0.95636	1.28688	12.68402	28.60381	6.87728	0.00000	2.7046	1.22152	-0.62304	0.04424	0.00000
66	Q13	69.6409	0.96132	1.28904	12.32435	27.80294	7.05065	0.00000	2.7046	-0.27598	2.66590	0.85721	0.00000
67	Q13	70.0270	0.96618	1.29137	13.12321	24.60965	7.54422	0.00000	2.7046	-1.82418	5.47962	1.70911	0.00000
68	O13	70.5242	0.97182	1.29499	15.01879	19.47218	8.39402	0.00000	2.7046	-1.98815	4.85276	1.70911	0.00000
69	Q14	70.8465	0.97512	1.29781	15.83790	17.03619	8.80915	0.00000	2.7046	-0.53445	2.80769	0.86812	0.00000
70	Q14	71.1668	0.97835	1.30094	15.69142	15.78778	8.94898	0.00000	2.7046	0.98554	1.11820	0.00000	0.00000
71	BP8	71.2765	0.97946	1.30205	15.47867	15.54639	8.94898	0.00000	2.7046	0.97188	1.10271	0.00000	0.00000
72	LS*	79.0117	1.10219	1.43482	7.96000	7.01563	8.94898	0.00000	2.7046	0.00000	0.00000	0.00000	0.00000
73	A20	79.0117	1.10219	1.43482	7.96000	7.01563	8.94898	0.00000	2.7046	0.00000	0.00000	0.00000	0.00000
74	REFL	158.0234	2.20438	2.86963	7.36998	5.92350	-0.00740	0.00000	5.4092	0.00000	0.00000	0.00000	0.00000

```
CIRCUMFERENCE =    474.0703 M        THETX =   6.28318542 RAD      NUX =    6.61313      DNUX/(DP/P) =   -8.42371
       RADIUS =     75.4506 M        THETY =   0.00000000 RAD      NUY =    8.60890      DNUY/(DP/P) =  -12.74768
 (DS/S)/(DP/P)=  0.0342301            TGAM=(   5.40500,     0.00000)

MAXIMA   --- BETX(  56) =   32.35226   BETY(   7) =   32.69482   XEQ(  71) =    8.94898   YEQ(  74) =   0.00000
MINIMA   --- BETX(  21) =    2.80939   BETY(  23) =    2.04436   XEQ(  34) =   -0.03856   YEQ(  74) =   0.00000
-----------------------------------------------------------------------------------------------------------------

***        FIN          // CORE USE SUMMARY                         MAXIMUM          USED     UNUSED
                           INFF   (ELEMENT DEFINITIONS)           4000(INFMAX)       128       3872
                           FLIB  (F.P. DATA AND STORAGE)          5000(IFLMAX)       206       4794
                           ILIB  (INTEGER DATA)                   1000  (IMAX)         3        997
                           CHLIB(CHARACTER DATA)                 15000(ICHMAX)       265      14735
                           SFLIB(F.P.CHARACTER DATA)              5000(ISFMAX)       206       4794
                           LQFIL(CALCULATED DATA)                60000 (IQMAX)      2169      57831
   ARRAY DIMENSIONS ARE SET IN COMMON FILES BINFF.CCC, BSTORE.CCC, CSTORE.CCC

=================================================================================================================
END OF SYNCH RUN
```

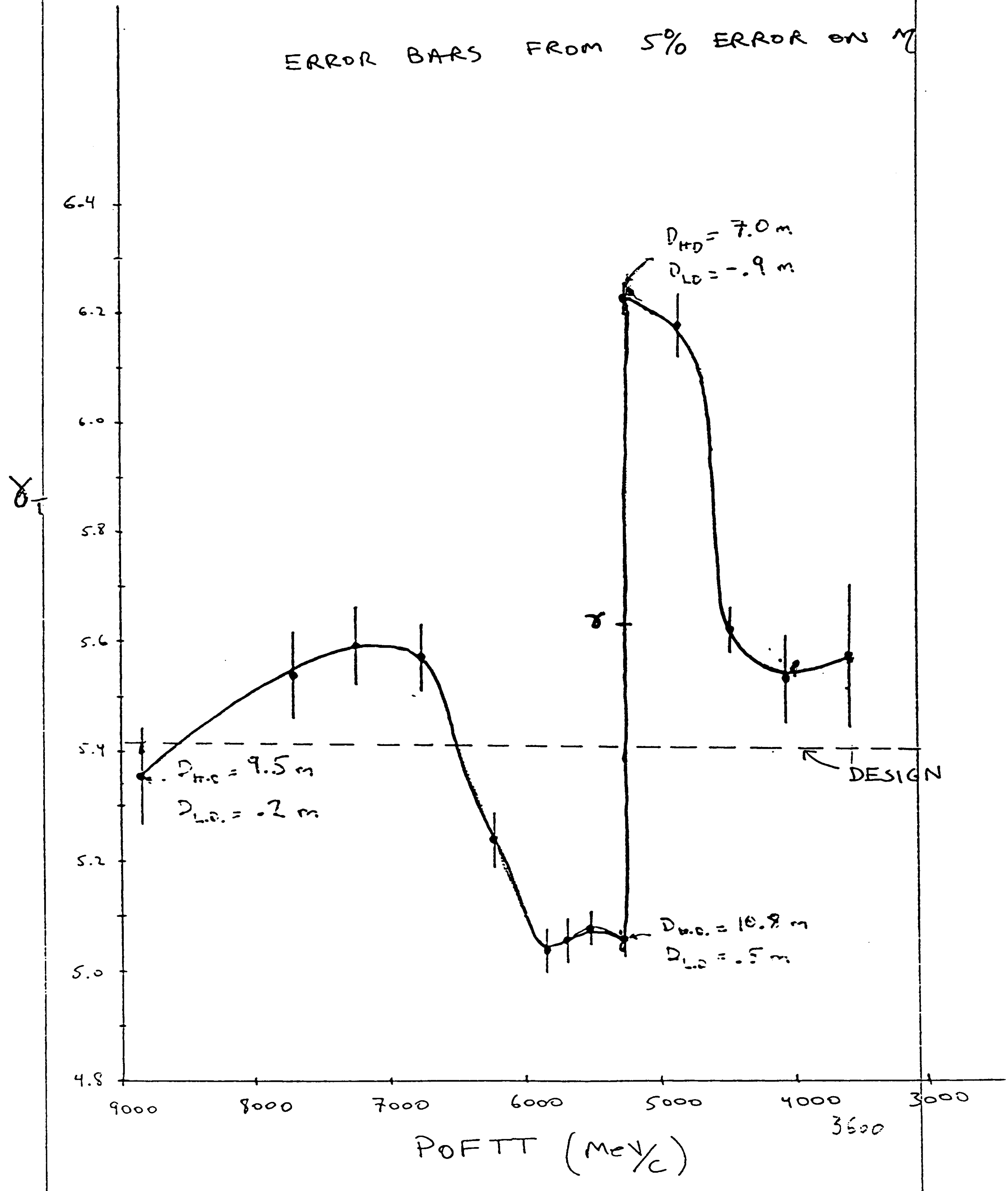

ERROR BARS FROM 5% ERROR ON M
6.4
6.2
6.0
5.8
5.6
5.4
5.2
5.0
4.8
γ_T
D_{HD} = 7.0 m
D_{LD} = -.9 m
γ
DESIGN
$D_{H.D.}$ = 9.5 m
$D_{L.D.}$ = .2 m
$D_{H.D.}$ = 10.8 m
D_{LD} = .5 m
9000
8000
7000
6000
5000
4000
3000
3600
POFTT (MeV/c)

Figure 2

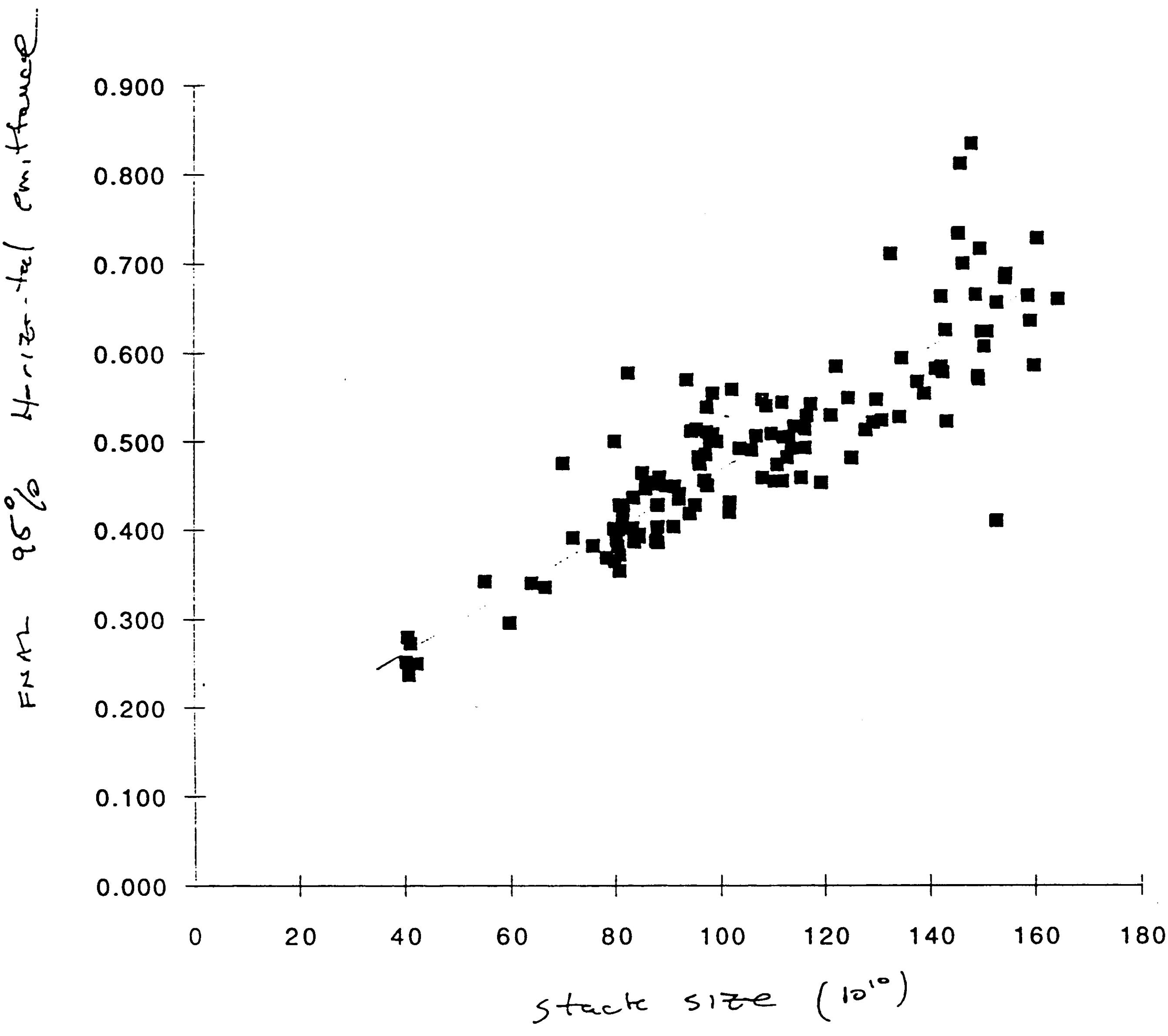
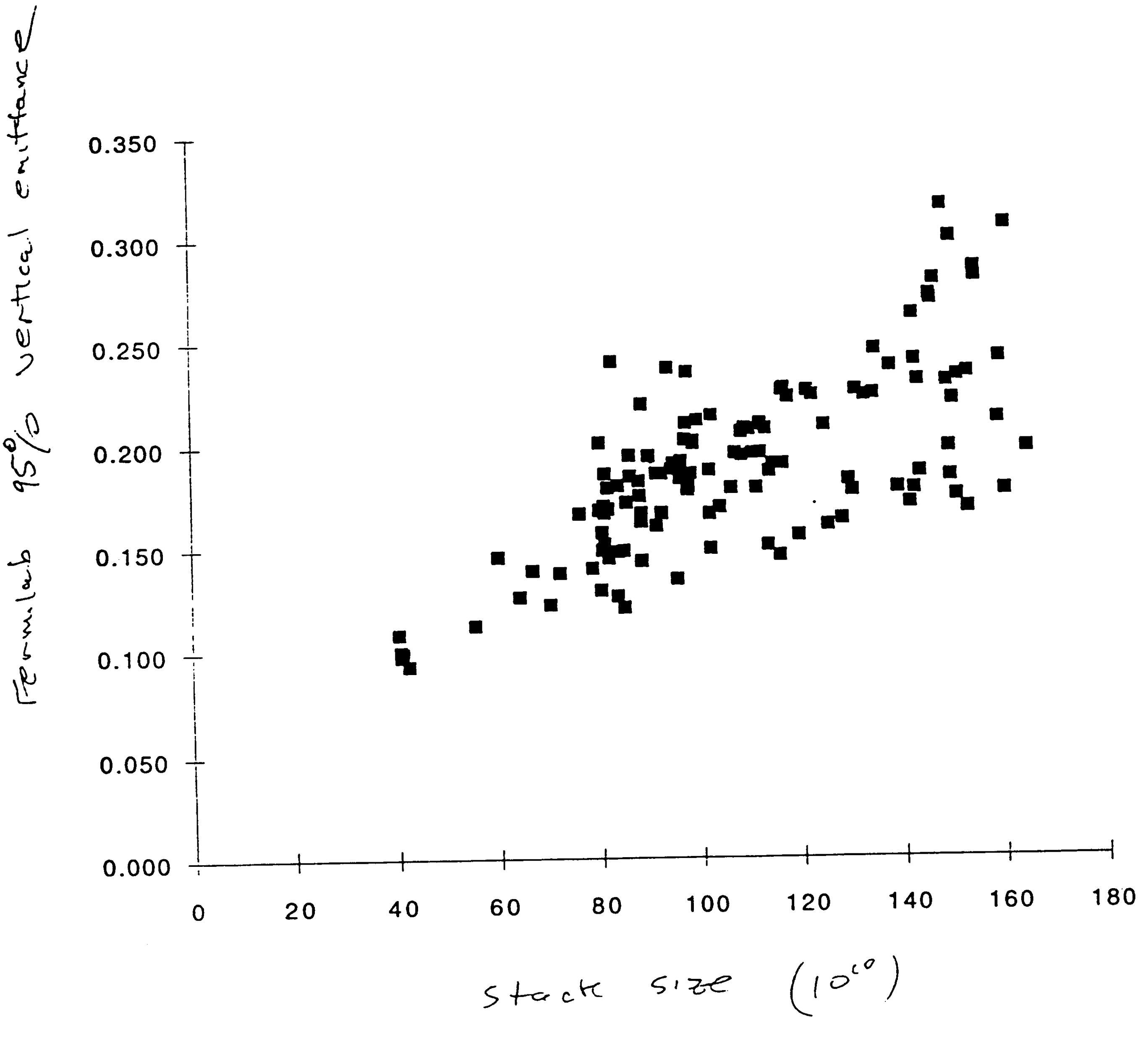
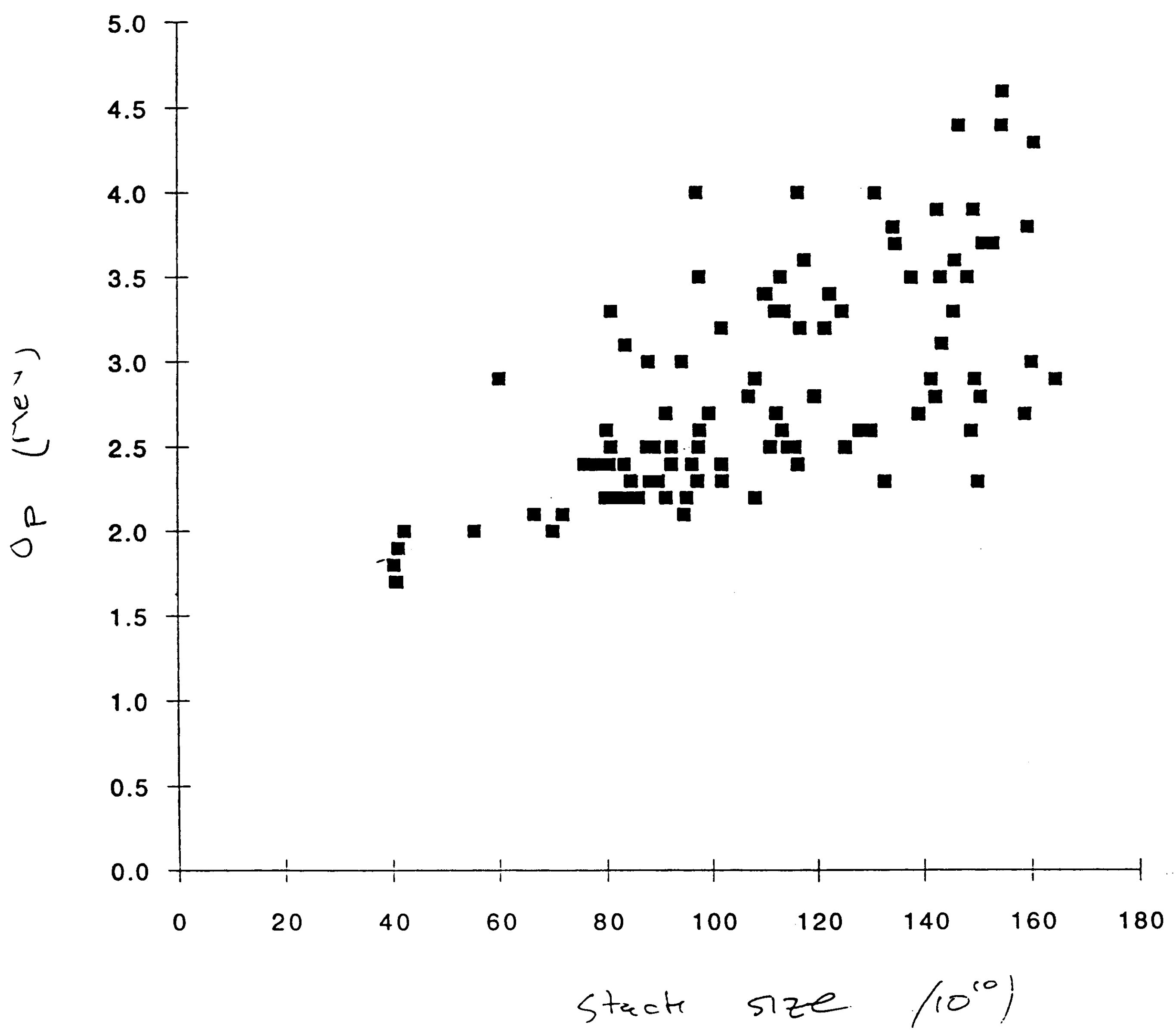

0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
0
20
40
60
80
100
120
140
160
180

figure 3



0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
0
20
40
60
80
100
120
140
160
180

figure 4



5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
100
120
140
160
180
stack size (/10^10)

figure 5