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MAGNETIZED ELECTRON BEAM COOLING TIME FOR HEAVY IONS

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1. INTRODUCTION

In this note we aim to give an approximate value of the expected cooling time of ions by magnetized electrons. The formulae taken from Ref. [1] are, as usual, asymptotic, i.e. quite accurate when the ion emittances are much larger or much smaller than the electron emittances.

This study is mostly oriented towards the cooling time of lead ions (Pb^{53+}) which are foreseen to be used at LEAR (Ref. [2]). In this case, due to the low energy, the asymptotic expressions will induce some errors in our expectations. Therefore, some approximations have to be made.

After some approximations and definitions, the expected cooling times will be computed using data, symbols, and parameters given in Section 2. Finally some possible improvements are discussed.

2. DATA, SYMBOLS, PARAMETERS

2.1 Data

q : Elementary charge, q [C] = 1.609×10^{19}

c : Light velocity, c [$\mathrm{m \cdot s^{-1}}$] = 3×10^{8}

m : Electron mass, m [kg] = 9.109×10^{-31}

m_p : Proton mass, m_p [kg] = 1.672×10^{-27}

ε_0 : Permittivity of vacuum, ε_0 [$\mathrm{F \cdot m^{-1}}$] = 8.854×10^{-12}, $1/4\pi\varepsilon_0 = 8.987 \times 10^{9}$ [$\mathrm{F^{-1} \cdot m}$]

r_e : Electron radius, r_e [m] = 2.817×10^{-15}

r_p : Proton radius, r_p [m] = 1.547×10^{-18}

2.2 Symbols

ε : Ion beam emittance

$\beta_{h,v}$: Betatron amplitudes [m], these functions are considered constant in the cooling space

θ_e : Electron beam rms divergence [rad]

θ : Ion beam rms divergence [rad], $\theta = \sqrt{\varepsilon/\beta_{h,v}}$

B : Solenoid longitudinal magnetic field [T]

η : Cooling length/Accelerator circumference

β = v/c, where v is the electron or the cooled ion mean longitudinal velocity

ω_c : Cyclotron frequency, ω_c [$\mathrm{s^{-1}}$] = qB/m

n : Electron beam density [$\mathrm{m^{-3}}$]

r_0 : Electron beam radius [m]

I : Electron beam intensity, I [A] = $qn\pi r_0^2 \beta c$

J : Electron beam current density, J [$\mathrm{A \cdot m^{-2}}$] = $I/\pi r_0^2$

ω_{pl} : Plasma frequency, ω_{pl} [s-1] = $\sqrt{nq/m\varepsilon_0} = \sqrt{Jq/m\beta c\varepsilon_0} = c\sqrt{4\pi r_e n}$

2.3 Lead Ion and Cooler Parameters

Type of ions: Lead. $A = 208$, $Z = 53$

E_c : Electron kinetic energy, $E_c = 2.32$ keV

β = 0.095, v [m·s^{-1}] = 2.85×10^7, $\gamma = 1.0045$

Proton equivalent momentum: $p = \gamma\beta(m_p c^2)/c = 89$ MeV/c

r_0 [m] = 2.5×10^{-2}

I [A] = 0.5, J [A·m^{-2}] = 2.54×10^2

n [m^{-3}] = 5.584×10^{13}

B [T] = 0.06, which is about twice larger than the nominal value

ω_c [s^{-1}] = 1.06×10^{10}, ω_{pl} [s^{-1}] = 4.2×10^8

Electron beam rms transverse temperature: $kT_{e\perp} = 0.1$ eV, (k = Boltzmann constant)

β_h [m] = 1.9, β_v [m] = 5.3, $\eta = 0.02$

θ_e [mrad] = 3

3. SOME ESTIMATES AND DEFINITIONS

Some definitions to be used in the next paragraph will be enumerated. As mentioned before, data, symbols and parameters are those of the previous section from which the numerical applications (N.A.) are deduced.

3.1 Divergences

The beam divergence θ of any particle is defined by the ratio (Fig. 1):

$$\theta = \frac{\text{Transverse velocity}}{\text{Longitudinal velocity}} = \frac{v_\perp}{v_\sigma} \cong \frac{v_\perp}{\beta c}$$

from which the electron and ion beam divergences can be deduced.

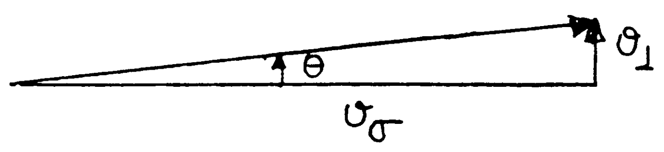

Fig. 1- Definition of the divergence θ

3.1.1 Electron divergence

The transverse energy $kT_{e\perp}$ determines the electron transverse velocity since

$$v_{e\perp} = \sqrt{\frac{kT_{e\perp}}{m}}$$

N.A.:

$$v_{e\perp}\ (\mathrm{m \cdot s^{-1}}) = 1.326 \times 10^5$$

Thus the electron transverse divergence is

$$\theta_e = \sqrt{\frac{kT_e}{2E_c}}$$

N.A.:

$$\theta_e = 4.642\ [\mathrm{mrad}]$$

As usual the longitudinal divergence of the electrons is not considered, since, due to the dynamic contraction, the longitudinal spread is relatively small, even if one takes into account the energy coupling from the transverse to the longitudinal plane.

3.1.2 Ion divergence

3.1.2.1 Transverse plane

The ion divergence is given by the transverse rms emittance since

$$\theta = \sqrt{\frac{\varepsilon}{\beta_h}}$$

N.A.:

$$\varepsilon = 50 \times 10^{-6}, \quad \beta_h = 1.9 \ \Rightarrow \ \theta = 5.12\ [\mathrm{mrad}]$$
$$\varepsilon = 100 \times 10^{-6}, \quad \beta_h = 10 \ \Rightarrow \ \theta = 3.162\ [\mathrm{mrad}]$$

3.1.2.2 Longitudinal plane

In this case, the relative energy spread is introduced and such as:

$$\frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta} = \gamma^2 \frac{dv}{v} = \gamma^2 \theta_\sigma$$

N.A.:

$$\frac{dp}{p} = 10^{-3} \Rightarrow \theta_\sigma = 1\ [\mathrm{mrad}]$$

Therefore, the initial proton divergences are of the same order as the electron beam divergence. The electron beam transverse divergence θ_e (or $v_{e\perp}/v$) is thus an important parameter in our estimation of the cooling force or time.

With a non-magnetized electron beam, the cooling friction force, versus the relative ion velocity v_i is represented in Fig. 2a. For $v_i >> v_e$ the friction force is proportional to $1/v_i^2$ while for $v_i << v_{e\perp}$ it increases linearly with v_i.

The cooling force with magnetized electron beams is represented in Fig. 2b. It is therefore expected, in such a case, that a drastic reduction of the cooling time for the ions having velocities equal to or lower than $v_{e\perp}$ will take place.

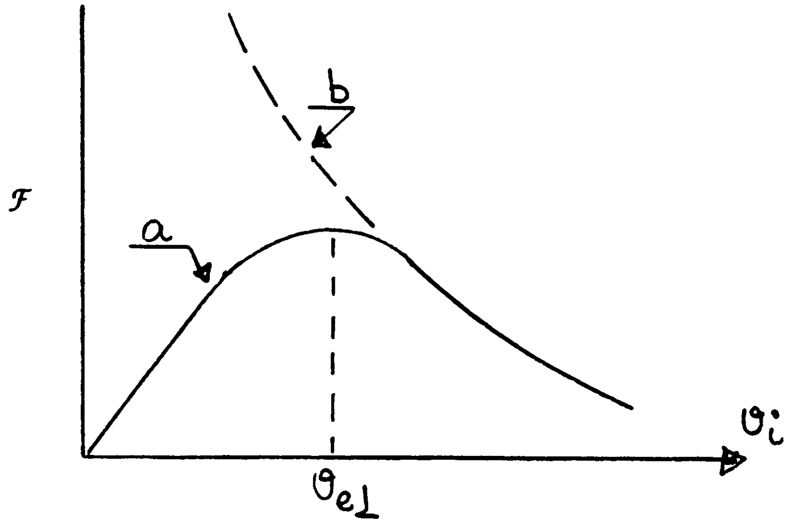

Fig. 2 - Shape of the cooling force $\mathcal{F}$ vs. the relative ion velocity v_i.
—— : non-magnetized, ----- : magnetized electron beam.

3.2 Condition for Magnetization

The parameters θ_e, n, and ω_c already defined are used, and a so-called Larmor wave-length is defined as:

$$\rho_l\,[\mathrm{m}] = \frac{v}{\omega_c} \quad (\mathrm{N.A.:}\ \rho_l = 5.66\times10^{-3})$$

The condition for the electron beam magnetization implies that $\theta_e \rho_l < n^{-1/3}$. This gives a lower boundary value to the longitudinal magnetic field:

$$B > B_{\min} = \frac{vm}{\rho_l q}$$

N.A.:

$$B_{\min}\,[\mathrm{T}] = 2.86\times10^{-2}$$

Taking $B = 0.06$ [T] the condition for magnetization is fulfilled.

4. COOLING TIME OF θ^2

The differential equation followed by θ^2 (and therefore by the ion emittance) is taken from Ref. [1]. It is shown that for magnetized electrons

$$\frac{d(\theta^2)}{dt} = -X\theta^2 F(\theta) \tag{1a}$$

with

$$X = \pi\eta\left[\frac{q}{4\pi\varepsilon_0}\frac{J}{mc^3}\right]\left(\frac{1}{\beta^4}\right)\left[\frac{cr_e m}{m_p}\right]\left(\frac{Z^2}{A}\right)$$

$$X\,[\mathrm{s}^{-1}] = \frac{\pi\eta r_e r_p J}{\beta^4}\left(\frac{Z^2}{A}\right)$$

$$F(\theta) \equiv F_1(\theta) = \frac{1}{\theta^3}\left(2L_f + L_m\right) \qquad \text{for } \theta \geq \theta_e$$

$$F(\theta) \equiv F_2(\theta) = \frac{L_f}{\theta_e^3} + \frac{L_a}{\theta_e^2\theta} + \frac{L_m}{\theta^3} \qquad \text{for } \theta < \theta_e$$

$$L_f = \ln\left[\frac{\beta^3 c}{\omega_c r_e}\frac{\theta^3}{Z}\right]$$

$$L_m = \ln\left[\frac{\omega_c}{\omega_{pl}}\frac{\theta}{\theta_e}\right]$$

$$L_a = \ln\left[\frac{\theta_e}{\theta}\right]$$

Remark: X is considered to be proportional to Z^2. Recent works [3] have shown that for small ion emittances Z^α should be considered, with $\alpha < 2$ instead. Since we deal with relative large ion emittances, this phenomenon will not be taken into account in the present paper.

4.1 Numerical Application

From the data and number given before, we get $X = 7.29 \times 10^{-8}$. For simplicity, θ and θ_e will be expressed in mrad. The differential equation then becomes:

$$\frac{d\left(\theta^2\right)}{dt} = -72.9\theta^2 F(\theta) \qquad (1b)$$

and

$$L_f = -1.286 + 3\ln(\theta)$$

$$L_m = 2.12 + \ln(\theta)$$

$$L_a = 1.1 - \ln(\theta)$$

If parameters different from those given in Section 2 were to be used, the constants should be scaled as follows:

$$X = 72.9\left(\frac{I}{0.5}\right)\left(\frac{Z}{53}\right)^2\left(\frac{208}{A}\right)\left(\frac{0.095}{\beta}\right)^4$$

$$L_f = -1.286 + 3\ln(\theta) + \left[3\ln\left(\frac{\beta}{0.095}\right) - \ln\left(\frac{B}{0.06}\right) - \ln\left(\frac{Z}{53}\right)\right]$$

$$L_m = 2.12 + \ln(\theta) + \left[\ln\left(\frac{B}{0.06}\right) - \frac{1}{2}\ln\left(\frac{I}{0.5}\right) - \ln\left(\frac{\theta_e}{3}\right)\right] \quad \text{(1c)}$$

$$L_a = 1.1 - \ln(\theta) + \left[\ln\left(\frac{\theta_e}{3}\right)\right]$$

The function $u(\theta) = -\theta^2 F(\theta)$ is plotted in Fig. 3a for $\theta_e = 5$ mrad or, equivalently, $kT_{e\perp} = 0.116$ eV. It shows a discontinuity for $\theta = \theta_e$ due to the fact that $F_1(\theta_e) \neq F_2(\theta_e)$. This is explained by the lack of an exact theory when the ion divergence θ is close to the electron beam divergence θ_e and when the electron beam is not fully magnetized. Indeed this discontinuity disappears for large values of the magnetic field (it is negligible for $B = 1$ T, see Annex 1) although the cooling time, as computed later on, is not drastically reduced. This is not the case in the classical theory (for non-magnetized electrons) where the cooling decrements are well defined. In any case, this discontinuity has no physical meaning and the above expression (1b) cannot be fully retained.

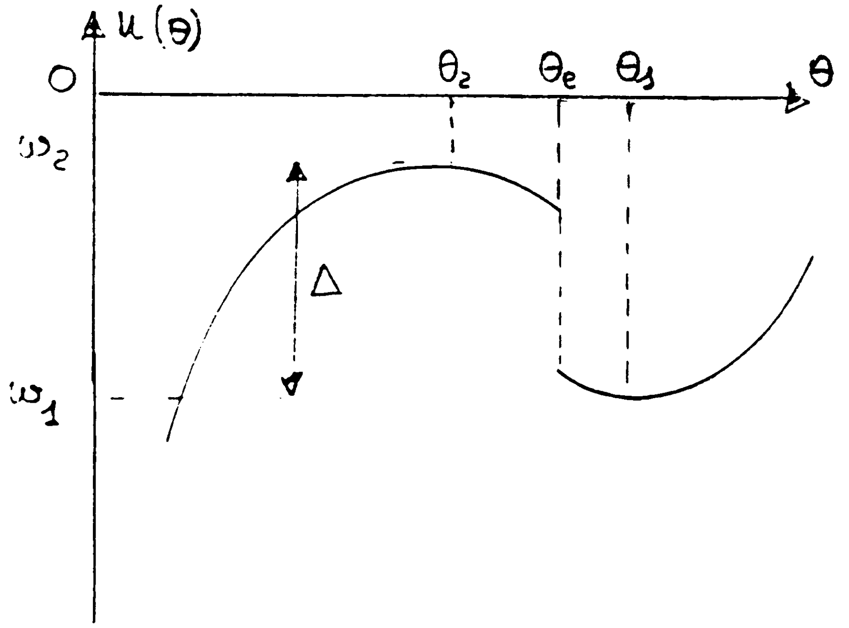

Fig. 3 a - Plot of $u(\theta) = -\theta^2 F(\theta)$ for $\theta_e = 5$ mrad

Therefore a smoother expression is chosen which is more consistent with the physical process, as described in Fig. 3b. We proceed in the following way:

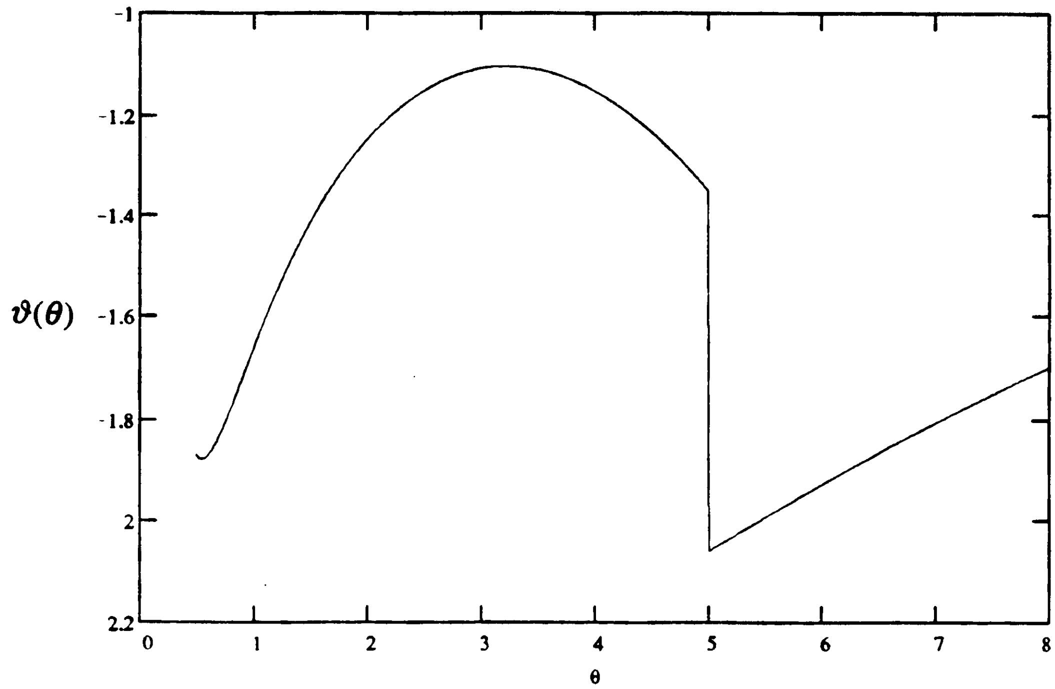

Fig. 3b - Explanation of the cooling force, or $u(\theta)$ smoothing process

a) One computes first (Fig. 3b)

- θ_1 such as $\frac{d}{d\theta}\left[-\theta^2 F_1(\theta)\right]_{\theta=\theta_1} = 0$, and $w_1 = -\theta_1^2 F_1(\theta_1)$
- θ_2 such as $\frac{d}{d\theta}\left[-\theta^2 F_2(\theta)\right]_{\theta=\theta_2} = 0$, and $w_2 = -\theta_2^2 F_2(\theta_2)$
- $\Delta = w_2 - w_1$

b) One considers now:

$$\frac{d(\theta^2)}{dt} = -X\theta^2 H(\theta) \tag{2}$$

with

$H(\theta) = F_1(\theta)$ for $\theta \geq \theta_1$

$H(\theta) = -\frac{w_1}{\theta^2}$ for $\theta_2 \leq \theta \leq \theta_1$

$H(\theta) = F_2(\theta_e) + \frac{\Delta}{\theta^2}$ for $\theta < \theta_2$

N.A.: $\Delta = 1.14$ for $\theta_e = 5$ mrad.

The function $\vartheta(\theta) = -\theta^2 H(\theta)$ is plotted in Fig. 3c for $\theta_e = 5$ mrad.

When using betatron amplitudes of the order of 10 m at the cooler level, Fig. 4b shows that the horizontal emittance is reduced from $\varepsilon = 100 \times 10^{-6}$ to $\varepsilon = 5 \times 10^{-6}$ in less than 100 ms. Therefore, increasing the cooling length by a factor two, and taking into account a penalty factor of two, implies that under all given conditions, a cooling time of about 100 ms can be reasonably expected.

REFERENCES

[1] I.N. Meshkov, *Electron Cooling: Status and Perspectives*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

[2] P. Lefèvre and D. Möhl, *Heavy Ion Accumulation Scheme for LHC*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

[3] G. Armbrüster, P.G. Reinhard and C. Toepffer, andG. Zwicknagel, *Molecular Dynamics Simulation of Electron Cooling*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

C. Ellert, M. Grieser, D. Habs, B. Hochadel, P. Repnow, D. Schwalm, and A. Wolf, *Non-Linear Effects on Ion Stopping in a Magnetized Electron Gas: Charge Dependence of the Electron Cooling Force Measured with Heavy Ions*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

K. Beckert, F. Bosch, H. Eickhoff, B. Franzke, M. Jung, O. Klepper, R. Moosham–mer, F. Nolden, H. Reich, P. Spädtke, and M. Steck, *Electron Cooling of Highly Charged Heavy Ion Beams*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

[4] V.I. Kudelainen, V.A. Lebedev, I.N. Meshkov, V.V. Parkhomchuk, and B.N. Sukhina, *Temperature Relaxation in Magnetized Electron Flux*, JETP v83, N6, pp. 2056-2064.

D. Habs, S. Pastuszka, D. Schwalm, A. Wolf, and S. Zwickler, *Cold Electron Beam for Cooling and Collision Experiments Obtained by Adiabatic Expansion in a Magnetic Field*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

H. Danared, *Electron Cooling at CRYING and a Technique to Produce Electron Beams*, Proceedings of the Workshop on Beam Cooling and Related Topics, Montreux, 4-8 October 1993.

ANNEX 1

1. Discontinuity between $F_1(\theta_e)$and $F_2(\theta_e)$

Using Eqs. (1) at $\theta = \theta_e$ gives:

$$F_1(\theta_e) = \frac{1}{\theta_e^3}\left[2L_f + L_m\right] \text{ and } F_2(\theta_e) = \frac{1}{\theta_e^3}\left[L_f + L_a + L_m\right]$$

Since $L_a(\theta_e) = 0$ is the discontinuity,

$$\Delta = F_1(\theta_e) - F_2(\theta_e) = \frac{L_f}{\theta_e^3}.$$

Considering now L_f in Eq. (1c) where only B differs from the initial value 0.06 T, then

$$\Delta = \frac{1}{\theta_e^3}\left[3\ln(\theta_e) - 1.286 - \ln\left(\frac{B}{0.06}\right)\right]$$

It is a decreasing function of B which cancels when

$$\ln\left(\frac{B}{0.06}\right) = 3\ln(\theta_e) - 1.286$$

N.A.:

$$\theta_e = 5 \text{ mrad}, \ \ln\left(\frac{B}{0.06}\right) = 3.54 \Rightarrow B \cong 2 \text{ T}$$

It is the value corresponding to $\rho_{\min} = \rho_f$ in Ref. [1], Eq. (11), and, therefore, the fast collision effects are negligible.

2. Asymptotic Cooling Time for Large θ

Considering now the case where $\theta > \theta_e$ where according to the previous formulae

$$F_1(\theta) = \frac{1}{\theta^3}\left[7\ln(\theta) - 0.452 - \ln\left(\frac{B}{0.06}\right)\right]$$

$$F_1(\theta) = \frac{1}{\theta_e^3}[7\ln(\theta) - 3.958] \text{ when } B = 2 \text{ T}$$

the derivative of

$$\theta^2 F_1(\theta) = \frac{1}{\theta}[7\ln(\theta) - 3.978] = \frac{1}{\left(\theta^2\right)^{1/2}}\left[3.5\ln\left(\theta^2\right) - 3.958\right]$$

versus θ^2 is about 0. Thus, for $\theta \geq \theta_e$ (and large B values): $\theta^2 F_1(\theta) \cong 1.3$, independently of θ^2. The differential equation (1) can be written in a simplified form:

$$\frac{d\left(\theta^2\right)}{dt} \cong -1.3X$$

Then

$$\theta^2 = \theta^2(t=0) - 1.3Xt$$

from which the cooling time can be deduced:

$$t \cong \frac{\theta^2(t=0) - \theta^2}{1.3X}$$

N.A.: $X = 72.9$, $\theta^2(t{=}0) = 10$, $\theta^2 = 0.5$ gives $t \cong 0.1$ s, which must be compared to $t = 0.065$ s, found by a more accurate approach.

Therefore, a rough approximation of the time t to cool from $\theta^2(t{=}0)$ to θ^2 with magnetized electron beams is:

$$t \cong \frac{\theta^2(t=0) - \theta^2}{1.3X} \cong \frac{\theta^2(t=0) - \theta^2}{1.3} \frac{\beta^4}{\pi r_e r_p J} \frac{A}{Z^2}$$