EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
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CERN - PS DIVISION

PS/ ARNote 93-08

NEUTRALISATION OF THE LEAR-ECOOL ELECTRON BEAM SPACE CHARGE

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10 mai 1993

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1. THE AIM AND PRINCIPLE OF NEUTRALISATION

Neutralisation of stored beams by ionised particles has been reported in many papers [1, 2]. In the present report we will see how the presence of a longitudinal magnetic field, used for electron cooling, modifies significantly the classical approach to this problem.

The space-charge effects of the dense electron beam used at the LEAR electron cooler (Ecool) may deteriorate the efficiency of the cooling process. These effects take place, mainly, in the drift space where the cooling electron beam current, of intensity I (A), at longitudinal velocity v, is mixed with the ion beam which has to be cooled. This has been detailed in Refs. [3, 4]. One can simply recall from Ref. [4] that:

a) the space charge will cause an increase of the electron transverse velocity v_d. This can be defined by (refer to Figs. 1a and 1b for symbols):

$$\theta_d = \frac{v_d}{v} = \frac{E_r - vB_\phi}{vB_0}\,[rad],$$

where

$E_r = -enr/2\varepsilon_0$ is the radial electrical field (V/m) at radius r due to the space charge of an electron beam of density n (m^{-3}) such that

$$n = n_e - n_+$$

$n_e = I/ev\pi a^2$ is the electron density of the beam of radius a and n_+ the positive ion density of the neutralisation charges inside the electron beam. The neutralising particles are created by ionisation of the residual gas as will be explained below.

$B_\phi = -\mu_0 I/2\pi r$ is the azimuthal magnetic field, due to the electron beam, at radius r.

B_0 is the longitudinal magnetic field of the electron cooler.

It is worth mentioning that the related space-charge force acting on an electron at radius r is given by:

$$f_r = \frac{e^2 n_e}{2\varepsilon_0} r \left(\frac{n}{n_e} - \beta^2 \right); \ \beta = \frac{v}{c}.$$

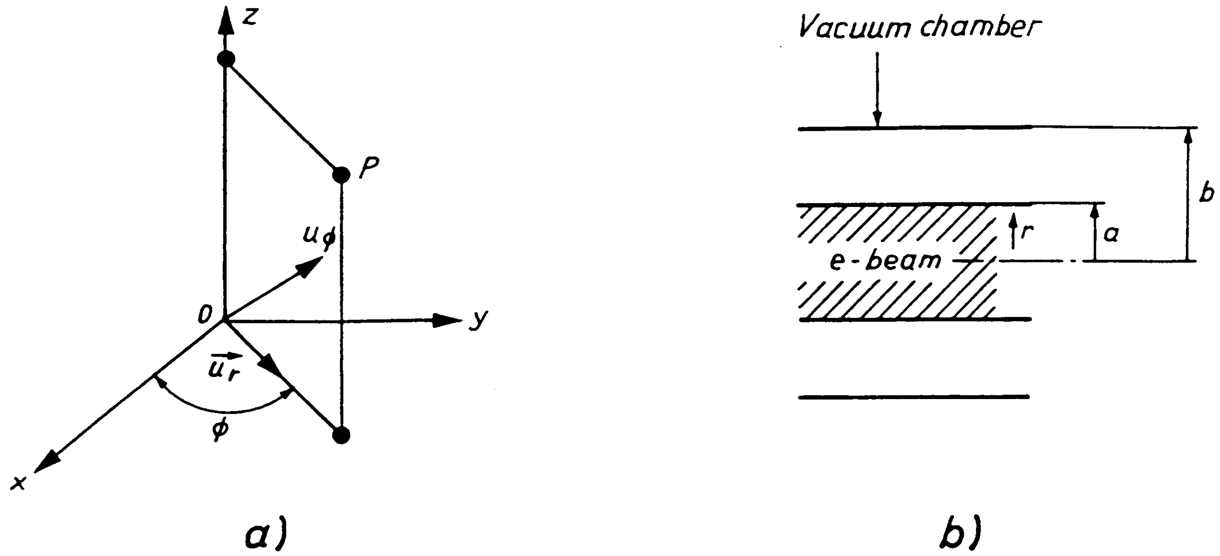

Fig. 1 . Definition of symbols

b) the space charge will induce a potential distribution U (volt) which, at radius r, is approximated by (Fig. 2):

$$U(r) = C_1 \begin{cases} 1 - \left(\frac{r}{a} \right)^2 + 2\ell n \left(\frac{b}{a} \right) & \text{for } 0 < r < a \\ 2\ell n \left(\frac{b}{r} \right) & \text{for } a < r < b \end{cases}$$

b is the vacuum-chamber (at potential 0) radius; $b = 70$ mm,
a is the electron-beam radius; $a = 25$ mm,
$C_1 = (ena^2)/4\varepsilon_0) = 30I/\beta$ (when $n = n_e$).

If the electron beam is accelerated by a potential U_0, the kinetic energy E_c of an electron at radius r will be:

$$E_c = e[U_0 - U(r)] = (\gamma - 1)mc^2 \cong \frac{mv^2}{2}$$

showing us that electrons at different radius will have different velocities. This is quite detrimental to electron cooling.

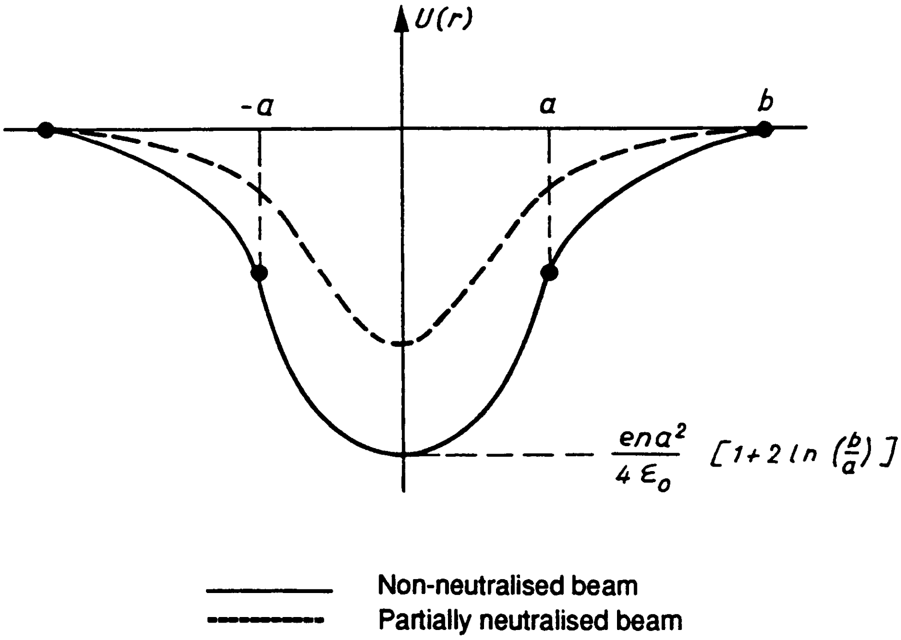

Fig. 2 - Potential distribution

Table 1 gives a numerical example for LEAR with a beam current I = 3 A when B_0 = 0.06 T (600 gauss) and $n = n_e$.

Table 1

Electron energy, E_c β	keV	30 0.3284	7 0.1638
Potential difference: $U(r{=}a)$-$U(r{=}0)$ $U(r{=}0)$	V	274 -836.3	549.45 -1680.1
Relative longitudinal velocity difference: $[v(r{=}a)-v(r{=}0)]/v(r{=}0)$		4.5×10^{-3}	3.9×10^{-2}
$E_r$max	V/cm	219.25	439.5
$B_\phi$max	gauss	0.24	0.24
$\theta_d$max	mrad	3.9	14.5

If the electron beam is neutralised, such that $n_e = n_+$, then the radial force and θ_d are drastically reduced, namely θ_d becomes equal to 0.4 mrad.

The method and the consequences will be explained later but from now on we can say that it consists of the trapping, inside the electron beam, of the positive ions created by the ionisation of the residual gas by the primary beam. The corresponding secondary electrons must be cleared away [5].

The neutralisation factor η is defined by:

$$\eta = \frac{n_+}{n_e}. \quad (1)$$

1.1 Neutralisation Time, Secondary Electron or Ion Current

We use the classical formula [1, 2] giving the increase of the ion density per unit of time (for single ionisation, see remark below):

$$\frac{dn_+}{dt} = \sigma_i n_r n_e v$$

where

$\sigma_i =$ ionisation cross section. $\sigma_i \propto E_c^{-1/2}$. For simplicity we take $\sigma_i \beta = 3.28 \times 10^{-18}$ cm^{-2},

$n_r =$ residual gas volumic density, $n_r \cong 3.5 \times 10^{16}$ $Ptorr$ cm^{-3} ($Ptorr$ expressed in torr).

Full neutralisation $n_+ = n_e$ is reached at the time $t = \tau_{neutr}$ given by:

$$\tau_{neutr} = \frac{1}{\sigma_i n_r v} = \frac{2.9 \times 10^{-10}}{Ptorr}, \quad [\mathrm{s}] \quad (2)$$

Example: If $Ptorr = 10^{-10}$ torr then $\tau_{neutr} = 3$ s.

Let N_i be the number of trapped positive charges. We have, therefore, the following identity:

$$\frac{dn_+}{dt} = \frac{1}{\pi a^2 L} \frac{dN_i}{dt}$$

$L =$ the ionisation length. $L \cong 3$ m in our practical case.

We may define the secondary current I_i by:

$$I_i = e\frac{dN_i}{dt} = \sigma_i n_r L I \cong 1 \times 10^2 \, Ptorr \, I$$

Example: $Ptorr = 10^{-10}$ torr, $I = 3$ A, $I_i \cong 30$ nA.

This would be the current on clearing electrodes which collect all the created ions.

Remark: In order to be more accurate one should take into account the ionisation to higher charge state of the ions already trapped, of density n_+, such that dn_+/dt should be written:

$$\frac{dn_+}{dt} = \left(\sigma_i n_r + \sigma_{ii} n_+\right) n_e v.$$

Usually $\sigma_i >> \sigma_{ii}$.

Even at full neutralisation when $n_+ = ne >> n_r$ we will neglect this secondary effect.

1.2 Instabilities

When the neutralised cooling electron beam current density J exceeds an upper limit or is smaller than a lower limit, the ion-electron system may become unstable [6, 7]. A short explanation is given in Appendix A.

The upper limit current density, described in Ref. [7], is expressed in practical units by:

$$J_{\max} \cong 0.05 \frac{E_c[\text{keV}] B_0[\text{kgauss}]}{\text{L[m]}}$$

where L is the neutralised beam length (about 3 m in our case).

The lower limit results from Landau damping properties of the ion cloud (see Appendix A). It is given, in practical units, by:

$$J_{\min} = 0.25\beta \frac{B_0^2\ [\text{kgauss}]}{M_i}$$

M_i being the ionised ion atomic mass.

Table 2 gives some values of $I_{\max}$ and $I_{\min}$ for a 5 cm diameter electron beam, and $L = 3$ m. These numbers must be taken as guidelines.

Table 2

Electron energy, T	keV	30	7	7
Magnetic field, B_0	gauss	600	600	200
$I_{\max}$	A	5.9	1.37	0.44
$I_{\min}$ (for $M_i = 14$)	mA	43.3	20.9	2.32

1.3 Influence on Pressure

Another effect of neutralisation is the worsening of the vacuum environment. Stored ions, inside the electron beam, are considered as an additional gas along the neutralisation length L. The increase of the average pressure will be:

$$\Delta P = 2.8 \times 10^{-17} n_+ \left(\frac{L}{C} \right)$$

where C is the storage ring circumference. For LEAR, when $n_+ = 10^8$ cm^{-3}, $\Delta P \cong 5 \times 10^{-11}$ torr.

2. TECHNIQUE OF NEUTRALISATION

The method is described in details in Refs. [5, 6]. It consists of two sets of neutralisation or trapping electrodes, one, El_g, placed at the gun output and the other, El_c, placed at the collector entrance (Fig. 3). Each electrode consists of two metallic half-cylinders separated by a "resistive insulator", like conducting glass (Fig. 4).

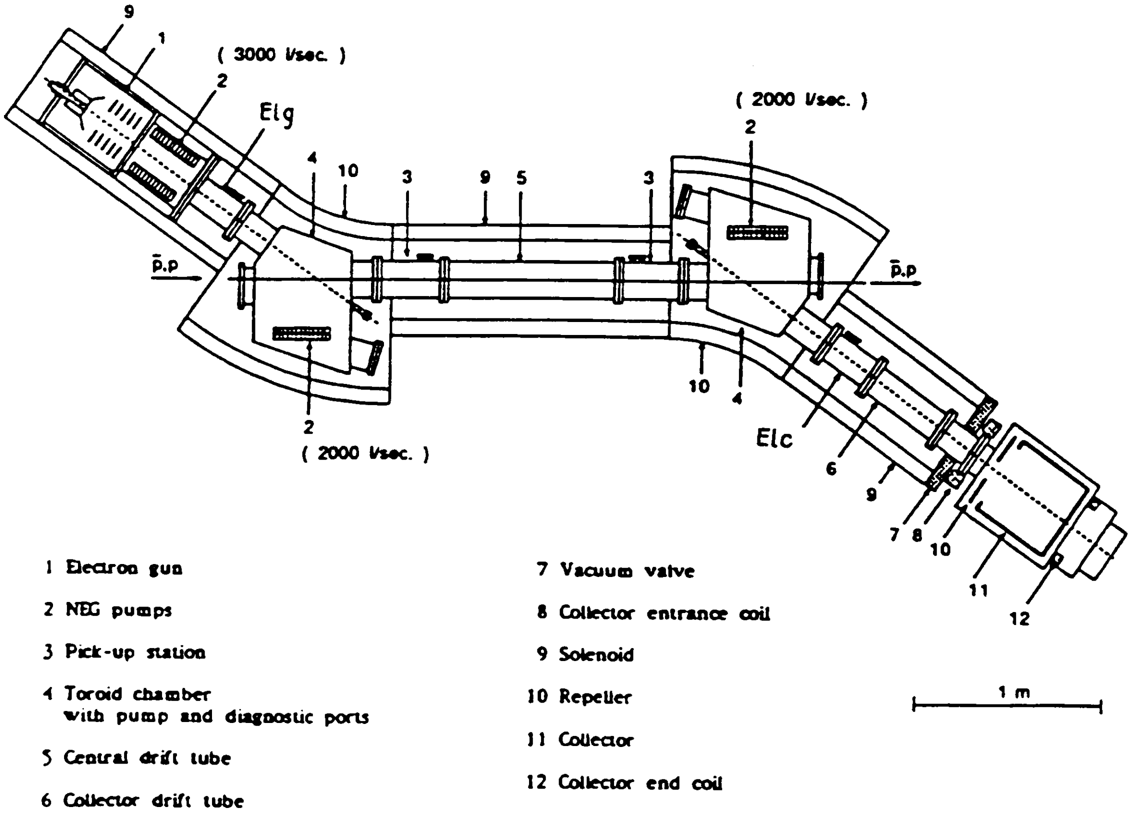

Fig. 3 - The electron cooling device with the implementation of El_g and El_c

One electrode is polarised by a positive (referred to ground) potential U_{e2}, the other is close to zero potential, $U_{e1} \cong 0$ V. This positive potential will keep the ionised ions trapped within the space between El_g and El_c, while the voltage difference (U_{e2} - U_{e1} ≈ 5 kV) will allow the collection of the secondary electrons. These electrons are drifting in crossed electric and magnetic fields (E_r, B_0) and are collected by the glass support of the neutralisation electrodes.

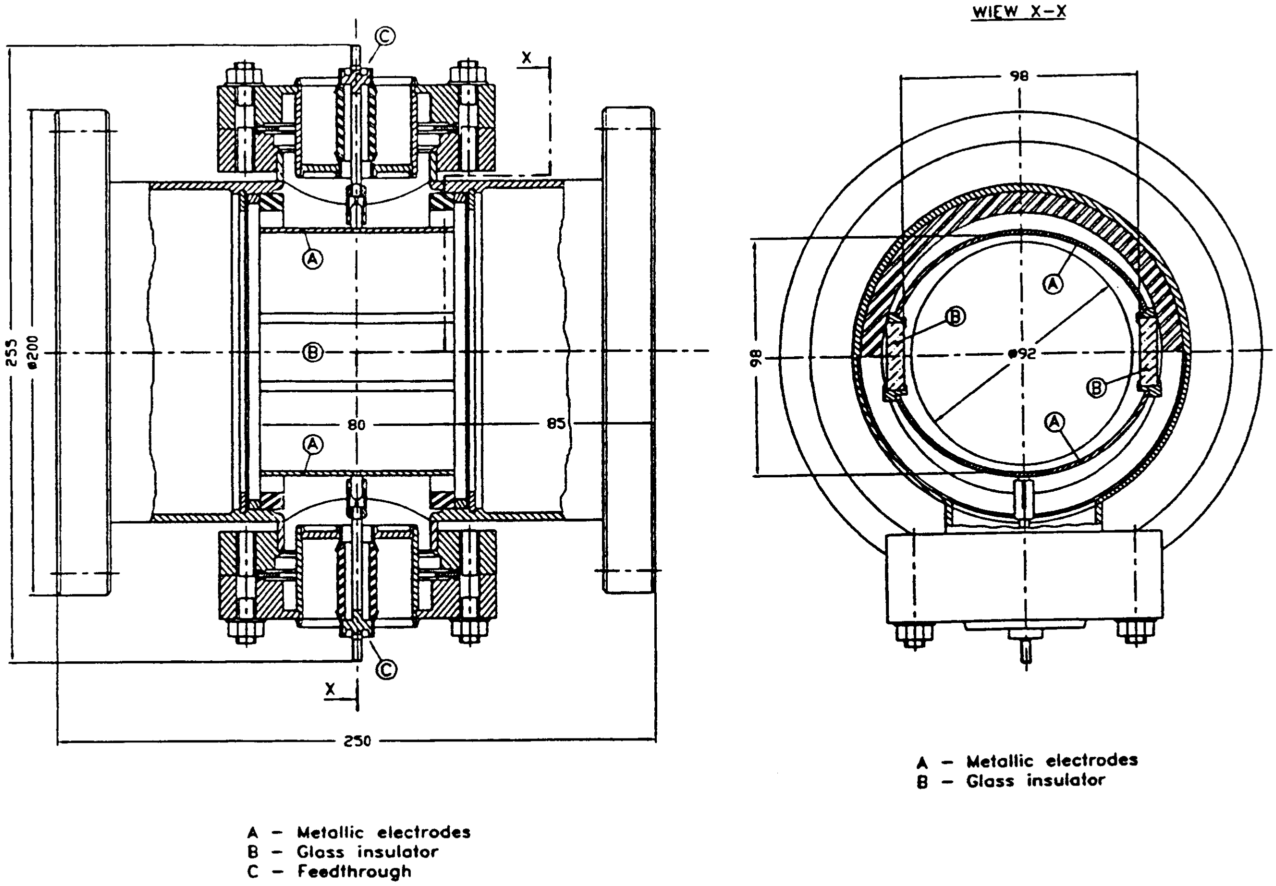

Fig. 4 - The neutralisation electrode

These neutralisation electrodes will be installed in place of the present gun and collector electron beam position pickups, as indicated in Fig. 3. Also represented in Fig. 3 are the pickups in the drift space to measure the position of both the electron beam and the LEAR ion beam to be cooled.

3. POTENTIAL DISTRIBUTIONS OF THE NEUTRALISATION ELECTRODES

The longitudinal and transversal equipotentials have been simulated.

3.1 Longitudinal Potential and Field

Figure 5 represents the equipotential surfaces inside the electron beam and the longitudinal electrical field, at r = 0, as a function of the longitudinal abscissa. We have considered here the case where $U_{e2} = U_{e1}$ and therefore where axial symmetry is of concern. The electron beam goes from the left to the right such that the ion density $n_+ = 0$ is at the electrode entrance, whereas the electron beam is fully neutralised, and $n_+ = n_e$ a few cm after the electrode.

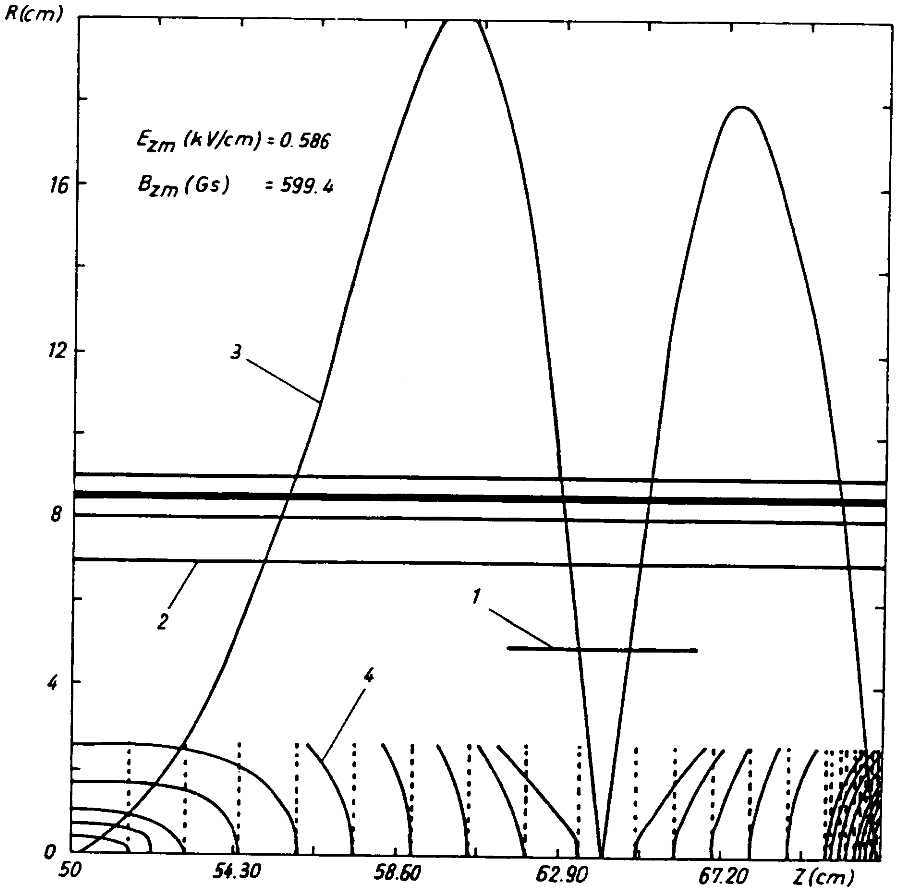

Fig. 5 - 1) Neutralisation electrode, 2) Vacuum chamber, 3) Module of the longitudinal electrical field $|E_z|$, 4) Equipotential surfaces for $eU = 7$ keV, $I = 3$ A, $U_{e2} = U_{el} = 5$ kV.

One can observe a strong radial electrical field at both electrode entrance and exit. This field will induce a harmful increment to the electron-beam transverse velocity. This increment must be estimated taking into account the longitudinal magnetic field and, therefore, using the techniques already developed for adiabatic guns [4].

In any case, a way to reduce the influence of these fields must be investigated such as, for example, an adequate shaping of the electrodes. One can also foresee two supplementary Helmholz coils for compensation at each end of the electrodes.

3.2 Transversal Potential

The potential distribution φ is taken at the cross section, in the middle of the electrodes, where the density is still $n = n_e$. It is given by the following expression:

$$\varphi(r,\psi) = \frac{U_{e2} - U_{e1}}{2}\left[1 - \frac{2}{\pi}\tan^{-1}\left(\frac{br\cos\psi}{b^2 - r^2}\right)\right] - U(r)$$

where the symbols are represented in Fig. 6.

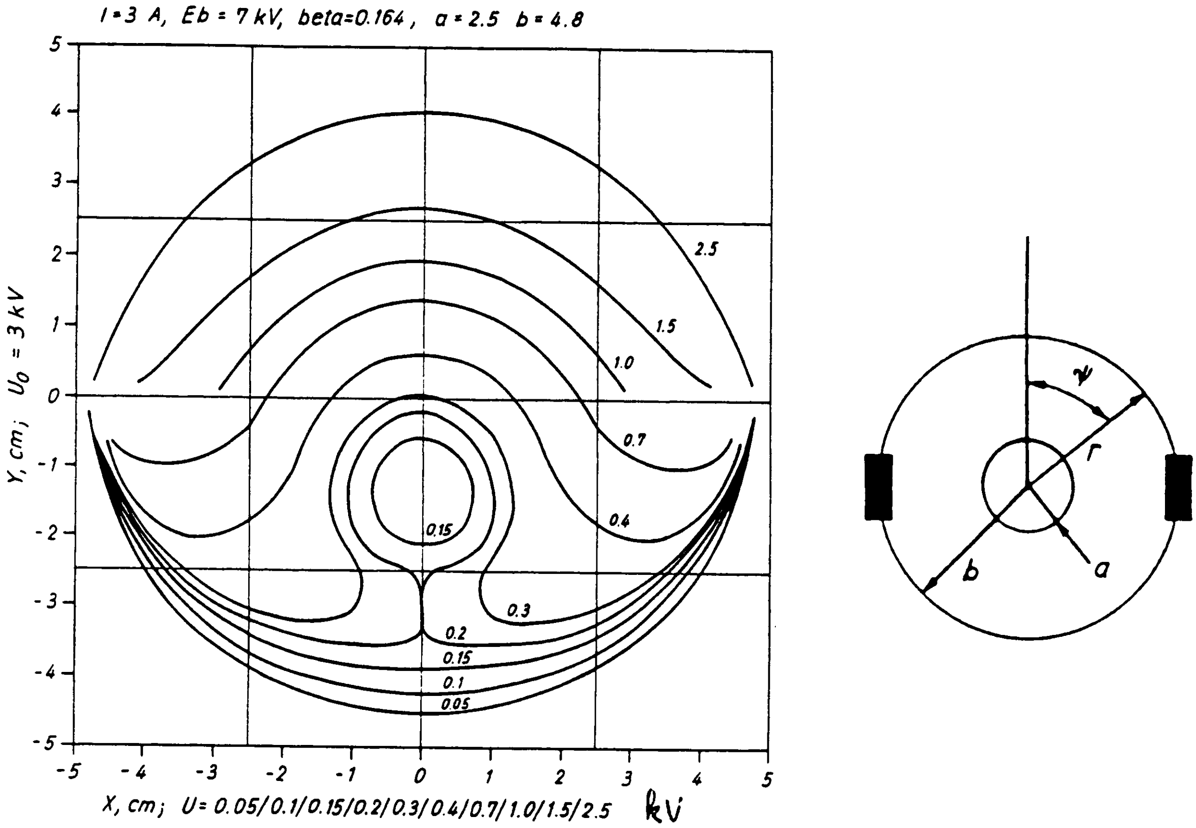

Fig. 6 - Transversal potential for U_{e2} = 3 kV, U_{el} = 0 V, eU = 7 keV, I = 3 A

In Fig. 6, $\varphi(r,\psi)$ is plotted for U_{e2} = 3 kV and U_{e1} = 0 V. One sees a curved potential which is closed in a certain region. In this case, some secondary electrons will stay inside the closed area and will perturb the goal of the trapping electrodes.

Figure 7 represents $\varphi(r,\psi)$ when U_{e2} = 5 kV and U_{e1} = 0 V. There is no closed potential and, therefore, all the secondary electrons will move to the electrode where they are collected. It is worth mentioning that the potential at the electron-beam centre is about 1.3 kV.

As a consequence, a potential U_{e2} > 5 kV is necessary to fulfill the neutralisation requirements.

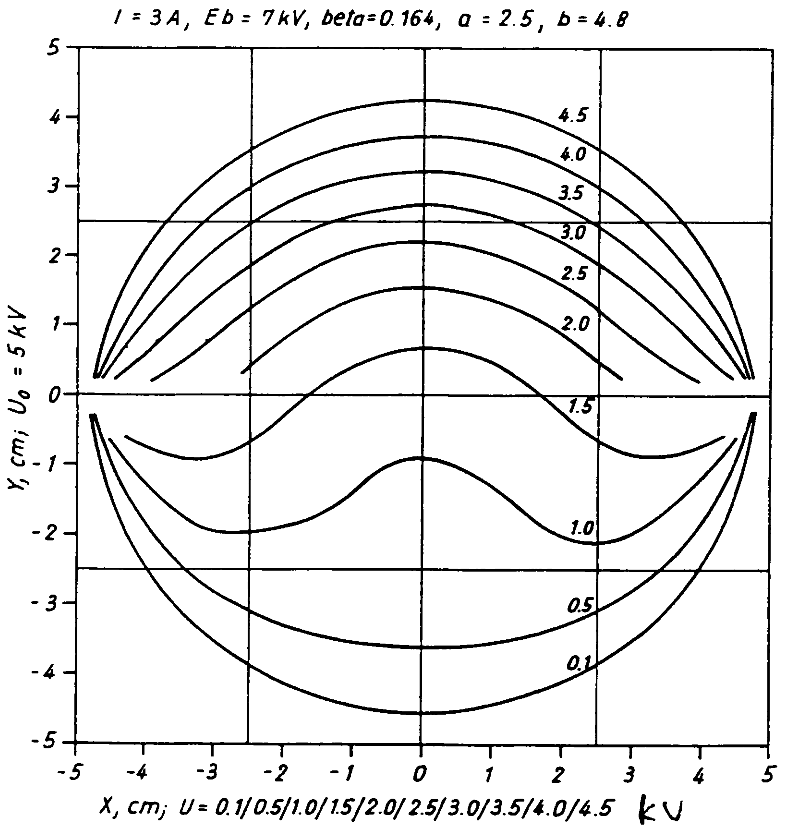

Fig. 7 - Transverse potential for U_{e2} = 5 kV, U_{e1} = 0 V, eU = 7 keV, I = 3 A

4. PHYSICS OF THE NEUTRALISATION PROCESS

The ion life time inside the electron is determinant for a better understanding of the neutralisation process. To this end one must analyse the electron-beam behaviour when interacting with slow ions.

The subject is quite wide. Some details are mentioned in Appendix B, where more physical than mathematical demonstrations are given on:

a) the dynamical equilibrium between the ionisation process and the ion escape from the beam,

b) the ion temperature inside the beam,

c) the time it takes for an ion to escape from the beam (a difference is shown between positively and negatively charged beams),

d) the fluctuations of the neutralisation coefficient.

Any estimation will depend on the actual cooling parameters. In practical cases (see Tables B.1 and B.2 of Appendix B), the electron-beam space charge can be neutralised, for an intensity of I = 3 A, to the level: $1-\eta \cong 2 \times 10^{-3}$.

The stability of such a system must conform to what has been detailed in 1.2 and for which an example is given in Table B.2 of Appendix B.

5. CONTROL OF THE NEUTRALISATION

It is important to have some measurement of the neutralisation coefficient η with and without electron cooling in operation.

Some methods, taken from Ref. [6], are mentioned hereafter. Methods 5.1 to 5.3 do not need the stored cooled ion beams as does method 5.4.

5.1 De-neutralisation Jump

When operating under neutralised conditions, the beam-position monitor (BPM) electrodes have a nominal charge, nearly zero, proportional to $(n_e\text{-}n_+)$; namely $Q1 = \lambda(n_e\text{-}n_+)$, where λ is constant.

If one sets one electrode (El_g or El_c) to the ground potential such that the neutralisation is suppressed, the electron-beam density will come to n_e and the charge on the BPM electrodes will come to $Q2 = \lambda(n_e)$. The integration during de-neutralisation of the BPM current by a capacitance C gives a voltage:

$$U_e = \frac{1}{C}\int i_{pu}\,dt = \frac{1}{C}\int \frac{\partial Q}{\partial t}\,dt = \frac{Q2 - Q1}{C} = \frac{\lambda n_+}{C} = kn_+.$$

The calibration factor k can be obtained with a de-neutralised beam when increasing or decreasing the electron current intensity by a well-measured amount, keeping the electron beam velocity constant.

This integration method is not sensitive to the charge distribution inside the beam.

5.2 Time of Flight Method (Fig. 8)

As explained in 1.b):

$$\frac{mv^2}{2} = e\left[U_0 - U(r)\right]$$

Since $U(r)$ is a function of n, the cooling electron velocity v will depend on $n = n_e - n_+$.

Let us consider a longitudinal velocity modulation of the electron beam at high frequency. This can be obtained by applying a voltage $U_1 = U_1 \cos\omega t$ on both electrodes of El_g or on the cathode. It corresponds to a density modulation of the cooling electrons. For the sake of simplicity we consider that this modulation is the same for all electrons moving at velocity v.

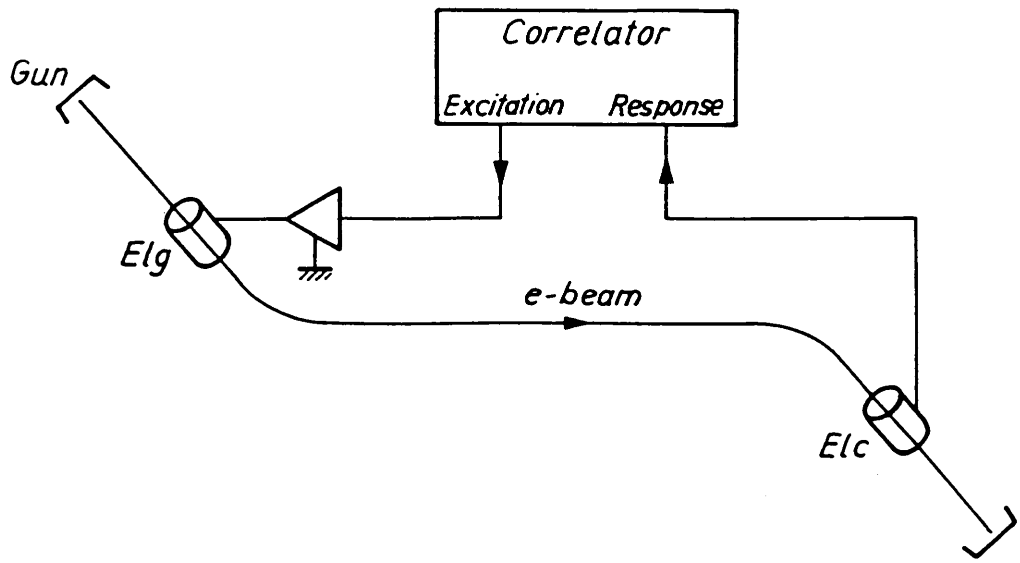

Fig. 8 - Time of flight method

The sum signal induced on the two electrodes of El_c is:

$$U_2 = U_2 \cos\omega\left(t - \frac{L}{v}\right),$$

L = distance between El_g and El_c.

If we perform the product:

$$U_{cor} = U_1 U_2 = \frac{U_1 U_2}{2}\left[\cos\omega\left(2t - \frac{L}{v}\right) + \cos\left(\frac{\omega L}{v}\right)\right]$$

and average it in time (correlation), we get:

$$\langle U_{cor} \rangle = \frac{U_1 U_2}{2}\cos\left(\frac{\omega L}{v}\right)$$

From the measurement of the phase $\theta = \omega L/v$ we can deduce v and therefore η.

Example: E_c = 30 keV, $\beta = 0.3284$, L = 3 m. We expect to be able to have a resolution of 1°. Then:

$$\theta = \frac{\omega L}{v} = \frac{\omega L}{v + \Delta v} \Rightarrow \Delta\theta = \left(\frac{\Delta L}{v}\right)\left(\frac{\Delta v}{v}\right) = \frac{\pi}{180}.$$

Let $\Delta v/v = 10^{-4}$, then the required frequency $f = \omega/2\pi$ = 90 MHz.

If accurate enough this technique can be used in a servo loop which aims to maintain the electron velocity constant (or its kinetic energy constant) when neutralisation is in use. However, for that purpose another method is foreseen.

5.3 Use of a Pencil Beam: p_b (Fig. 9)

A pencil beam, generated by a gun (p_{bgun}) and falling on a collector is flowing inside the drift space, parallel to the electron-beam axis at a distance r. Its velocity is v_{pb}. The pencil beam will sense the radial electrical field E of the electron beam being neutralised. Indeed the E-field will depend on the density $n = n_e - n_+$ and, therefore, on η, and, of course, on r which is fixed.

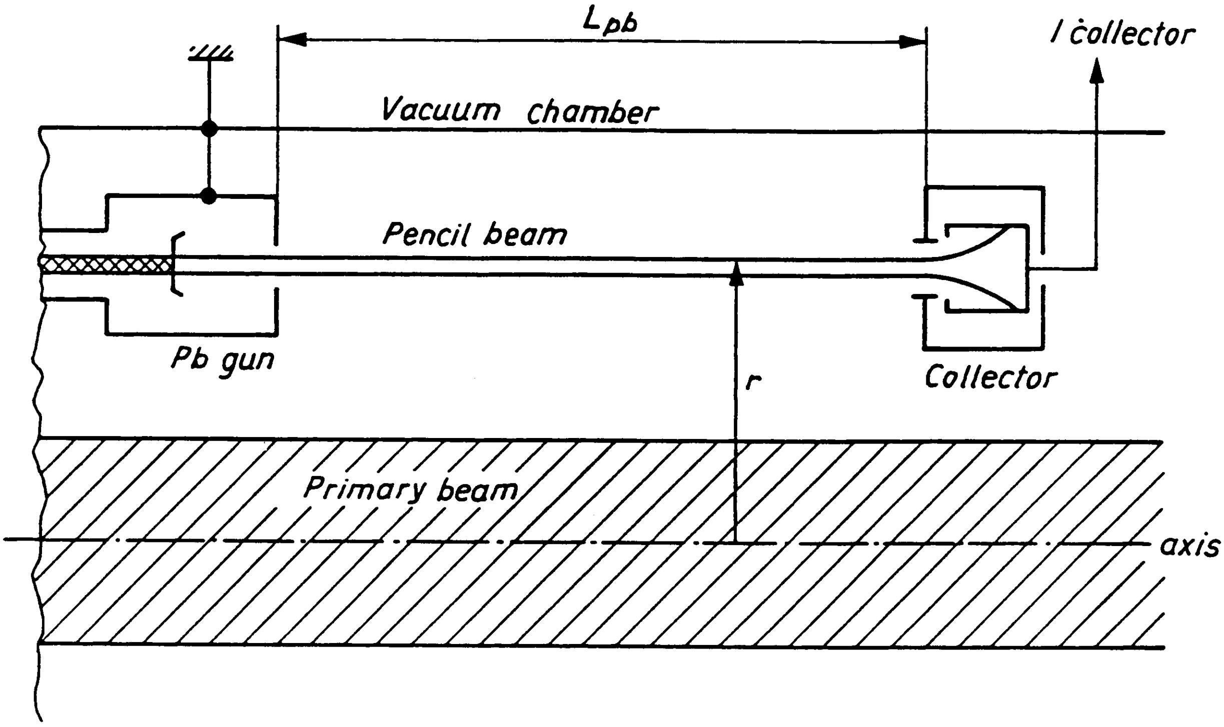

Fig. 9 - Principle of the pencil-beam η measurement

The pencil beam, p_b, will drift in the crossed E_r and B_0 fields. Its displacement (in the azimuthal direction) is:

$$\Delta\phi \propto \frac{E_r}{B_0} \frac{L_{pb}}{v_{pb}}$$

The p_b position at the gun output (p_{bgun}) is adjusted by a displacement so as to cancel $\Delta\phi$ in order to bring the beam right onto the collector input hole.

5.4 Using Electron Cooling

It is well known that electron cooling will impose, on average, the ion beam velocity to be the same as that of the electron beam. Longitudinal cooling is easily observed by displaying the longitudinal Schottky pickup signal on a spectrum analyser. The relative accuracy $\Delta v/v_0$ can be of the order of 10^{-5}.

The expression of the electron velocity v has been given in Section 1.b.

$$\frac{mv^2}{2} = e[U_0 - U(r)]$$

such as for $r = 0$:

$$\frac{mv^2}{2} = e\left\{U_0 - \frac{ena^2}{4\varepsilon_0}\left[1 + 2\ell n\left(\frac{b}{a}\right)\right]\right\}$$

where $n = n_e - n_+$, as defined before.

We consider a fixed velocity (easily observed on the spectrum analyser) $v = v_0$ and adjust $U_0 = U_\eta$, such that:

$$\frac{mv^2}{2} = e[U_\eta - C_2(n_e - n_+)]$$

C_2 = constant.

From the measurement of U_η we can deduce $(n_e - n_+)$. Then, by measuring the electron current intensity I (from which we can deduce n_e), it is easy to retrieve:

$$\eta = \frac{n_+}{n_e}$$

Example: According to Table 1, at 7 keV and $I = 3$ A, the voltage change of $U\eta$ is 1680 V when passing from a non-neutralised beam to a fully neutralised beam (or conversally).

6. CONCLUSION

The goal and technique for neutralisation have been described. Some physical processes related to the beam stability and to the limit of the neutralisation coefficient have been more described than demonstrated.

Many studies remain to be made related to e.g. the effects on the electron-beam temperature and on the measurement techniques.

Experimental results will certainly ask for some refinement of the rough theory developed in this paper.

7. REFERENCES

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APPENDIX A

1. UPPER CURRENT DENSITY THRESHOLD

Let us start from a well neutralised beam ($n_e = n_+ = n_0$). Let the electron beam centre have a small displacement r with respect to the motionless ion beam. As a consequence (see Fig. A.1) of this displacement, the centre of mass of the electrons will be submitted to an electrical field, and consequently to a radial force:

$$\vec{f}_r = -\frac{(n_0 e)^2}{2\varepsilon_0}\vec{r}.$$

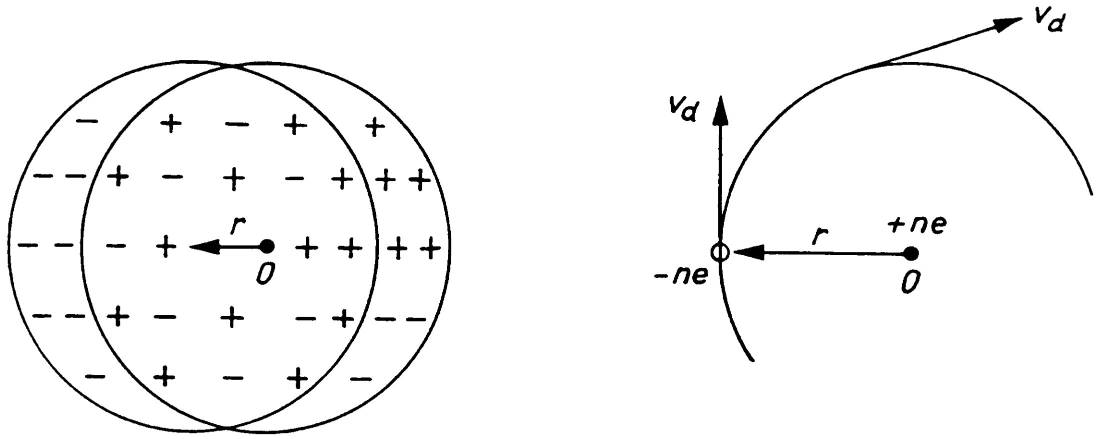

Fig. A.1 - Relative displacement of the electron beam

Taking into account the magnetic field the electron beam will rotate around the axis with a drift velocity v_d such that the electrical and magnetic forces cancel:

$$\frac{n_0 e}{2\varepsilon_0} r = v_d B_0$$

$$\Omega_d = \frac{v_d}{r} = \frac{n_0 e}{2\varepsilon_0 B_0}.$$

This polarisation displacement travels along the beam as a wave with a phase velocity:

$$v_{phase} = \frac{\omega}{k}.$$

Resonance occurs when $v_{phase} = v_0$. So, equalising the wave frequency and the angular drift velocity Ω_d one can have a resonance at $k = 2\pi/\lambda = 2\pi/L$, such that:

$$v_0 \frac{2\pi}{L} = \frac{n_0 e}{2\varepsilon_0 B_0}$$

or, for a threshold density:

$$(n_0)_{thr} = 4\pi\varepsilon_0 \frac{v_0 B_0}{eL} = \frac{I_{thr}}{e v_0 (\pi a)^2}.$$

This will define a threshold current density:

$$J_{thr} = \frac{(4\pi\varepsilon_0) v_0^2 B_0}{L} \propto \frac{E_c B_0}{L}$$

2. LOWER THRESHOLD DENSITY

We have now to consider the ion-ion collisions when the beam is almost neutralised. The collision time is given by the inverse of the plasma frequency

$$\tau_{col} = \omega_{pl}^{-1} = \left(\frac{n_+ e^2}{m_i \varepsilon_0} \right)^{-1/2}.$$

If the collision time is smaller than the Larmor rotation period which is equal to about $(\Omega_d)^{-1}$, the collision dynamics are not influenced by the magnetic field; we are in the case of a free plasma where the Landau damping is effective.

Conversely, the beam may become unstable if:

$$\tau_{col} \equiv \omega_{pl}^{-1} > \tau_{Larmor} \equiv \Omega_d^{-1},$$

or

$$\left(\frac{m_i \varepsilon_0}{n_+ e^2} \right)^{1/2} > \frac{m_i}{e B_0},$$

$$n_+ < \frac{\varepsilon_0 B_0^2}{m_i}.$$

Since $n_e \cong n_+$ the lower level of the electron current density is given by:

$$J_e < J_{low} = e n_e v_0 = \frac{e \varepsilon_0 B_0^2 v_0}{m_i} \propto B_0^2.$$

APPENDIX B

1. DYNAMICAL EQUILIBRIUM OF AN ELECTRON BEAM INTERACTING WITH THE RESIDUAL GAS

In a physical system where the cooling electron beam is mixed with slow ions, the equilibrium between the ionisation process and the ion escape process is given by

$$\frac{dn_+}{dt} = \frac{n_+}{\tau_{neutr}} - \alpha_r n_+ n_e - \frac{n_+}{\tau_{life}} = 0 \qquad \text{(B.1)}$$

where

n_+ is the positive ion density, expressed in m^{-3}, within the electron beam,
τ_{neutr} is the neutralisation time expressed in s (see Formula 2),
α_r is the recombination coefficient, for the neutralisasing ions
n_e is the electron beam density, expressed in m^{-3},
τ_{life} is the ion lifetime within the electron beam.

The influence of the recombination can be neglected since the slow secondary electron density is small (see 1.1). Also the cooling electrons are too fast to be captured by the ions. As a consequence, the equilibrium is achieved when

$$\tau_{neutr} = \tau_{life} \qquad \text{(B.2)}$$

2. ION TEMPERATURE

Since we know the ion lifetime, its energy kT_i (or simply T_i, where T_i is expressed in Joules or eV, k = Boltzmann constant = 8.62×10^{-5} eV·K^{-1}) can be determined. Immediately after ionisation the ion is almost at rest. However, during τ_{life} the ion will be heated by the electron beam and will acquire an energy kT_i. We can use the classical formula [Ref. 1, and remark below] for the temperature increase:

$$\frac{dTi}{dt} = \frac{4\pi n_e e_0^4 Z^2}{m_i v} L_c \quad [J \cdot s^{-1}] \qquad \text{(B.3)}$$

where

n_e, v are the electron-beam nominal density and velocity,
m_i is the ion mass (kg), $m_i = M_i/No$, $M_i \equiv$ ion atomic mass, No = Avogadro number,
L_c is the classical Coulomb logarithm of the order of 10 for fast electrons and slow ions,
$e_0^2 = e^2/4\pi\varepsilon_0 = 2.31 \times 10^{-28}$ m^3·kg·s^{-2}.

Remark: The factor Z^2 comes from the fact that fast electrons will penetrate the atomic electron cloud and, therefore, interact with the nucleus itself of charge (Ze). Then the usual factor $(e_0)^4$, used in the electron cooling literature, will become $e_0^2(Ze_0)^2$.

Then, for $\Delta T_i = \dot{T}_i \tau_{neutr}$, one gets:

$$\begin{aligned}\Delta T_i &\cong \frac{4\pi r_e c e_0^2 Z^2}{\beta}\frac{m}{m_i}L_c n_e \tau_{neutr} \cong 8.34\times 10^{-3}\left(\frac{Z^2 n_e^* \tau_{neutr}}{M_i \beta}\right) \quad [eV] \\ &\cong \frac{2\pi e_0^4 Z^2}{\sigma_i E_c n_r}\frac{m}{m_i}L_c\end{aligned} \tag{B.4}$$

where

r_e is the electron radius,
n_e^* is expressed in units of 10^8 cm^{-3},
n_r is the residual gas density,
σ_i is the ionisation cross section given in 1.b.

A numerical example is given in Table B.1 for $P \cong 1 \times 10^{-10}$ torr (nitrogen), $n_r \cong 2 \times 10^6$ cm^{-3}, $M_i = 14$, $Z = 7$ and $I = 3$ A.

Table B.1

Electron energy	keV	30	7
β		0.3284	0.1638
Electron beam density	cm^{-3}	0.97×10^8	1.94×10^8
Neutralisation time	s	3	3
Ion temperature	eV	2.6×10^{-1}	1.04

2.1 Some Comments on the Ion Temperature

a) The ion will be above the room temperature since $kT_i = 8.62\times10^{-5}\times273 = 2.35\times10^{-2}$ eV.

b) To a nitrogen ion having a temperature of 1 eV, will correspond a velocity $v_i = (kT_i/m_i)^{1/2} = 2625$ ms^{-1}. If v_i is the longitudinal velocity the transit time between the two trapping electrodes $\tau_{tr} = L/v_i = 1.14$ ms.

c) If v_i is again the longitudinal velocity, a difference of potential of about 1 V, crossed by the ion will be sufficient to make it reflected.

d) The ion beam is supposed to be "thermalised". In other words, we suppose the transverse energy = longitudinal energy = kΔT_i.

3. ION ESCAPE PROCESS AND DYNAMICS OF NEUTRALISATION

Let us consider the neutralisation process. Just after the electron beam is switched on, its potential φ is negative (see Fig. B1, point 1). During the time the beam is being neutralised φ is increasing until the time $t = t_{neutr}$, where φ is close to 0 V (point 2, Fig. B1). As explained before, during this time the ions are heated up to a temperature equal to ΔT_i and presumed to be thermalised (pointB 2.1d).

We will be in a stationary regime when the rate of ions escaping the e-beam, and therefore hitting the vacuum chamber, is equal to the rate of the ions being ionised. This occurs when:

$$\tau_{escape} = \tau_{neutr}.$$

The stationary regime is reached when the beam is almost neutralised so that the potential barrier φ, which prevents the ion from escaping, is drastically reduced to the level:

$$|e\varphi| < \Delta T_i \qquad \text{(B.5)}$$

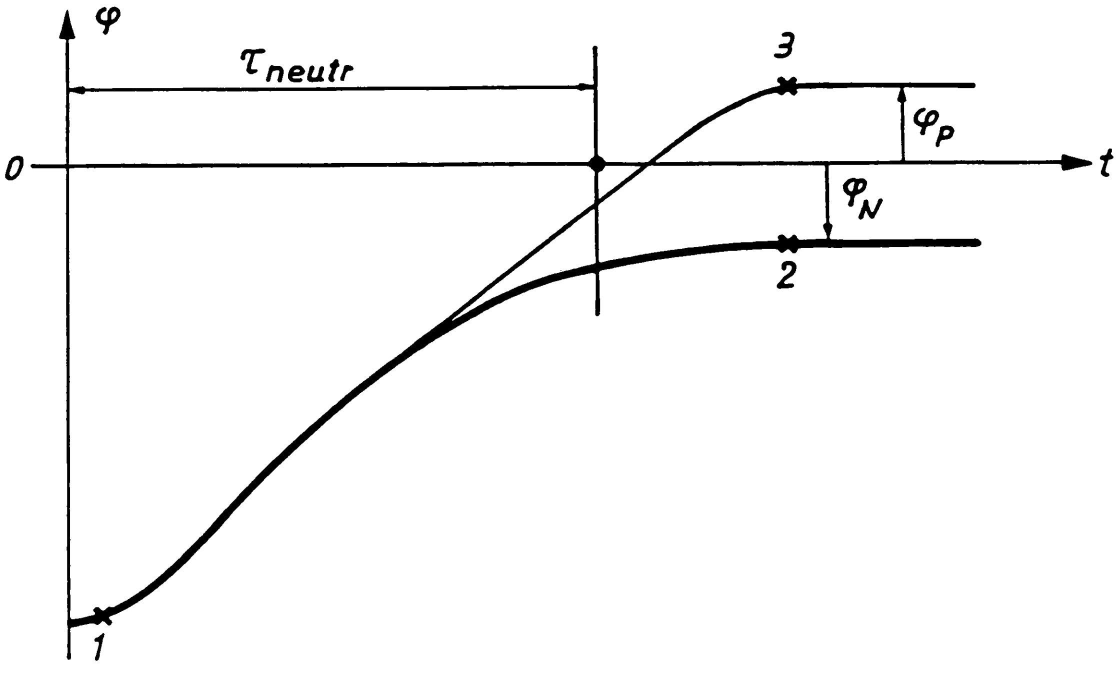

Fig. B.1 - Electron-beam potential versus time during neutralisation

Even a non-zero potential φ (or non-zero electrical field within the beam) cannot explain any escape of the ions from the beam. This is due to the magnetic field.

Consequently, we must introduce some diffusion mechanism.

We thus consider the diffusion of an ion in a magnetic field. The increase of the ion rms distance $\langle r^2 \rangle$, from the centre, can be roughly expressed by:

$$\langle r^2 \rangle = \rho_\perp \frac{t}{\tau_\parallel} \tag{B.6}$$

where: $\rho_\perp$ is the cyclotron radius,

$\tau_\parallel$ is the time between two consecutive random scatterings.

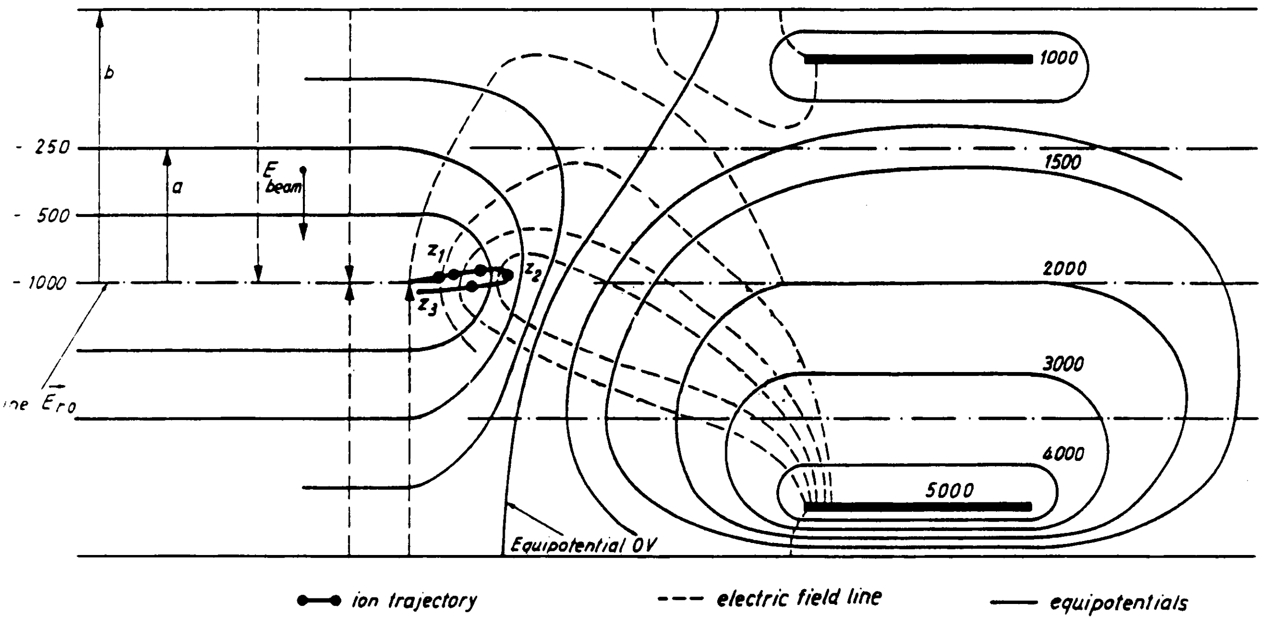

Fig. B.2 - Sketch of equipotential, electric field, ion trajectory for a non-neutralised electron beam

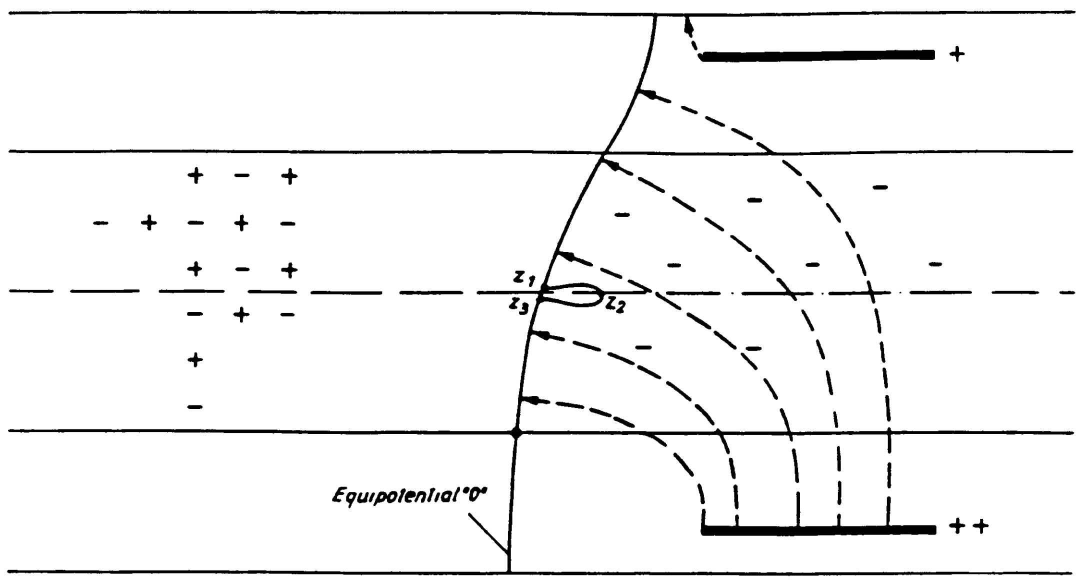

Fig. B.3 - Sketch of equipotential, electric field, ion trajectory for a neutralised electron beam

This scattering occurs, during the ion-ion collision, but mainly when reflected by the electrical field of the electric field of the neutralisation electrodes (Figs. B.2 and B.3).

Consequently, the escape time is given from B.6 when $\langle r^2 \rangle = b^2$ and so:

$$\tau_{escape} \cong \frac{b^2}{\rho_\perp^2} \frac{L}{v_T} \qquad \text{(B.7)}$$

L: distance between E_{lg} and E_{lc}

$v_T = v_\parallel$: ion longitudinal velocity $m_i v_T^2 = k\Delta T_i$.

Depending on the sign of φ, $\rho_\perp$ will be different.

a) $\varphi < 0$

In this case $\rho_\perp$ is the cyclotron radius

$$\rho_\perp = \frac{m_i v_T}{eB_0} \qquad \text{(B.8)}$$

and consequently:

$$\left(\tau_{escape}\right)_N = \left(\frac{eB_0 b}{m_i}\right)^2 \frac{L}{v_T^3} \qquad \text{(B.9)}$$

It is clear that the hypothesis of a slightly negatively charged beam implies that

$$\tau_{escape} \leq \tau_{neutr} \qquad \text{(B.10)}$$

and so

$$\varphi_N \cong -\frac{\Delta T_i}{e} \qquad \text{(B.11)}$$

From these formulae, we can find the critical magnetic field B_1

$$B_0 \leq B_1 = \frac{m_i}{eb} v_T^{3/2} \sqrt{\frac{\tau_{neutr}}{L}} \qquad \text{(B.12)}$$

b) $\varphi > 0$

When $B_0 > B_1$ the ion cannot escape quickly enough and the ion storing process will continue until the beam becomes positively charged (point 3 on Fig. B.1). The ion will acquire a cycloidal motion in the crossed fields B_0 and $E \cong \varphi / a$.

Now we must take into account the drift velocity v_d (Fig. B.4) such that

$$\rho_\perp = \frac{m_i v_d}{eB_0} \quad \text{with } v_d = \frac{E}{B_0} \cong \frac{\varphi}{aB_0} \qquad \text{(B.13)}$$

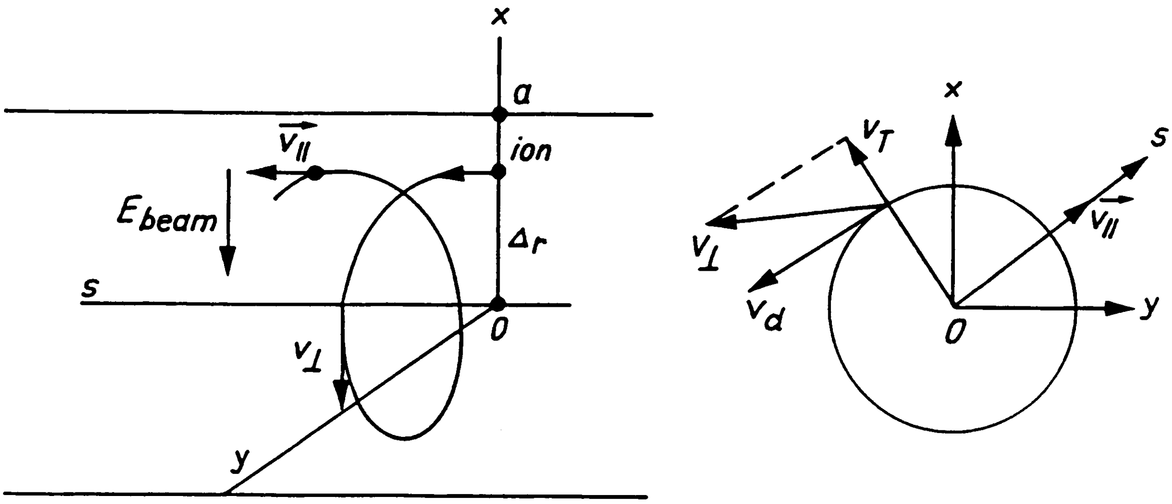

Fig. B.4 - Definition of symbols

In this new regime ($\varphi > 0$) the condition for escaping (B.10) is given by the substitution of (B.13) into (B.7):

$$\tau_{neutr} \cong \tau_{escape} \cong \left[\frac{eab}{m_i\varphi}\right]^2 B_0^4 \frac{L}{v_T} \tag{B.14}$$

and so

$$\varphi = \frac{eab}{m_i} B_0^2 \sqrt{\frac{L}{v_T \tau_{neutr}}} \tag{B.15}$$

$$B_0 > B_1$$

Table B.2 gives some values for: A = 14. L = 3 m, b = 10 cm, a = 2.5 cm.

Table B.2

Electron energy	keV	30	7
Neutralisation time,	s	3	3
Ion temperature,	eV	0.26	1.04
Ion longitudinal velocity	$m \cdot s^{-1}$	1.5×10^3	3×10^3
Critical B_1,	T	0.05	0.14

For the LEAR electron cooler, the magnetic field B_0 should not exceed 0.06 T. This means that we expect the beam to be negatively charged with:

$$\varphi \cong -\frac{\Delta T_i}{e} \cong \div\ 1\ \mathrm{V}.$$

If we take $\Delta T_i = 1$ eV and the values of V taken from Table 2, we obtain

$$\eta \cong \frac{1}{274}. \div \frac{1}{550}.$$

However, one can expect that for unforeseen reasons, we might not come to such a degree of neutralisation.