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# PROTON-ANTIPROTON INSTABILITY IN THE AA

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## 1. Introduction

Coherent instabilities on the lowest dipole mode (hiccups) leading to beam growth of dense and cool stacks have been identified in the AA. These instabilities limit the smallest vertical emittance achievable with stacks of more than typically $2 \times 10^{11}$ $\bar{p}$'s. The same is true in the horizontal plane above $3.8 \times 10^{11}$ $\bar{p}$'s.

Protons resulting from ionization by the antiprotons of the residual molecules (mainly $H_2$) and trapped in the beam potential wells of the AA long straight sections are thought to be responsible for these instabilities. Their coherent bounce frequencies correspond to the lowest transverse dipole mode of the beam (n = 3). The process is similar to the electron proton instability already seen in the ISR[1] and studied by H. Hereward[1], E. Keil and B. Zotter[2-3].

Application of E. Keil and B. Zotter's linear theory with no frequency spread (i.e. no Landau damping) to the AA observations reveals the seriousness and difficulty of ion clearing: a single neutralization pocket created by a chamber enlargement of a factor 5 over 5 cm can provoke the observed proton antiproton instability!

However, the occurrence of transverse instabilities at higher beam emittances in the process of stack cool down ("first" H, V or H & V hiccup), suggests the existence of an abnormally large neutralization pocket, with ion filling times of the order of tens of minutes. The possibility for it to be due to the 2.5 meters of ceramic chamber of the stack tail kicker system in straight section 12 is examined.

Finally, the problem of improving the ion clearing in the AA is discussed, together with some ideas to suppress these instabilities (or rather push further their threshold), which require -of course- experimental proof.

## 2. Ion Production and Trapping in the AA

Observation of ion production and trapping in the AA results essentially from two types of measurements: recording of the clearing current on a clearing electrode (QDN 13) placed in the middle of one of the two long straight sections with zero dispersion (SS 12 & 13) and tune shifts measurements with the clearing electrodes turned on or off (cf. Annex 1, Fig. 1 and Ref. 3).

The normal vacuum situation is such that with an average gauge pressure of $5 \times 10^{-11}$ Torr with 90% $H_2$ and 10% of mass 28 (CO or $N_2$), it takes about 26 seconds for an antiproton to produce an ion, with roughly equal probability for it to be $H^+$ or $CO_2^+$ (or $N_2^+$. Unfortunately, little is known on the relative fraction of protons directly produced in the ionization process. However, given the energy of the ionizing antiprotons, certainly a fraction of protons are directly created from the hydrogen molecules (cf. mass spectrometer cracking patterns and Ref. 4), this being even more true in neutralization pockets due to chamber enlargements,where ions sojourn times should be such as to give them a non-negligible chance to be further ionized. In fact, a simple analysis of the process of ion formation and clearing in pockets shows that these are finally mainly neutralized with protons after a time shorter than the primary ionization time (Annex 1). Despite an effort to have a smooth chamber together with clearing electrodes judiciously placed to minimize the residual neutralization, cross-sectional changes -and thus pockets- are hardly avoidable and exist in the AA, particularly in the long straight sections (vacuum valve bodies, scraper tanks, bellows, etc.)

## 3. Ion Bounce Frequencies and Minimum Clearing Voltages

Dipole transverse instabilities induced by ions occur when the coherent ion bounce frequency $Q_i\Omega$ in the beam potential well is roughly equal to the beam's natural transverse frequencies $(n - Q)\Omega$, and when the force exerted by the ions (i.e. their relative number) is sufficiently high to overcome the beam damping mechanisms (damper + Landau).

$Q_i$, the ion bounce number, does not depend on neutralization as long as there is only ion specie i. The instability mechanism requires some coherence in the ion motion: the ion cloud oscillates as a whole.

Annex 2 gives the ion bounce frequencies ($H_2^+$ and $H^+$) for the typical AA numbers and stack intensities for which Hiccups appear. It is clearly seen that coherence with the lowest beam modes (3-Q), which are typically around 1350 to 1400 kHz ($Q_H$, $Q_V \simeq 2.26$) are only possible with protons bouncing in the long straight sections where the beam size is smallest, at least with the actual AA intensities. Also given are the minimum clearing voltages required to effectively clear the beam around the machine, for typical chamber heights (one clearing electrode is usually placed on the bottom of the chamber).

## 4. Instability Threshold

One Hiccup lasts typically 1 minute during which the emittance increases by 10%. It consists of several (5 to 10) microinstabilities, each increasing the emittance by ~1% stepwise, until the neutralization threshold with ions of lower frequencies cannot be reached any more. The cooling system then reduces the beam size to the initial threshold and the same process starts again. The process is illustrated by the recording of AA stack emittances during cooldown shown in Fig. 2. Figure 3 shows spectrum analyser photographs of the coherent signals.

The average neutralization of the AA stack is small with the clearing system on (certainly less than 1%, cf. Annex 1). The fact that the interval between each microinstability is between 5 and 10 seconds points at residual neutralization pockets as being responsible for the instability. Ion bounce frequency calculations point at pocket with large numbers of protons in the long straight sections. This hypothesis is further supported by the analysis of the clearing current collected on the QDN 13 clearing electrode located in the middle of SS 12 and 13, during a hiccup (Fig. 5). As the instability is a clearing mechanism for pocket's protons causing it, the pocket potential deepens and the flow of ions to the clearing electrode is momentarily diminished (protons passing through the pocket are ejected). It thus seems that clearing current monitoring is an attractive way to locate eventual harmful pockets around the machine, and we plan to equip the AA with a full set of clearing current monitoring devices for the end of 1985.

The following threshold calculation using Keil and Zotter's[2] formulation without spread in frequencies shows that only one reasonable neutralization pocket is enough to explain the phenomenon (this in turn justifies the neglect of frequency spreads). With only one pocket around the machine participating into the instability, the treatment considers only the 0th harmonic of the force exerted by the pocket on the antiproton and neglects the effect of bouncing ions outside the long straight sections, whose frequencies are well out of the beam modes, at least for the number of $\bar{p}$'s and emittance considered.

To pursue the simple case (hypothesis) of a single pocket yielding the phenomenon, we start from the beam conditions leading to the instability shown in Fig. 3, namely:

$$N_{\bar{p}} = 3.11 \cdot 10^{11}$$

$$\text{emittances at onset} \begin{cases} \varepsilon_H = 3.9\pi \text{ mm.mrad} \\ \varepsilon_V = 1.9\pi \text{ mm.mrad} \end{cases}$$

$$\text{betatron tunes} \begin{cases} Q_V = 2.2575 \\ Q_H = 2.2651 \end{cases}$$

From Reference 2, the complex frequency of the system of 2 oscillators (beam + protons in pocket) is:

$$\frac{\omega}{\Omega} = Q_i + \varepsilon = 3 - Q_V + \varepsilon$$

with

$$\varepsilon = \frac{(3 - Q_i)^2 - Q_q^2}{4(3 - Q_i)} \pm \left[ \frac{[(3 - Q_i)^2 - Q_q^2]^2}{16(3 - Q_i)^2} - \frac{Q_i Q_p^2}{4(3 - Q_i)} \right]^{1/2} \qquad (1)$$

and $\Omega$ = the angular frequency of the circulating antiprotons,
$Q_i = 2\pi f_i/\Omega$ = the proton bounce number in the antiproton potential well,
$Q = (Q_V^2 + Q_p^2)^{1/2}$,
$Q_p$ = the antiproton bounce number in the proton potential well.
$\Delta$ = the determinant in expression 1).

Photo 1 of Fig. 3 gives the frequency of the unstable mode $(3 - Q_V)f_r$ = 1 379.5 kHz, normally $(3 - Q_V)f_r$ = 1 377 kHz; $Q_V$ is thus shifted by $\Delta Q_V = 1.2 \cdot 10^{-3} = -\varepsilon$.

Working backwards the ion frequency at the onset of instability for which $Q_p$ is minimum and the determinant $\Delta$ in (1) is zero:

$$\varepsilon = \frac{(3 - Q_i)^2 - Q^2}{4(3 - Q_i)} = -1.2 \cdot 10^{-3}$$

yields $Q_i$ = 0.74 (i.e. a proton frequency of 1 372.7 kHz, which is well within long straight section frequencies as can be seen from the corresponding table 4 of Annex 2).

With this frequency, one finds the value of the antiproton bounce number (a measure of the force exerted by the protons) $Q_p$ which vanishes the determinant $\Delta$ of (1), i.e. at the onset of the instability ($Q \approx Qv$):

$$Q_p > \frac{(3 - Q_i)^2 - Q_V^2}{2 \sqrt{Q_i}\,(3 - Q_i)} = 0.0044 \; .$$

With Ref. 2's definition of proton and antiproton bounce numbers:

$$Q_i^2 Q^2 = \frac{2N_p\, r_p\, c^2}{\pi\, b(a + b)R}$$

$$Q_p^2 Q^2 = \frac{2N_{\bar{p}}\, r_p\, c^2}{\pi\, b(a + b)\gamma R}$$

$\eta = N_p/N_{\bar{p}}$ (average neutralization coefficient for protons entering the instability), the neutralization required is ($\gamma$ = 3.77 for AA stack core) thus:

$$\eta_{\text{threshold}} = \gamma \frac{Q_p^2}{Q_i^2} = 1.3 \times 10^{-4} \; . \qquad (2)$$

As already said, microinstabilities (spaced by about 10 seconds) point at pockets with high neutralization. If one takes a proton pocket filling time of 10 seconds , the proton neutralization is ~40% (cf. Annex 1).

An average neutralization of $1.3 \times 10^{-4}$ is thus produced by a pocket:

$$x = 156 \times \frac{1.3 \times 10^{-4}}{0.4}\ \text{m}$$

$$\approx 5\ \text{cm long !}$$

(156 m is the AA circumference). Then, what could be the chamber enlargement, 5 cm long, corresponding to that pocket able to produce the threshold neutralization?

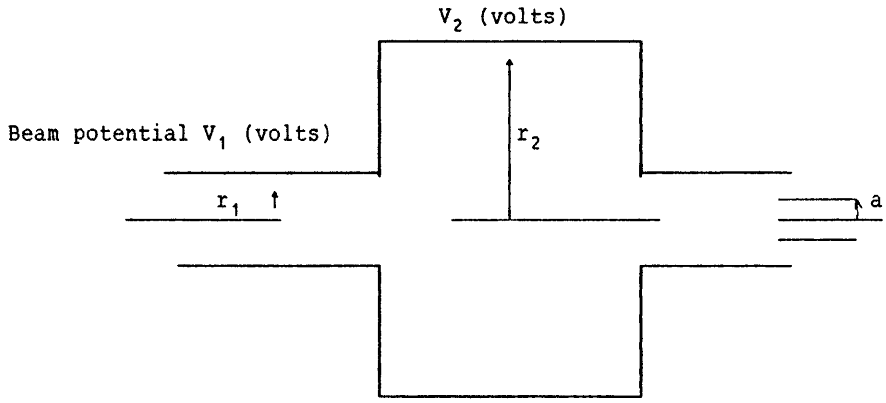
A reasonable estimate is (cf. Annex 1), an equilibrium ion population of more than 80% protons. Thus the total neutralization should be about 50%. This means that the chamber enlargement deepens the beam potential twofold. With a 4 mm diameter beam in a round chamber 60 mm in diameter, as in QDN 13 region, this means a chamber enlargement of a factor roughly 5, as the beam potential is:

$$V = \frac{\lambda}{2\pi\, \varepsilon_0} \left(L_n \frac{r_0}{a} + \frac{1}{2}\right) \qquad (3)$$

(a = beam radius, $r_0$ = chamber radius, $\lambda$ = $\bar{p}$ line density).

that is, ≈300 mm.

Such objects exist in AA long straight sections: scrapers and flying wire tanks, valve body and BLG square pick-ups when they were fully polarized to -24 volts. Of course, if the proton production in pockets is less abundant than estimated in Annex 1, the enlargement does not have to be as big to explain the mechanism, but the pocket will have to be simply longer. The effect of introducing spread in frequencies would -if anything- increase the threshold, that is the length or the number of pockets participating in the instability. Note that the threshold $Q_p$, that is essentially the neutralization, will depend upon the beam size in the chamber enlargement:




By virtue of (3) the maximum neutralization will be:

$$\eta = \frac{V_2 - V_1}{V_2} = \frac{L_n(r_2/r_1)}{L_n(r_2/a) + 1/2} .$$

If a is small compared to $r_1$ and $r_2$, $\eta$ will depend weakly on it. But if the beam nearly fills the aperture, the neutralization in the pocket can reach very high values.

## 5. Instability at High Threshold: the First "Hiccup"

Numerous observations of stack emittances during the cooldown sequence point at instabilities occurring at higher threshold, such as could be produced by a long pocket nearly fully neutralized with protons (see Fig. 2).

The threshold curve of the vertical instability is given in Fig. 4. The ion bounce number for a typical stack at which the first hiccup occurs is (see Table 1 in Annex 2):

$$Q_i = 0.65 \quad (f_i \simeq 1\ 200\ \text{kHz})$$

for which the threshold is (see Fig. 4) :

$$Q_p \simeq 0.018 .$$

Expression (2) gives the threshold average neutralization:

$$\bar{\eta}_{th} = 2.9 \times 10^{-3} ,$$

that is, for a fully neutralized pocket a length of:

$$L \simeq 1.2 \text{ m} .$$

The fact that following this instability (which discharges the pocket), the phenomenon does not occur after minutes of cooldown of emittances down to lower values than at initial threshold, could point at a pocket with unusually long filling times of the order of tens of minutes, or to a pocket which disappears after the first discharge.

Such an object could exist in long straight section 12, if the ceramic chambers (4 of 0.6 m long each) were allowed to charge negatively or positively depending on the energy of the impinging electrons from ionization of the residual gas (indeed, the initial capacitive potential well due to the ceramic and the chamber enlargement is small and would allow only a few percent neutralization).

After a careful examination of the literature[5] on electron surface interactions, it so happens that the sign of the charge taken by the ceramic will depend critically on the energy of the electrons.

From secondary emission measurements of N. Hilleret (LEP Division) on aluminium, the cross-over for negative to positive yield on an alumina surface is around 15 eV:

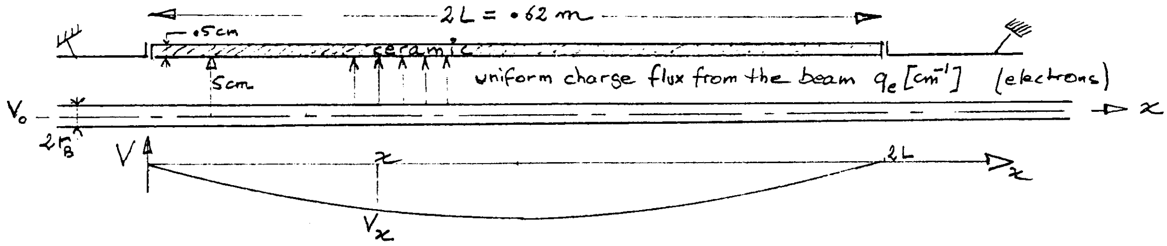
yield of secondary electrons $\eta_e < 1$ for $E < 15$ eV → negative charging of the ceramic,

$\eta_e > 1$ for $E > 15$ eV → positive charging of the ceramic.

The bulk of the spectrum of electron energies upon ionization is below 10 eV[6]. If there is little accelerating field to the chamber wall either because at the beginning of accumulation the beam potential is weak or in a condition of beam already partly neutralized, the chamber will charge negatively and neutralization will thus increase.

The process of pocket formation, that is of potential deepening due to charge accumulation is very slow, due to the very low flux of sticking negative charges and the high capacity of the chamber.

To illustrate this, we take the following example. For a given uniform electron flux and sticking coefficient, the equilibrium potential will be given by the ohmic resistance of the ceramic tube and will have a parabolic distribution along the tube:




$$V_x = q_e R[x - x^2/2L]$$

with R the resistance of the length L of the tube.

$$R = (L/S) \cdot \rho$$

with $\rho = 10^{14}$ Ωcm, ceramic resistivity,

$S = 2\pi r \cdot 0.5$ cm$^2$, surface.

The minimum voltage will be:

$$V_L = \frac{q_e RL}{2} .$$

Numerical Application

With $N_{\bar{p}} = 3.11 \times 10^{11}$, $r_B = 2$ mm, $R = 2 \times 10^{14}\ \Omega$, $L = 31$ cm, $\tau_i$ the average ionization time = 10 s, s the negative charge balance at equilibrium = 0.01, one has:

$$q_e = \frac{\text{proton line density } (\lambda)}{\tau_i \times s^{-1}} \simeq 3 \times 10^{-15}\ [\mathrm{A.cm^{-1}}]$$

and

$$V = -10 \text{ volts} .$$

The surrounding potential of the beam is:

$$V_0 = \frac{\lambda}{2\pi\varepsilon_0} \left(\log_n \frac{r}{r_B} + \frac{1}{2}\right) = -15 \text{ volts} .$$

The potential well thus created will allow 15/25 = 60% neutralization at equilibrium. With the above flux of charge ($3 \times 10^{-13}$ A/m$^{-1}$), and a capacity of around $550 \times 10^{-12}$ F/m$^{-1}$, the rate of voltage change on the ceramic will be:

$$\frac{dV}{dt} = \frac{I}{C} = -5.4 \times 10^{-4} \text{ volts } s^{-1}$$

that is, 5 hours will be necessary to reach -10 volts, or to create a pocket of 60% neutralization !

Thus the process of pocket formation by accumulation of negative charges on the ceramic is a very slow one, and this may explain the fact that one sees only one first hiccup with high threshold during stack cooldown (emittance threshold is passed before the pocket has filled up).

Another interesting point is worth mentioning, as it links to the possibility of positive charging of the ceramic: for some first hiccups a subsequent permanent drop in the clearing current collected by QDN 13 electrode (located 10 cm downstream of the ceramic chamber) is observed (Fig. 5), as if a positive potential barrier had developed in SS 12, thus preventing the normal drifting of positive ions to the clearing electrode where the beam potential is normally the lowest.

No doubt that under certain beam conditions (high intensities), after the first discharge at high threshold of the pocket in the ceramic chamber, the accelerating field of the beam with low potential to the chamber wall could be such as to communicate to the electron an energy above the threshold for positive charging of the ceramic.If during the instability the flow of positive charges has been sufficient to discharge the ceramic wall, this process could develop a positive potential barrier that we identify as a drop in clearing current.

Finally to be complete about this first hiccup, one must mention the fact that it may happen horizontally, vertically or both. In this latter case, the growth in vertical amplitude is smaller than in the horizontal plane, and this suggests strongly a relationship between the vertical oscillation of the ions and the horizontal mode of the beam, possibly through residual coupling in the machine. However, this deserves further study.

## 6. Oscillation Amplitudes and Growth Rate

From Ref. 2, the ratio between antiproton and proton amplitude is:

$$A = \frac{Y_i}{Y_p^-} = \frac{Q_i^2}{Q_i^2 - x^2}$$

with $x = \omega/\Omega = Q_i + \varepsilon$.
For small $\varepsilon$ this can be written:

$$A \simeq \frac{Q_i}{2\varepsilon} \quad .$$

For our typical instability with last paragraph's numbers, at onset of instability,

$$A \simeq 310 \quad .$$

As already seen, one microinstability yields a ~1% growth in emittance, i.e. a 0.5% growth in vertical amplitude. With our numbers (1.7 mm vertical RMS size) the antiproton coherent amplitude grows to $8.5 \times 10^{-3}$ mm while the proton's one is 2.62 mm, that is just out of the beam. Thus the instability stops when coherence is lost due to non-linearity of the proton motion; the pocket is cleared, ion accumulation can start again and the process repeat itself provided the threshold for the next microinstability has decreased, since for this one slightly higher emittances mean lower proton bounce frequencies. This is the case if some Landau damping is introduced by ion neutralization itself: Q shift due to ions is not uniform across the stack momentum distribution. After successive hiccups, the beam becomes hollow and the Q spread due to ions is reduced as their transverse density has decreased. The process finally stops when the threshold for $Q_p$ cannot be reached anymore, due to lower proton bounce frequencies and residual damping.

The growth rate of the instability is given by the positive imaginary part of the frequency shift in expression (1). As the beam size goes down with cooling, $Q_i$ and $Q_p$ change so that at onset of the instability the growth rate is zero. The $Q_i$ and $Q_p$ values reached after the time necessary for the amplitude to exceed the noise level will give the growth rate. Its maximum value could be, if by some mean one could damp the mechanism until the ions bounce frequency is equal to the beam transverse mode frequency, for a fiven $Q_p$:

$$\Omega \, I_m \, (\varepsilon) = \Omega \, \frac{Q_p}{2} \sqrt{\frac{Q_i}{3 - Q_i}} \qquad \text{(see (1))}$$

which, with our numbers, is:

$$\frac{1}{\tau} = 1.5 \times 10^4 \; s^{-1} \; ,$$

or a growth time of ~0.1 ms, 100 times smaller than the observed e folding time (Fig. 3, photos 5, 6, 7).

Attempts to use the transverse damper to control hiccups by increasing its gain are shown in photos 6, 7. Its effect is efficient on damping the instability once the ions are removed. However, it is clear that more gain is needed to effectively control the phenomenon.

## 7. AA Ion Clearing Improvements: Things Reasonable to do and Possible Artifices

At time of writing, already large improvements have been made to the clearing system -mainly dictated by other AA more tricky phantoms such as dust particles trapped in the beam: correct polarization of pick-ups in magnets, 5 new electrodes, higher clearing voltage, computer control of the clearing system. Obviously, one has to continue this effort with the future big stacks with ACOL in mind: Table 5 in Annex 2 shows that neutralization pockets all around the machine will be harmful at $10^{12}$ $\bar{p}$'s. This means avoiding where possible chamber cross-section changes and installation of new electrodes. Ceramic chambers in SS 12 should be metallized.

To mention 2 possible artifices to be soon tested, all aimed at increasing the threshold of the proton-antiproton oscillation:

a) Strong damper gain increase with a narrow active filter in the concerned beam mode frequency range.

b) Use of the skew quadrupole to increase the threshold by introducing coupling and spread in characteristic frequencies.

No doubt also that numerous experiments are still necessary to fully check some suspicions presented here:

a) proton formation in pockets (can be checked with a plasma monitor in place of a clearing electrode),

b) ceramic charging ("monitor" the first hiccup, and ... metallize !),

c) last but not least: monitor the clearing current on each clearing electrode. This will point at eventual pockets responsible for instabilities. One plans to use the electrometers of the AA ion gauge power supplies for this experiment, in the second half of 1985.

Reported by A. Poncet

## References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2. E. Keil and B. Zotter, Landau Damping of Coupled Electron-proton Oscillations, CERN/ISR-TH/71-58 (1971).

3. A. Poncet, Measurement of the Ion Clearing Electrode Current at Exit of BST5, PS/AA/ME Note 71, 22.9.1983

4. Institute of Plasma Physics, Cross-Sections for Atomic Processes, Nagoya University, Nagoya, Japan.

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6. Y. Baconnier, Neutralization of Accelerator Beams by Ionization of the Residual Gas, CERN/PS/PSR 84-24 (1984).

7. F. Rieke and W. Prepejchal, Ionization Cross-Sections of Gaseous Atoms and Molecules for High Energy Electrons, Physical Review A, Vol. 6, No. 4, October 1972.

# A N N E X  2

### *HP 9845B COMPUTER PROGRAM FOR AA ION BOUNCE FREQUENCIES AS A FUNCTION OF $N_{\bar{p}}$, TRANSVERSE AND LONGITUDINAL EMITTANCES (HICCUPS...) AND MINIMUM CLEARING VOLTAGES AND BEAM POTENTIALS*

# The beam has gaussian distributions in all 3 planes:

1. Longitudinal RMS size: $\sigma_p$
2. Vertical RMS size : $\sigma_V$
3. Horizontal RMS size : $\sigma_{HP} = (\sigma_H^2 + \sigma_p^2)^{1/2}$

<u>Remarks</u>

1. $\sigma_p$ is given directly by Simon's program: from the stack RMS frequency spread $\sigma_s$ (hertz).

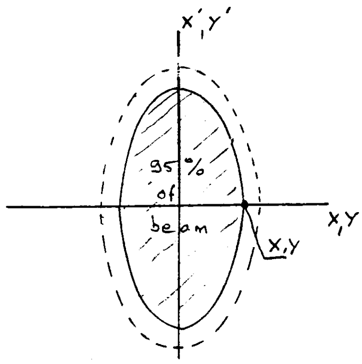
$$\sigma_p = \alpha_p \frac{\Delta p}{p} = \alpha_p \frac{1}{\eta} \cdot \frac{\sigma_s}{1.855 \times 10^6} \; ; \qquad \eta = 0.0853 \; .$$

2. $\sigma_V$ and $\sigma_H$ are obtained from 95% emittances $E_H$, $E_V$ (scaled from scraper X, Y measurements at ~QDN 13).

vertical size: $Y = \left(\frac{\beta_V E_V}{\pi}\right)^{1/2}$ ; horizontal: $X = \left(\frac{\beta_H E_H}{\pi}\right)^{1/2}$

The equation of the ellipse at QDN 13 containing 95% of particle $(\alpha_{\substack{V\\H}} \simeq 0)$ is:

$$\frac{X^2}{\sigma_H^2} + \frac{X'^2}{\sigma_H'^2} = 2.477^2 \; ; \quad \text{same for } Y.$$




Thus
$$\begin{cases} X = 2.4477\ \sigma_H \\ Y = 2.4477\ \sigma_V \end{cases} \Rightarrow \begin{cases} \sigma_H = \dfrac{X}{2.4477} \\ \sigma_V = \dfrac{Y}{2.4477} \end{cases}$$

# Ion bounce frequencies for small amplitudes around the beam axis:

$$f = \frac{1}{2\pi}\sqrt{\frac{Ke}{M}} \quad \text{[hertz)}$$

with

$$K = \frac{\partial E}{\partial x} \text{ or } \frac{\partial E}{\partial y}\bigg|_{x=y=0}$$

$$\begin{cases} M = 1.67 \times 10^{-27} \text{ kg for } H^+ \\ \quad\; = 2.34 \times 10^{-27} \text{ kg for } H_2^+ \\ e = 1.6 \times 10^{-19} \text{ coulombs} \end{cases}$$

The field gradient is, for a gaussian beam (around the beam axis):

$$\frac{\partial E}{\partial x} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{\sigma_{HP}(\sigma_{HP} + \sigma_V)}$$

$$\frac{\partial E}{\partial y} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{\sigma_V(\sigma_{HP} + \sigma_V)}$$

$\lambda = N_p \cdot e/2\pi R = N_p \cdot e/157.$

# Maximum vertical field inside the Bi gaussian beam of elliptical cross-section.

With:

$$\varrho(x,y) = \frac{\lambda}{2\pi\sigma_V\sigma_{HP}} e^{-x^2/2\sigma_{HP}^2 - y^2/2\sigma_V^2}$$

The field is (CERN/ISR-GS/75-36 by B.W.Montague):

$$E_y = \frac{\lambda}{4\pi\varepsilon_0} \int_0^\infty \frac{2\,y\,e^{-\frac{x^2}{2\sigma_{HP}^2+t} - \frac{y^2}{2\sigma_V^2+t}}}{(2\sigma_V^2+t)\sqrt{(2\sigma_V^2+t)(2\sigma_{HP}^2+t)}}\,dt$$

The vertical field is maximum for:

$$y \simeq \sigma_V$$
$$x = 0$$

Thus

$$E_{ymax} = \frac{\lambda}{4\pi\varepsilon_0} \int_0^\infty \frac{2\,\sigma_V\,e^{-\frac{\sigma_V^2}{2\sigma_V^2+t}}}{(2\sigma_V^2+t)\sqrt{(2\sigma_V^2+t)(2\sigma_{HP}^2+t)}}\,dt$$

the precision in the integration is found satisfactory if:

$$\Delta t < \min(\sigma_V/10;\ \sigma_{HP}/10)$$

and an upper limit of integration:

$$t_{max} > \max(10*\sigma_V;\ 10*\sigma_{HP})$$

From $E_{ymax}$ one obtains the minimum clearing voltage required for 100% clearing:

$$V_{CE} = E_{ymax}\,De$$

(De being the spacing between clearing plate and chamber wall)

(The length of the plate with $V_{CE}$ is always sufficient in practical cases if it is larger than the spacing,with transverse velocities larger than longitudinal ones)

The beam potential is approximated for a round uniform beam in a round chamber by:

$$V = \frac{\lambda}{2\pi\varepsilon_0}\left(\mathrm{Ln}\left(\frac{2\,De}{\sigma_V + \sigma_{HP}}\right) + \frac{1}{2}\right)$$

Annex 2

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

TABLE 3 :

(First Hiccup CONDITION)

95% HORIZONTAL EMITTANCE= 4.90E PI.mm.mrad

95% VERTICAL EMITTANCE = 2.40E PI.mm.mrad

RMS LONGITUDINAL SPREAD = 1.074 o/oo in DeltaP/P

NUMBER OF P-BARS........= 3.11E+11

| Dist. (m) | BetaH (m) | BetaV (m) | Alfap (m) | RMS Half width (mm) | RMS Half height (mm) | Horizontal frequency H+ (Khz) | Horizontal frequency H2+ (Khz) | Vertical frequency H+ (Khz) | Vertical frequency H2+ (Khz) | Max Field Kv/m | Min Clrg Voltg | Beam Potential (volts) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONG STRAIGHT SECTION--------(E DISPERSION) | | | | | | | | | | | | |
| 0.00 | 7.36 | 8.32 | 0.00 | 2.45 | 1.83 | 1147.8 | 811.8 | 1330.7 | 941.1 | .96 | -58 | -17.8 |
| .38 | 7.54 | 8.13 | 0.00 | 2.48 | 1.80 | 1139.8 | 806.0 | 1337.0 | 945.5 | .96 | -58 | -17.8 |
| .66 | 8.00 | 7.70 | 0.00 | 2.56 | 1.76 | 1119.6 | 791.8 | 1351.1 | 955.5 | .96 | -77 | -17.7 |
| 2.38 | 13.16 | 6.77 | 0.00 | 2.88 | 1.65 | 1029.3 | 727.9 | 1361.8 | 963.1 | .92 | -74 | -17.5 |
| 4.38 | 14.10 | 6.63 | 0.00 | 3.40 | 1.63 | 900.3 | 636.7 | 1299.6 | 919.1 | .83 | -67 | -16.9 |
| 6.14 | 13.69 | 7.53 | 0.00 | 3.91 | 1.74 | 791.6 | 559.8 | 1187.6 | 839.9 | .73 | -58 | -16.2 |
| 6.89 | 13.61 | 9.48 | 0.00 | 3.90 | 1.95 | 778.5 | 550.6 | 1101.5 | 779.0 | .69 | -55 | -16.0 |
| BLG MAGNET | | | | | | | | | | | | |
| 7.29 | 17.40 | 11.05 | 0.00 | 3.77 | 2.10 | 789.9 | 558.6 | 1057.7 | 748.1 | .67 | -60 | -16.0 |
| 8.30 | 12.50 | 21.00 | .50 | 3.24 | 2.90 | 833.4 | 589.4 | 881.1 | 623.2 | .59 | -53 | -15.7 |
| 12.20 | 7.71 | 33.66 | 1.63 | 3.06 | 3.67 | 819.2 | 579.3 | 748.0 | 529.0 | .49 | -59 | -15.2 |
| END OF BLG MAGNET | | | | | | | | | | | | |
| 12.66 | 7.23 | 34.80 | 1.91 | 3.18 | 3.73 | 792.9 | 560.7 | 731.9 | 517.6 | .47 | -57 | -15.1 |
| 13.40 | 7.10 | 33.91 | 2.54 | 3.64 | 3.69 | 720.1 | 509.3 | 715.7 | 506.2 | .44 | -53 | -14.7 |
| 19.20 | 15.87 | 11.52 | 8.14 | 9.50 | 2.15 | 353.5 | 250.0 | 743.4 | 525.8 | .32 | -19 | -12.1 |
| 20.00 | 15.81 | 11.02 | 8.37 | 9.73 | 2.10 | 346.7 | 245.2 | 746.1 | 527.6 | .31 | -19 | -12.0 |
| 25.70 | 5.66 | 21.87 | 6.23 | 7.09 | 2.96 | 440.7 | 311.7 | 682.0 | 482.3 | .33 | -30 | -12.9 |
| 26.45 | 5.75 | 21.89 | 6.23 | 7.09 | 2.96 | 440.4 | 311.4 | 681.6 | 482.0 | .33 | -30 | -12.9 |
| 26.80 | 7.17 | 20.95 | 6.36 | 7.25 | 2.90 | 433.6 | 306.7 | 686.0 | 485.1 | .33 | -30 | -12.9 |
| 29.70 | 11.91 | 11.63 | 7.99 | 9.13 | 2.16 | 366.2 | 259.0 | 753.3 | 532.8 | .33 | -20 | -12.3 |
| 32.20 | 13.63 | 5.28 | 10.02 | 11.45 | 1.45 | 305.9 | 216.4 | 858.5 | 607.1 | .31 | -19 | -11.5 |
| 33.00 | 13.76 | 4.74 | 10.02 | 11.46 | 1.38 | 306.7 | 216.9 | 884.4 | 625.5 | .32 | -29 | -11.5 |
| 35.50 | 12.67 | 6.63 | 7.97 | 9.15 | 1.63 | 374.6 | 264.9 | 887.4 | 627.6 | .37 | -33 | -12.5 |
| 38.40 | 8.25 | 10.54 | 6.33 | 7.28 | 2.05 | 451.2 | 319.1 | 849.2 | 600.6 | .41 | -37 | -13.3 |
| 38.74 | 7.84 | 10.98 | 6.20 | 7.13 | 2.10 | 458.7 | 324.4 | 845.6 | 598.0 | .41 | -25 | -13.4 |
| 39.10 | 7.65 | 11.22 | 6.13 | 7.04 | 2.12 | 462.9 | 327.3 | 843.7 | 596.7 | .41 | -25 | -13.5 |

$(3-Q_V) f_r = 1'377\ kHz$

$(3-Q_H) f_r = 1'350\ kHz$

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

TABLE 4 :

(SUBSEQUENT VERTICAL Hiccups Conditions)

95% HORIZONTAL EMITTANCE= 3.90E PI.mm.mrad

95% VERTICAL EMITTANCE = 1.90E PI.mm.mrad

RMS LONGITUDINAL SPREAD = 1.074 o/oo in DeltaP/P

NUMBER OF P-BARS........= 3.11E+11

| Dist. (m) | BetaH (m) | BetaV (m) | Alfap (m) | RMS Half width (mm) | RMS Half height (mm) | Horizontal frequency H+ (Khz) | Horizontal frequency H2+ (Khz) | Vertical frequency H+ (Khz) | Vertical frequency H2+ (Khz) | Max Field Kv/m | Min Clrg Voltg | Beam Potential (volts) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONG STRAIGHT SECTION--------(E DISPERSION) | | | | | | | | | | | | |
| 0.00 | 7.36 | 8.32 | 0.00 | 2.19 | 1.62 | 1287.4 | 910.4 | 1494.4 | 1056.9 | 1.10 | -66 | -18.4 |
| .38 | 7.54 | 8.13 | 0.00 | 2.22 | 1.61 | 1278.3 | 904.0 | 1501.5 | 1061.9 | 1.10 | -66 | -18.4 |
| .66 | 8.00 | 7.70 | 0.00 | 2.28 | 1.56 | 1255.6 | 888.0 | 1517.4 | 1073.1 | 1.10 | -88 | -18.4 |
| 2.38 | 13.16 | 6.77 | 0.00 | 2.57 | 1.47 | 1154.3 | 816.3 | 1529.2 | 1081.5 | 1.06 | -85 | -18.1 |
| 4.38 | 14.10 | 6.63 | 0.00 | 3.03 | 1.45 | 1009.6 | 714.0 | 1459.3 | 1032.0 | .95 | -76 | -17.5 |
| 6.14 | 13.69 | 7.53 | 0.00 | 3.49 | 1.55 | 887.6 | 627.7 | 1333.6 | 943.1 | .84 | -67 | -16.9 |
| 6.89 | 13.61 | 9.48 | 0.00 | 3.48 | 1.73 | 873.0 | 617.4 | 1236.9 | 874.7 | .79 | -63 | -16.7 |
| BLG MAGNET | | | | | | | | | | | | |
| 7.29 | 17.40 | 11.05 | 0.00 | 3.37 | 1.87 | 885.9 | 626.5 | 1187.8 | 840.0 | .78 | -70 | -16.6 |
| 8.30 | 12.50 | 21.00 | .50 | 2.90 | 2.58 | 932.2 | 659.3 | 988.7 | 699.2 | .69 | -62 | -16.4 |
| 12.20 | 7.71 | 33.66 | 1.63 | 2.84 | 3.27 | 892.2 | 631.0 | 832.4 | 588.7 | .57 | -68 | -15.8 |
| END OF BLG MAGNET | | | | | | | | | | | | |
| 12.66 | 7.23 | 34.80 | 1.91 | 2.99 | 3.32 | 856.9 | 606.0 | 812.4 | 574.6 | .55 | -66 | -15.6 |
| 13.40 | 7.10 | 33.91 | 2.54 | 3.47 | 3.28 | 767.8 | 543.0 | 790.3 | 558.9 | .51 | -61 | -15.2 |
| 19.20 | 15.87 | 11.52 | 8.14 | 9.35 | 1.91 | 362.4 | 256.3 | 801.6 | 566.9 | .34 | -20 | -12.3 |
| 20.00 | 15.81 | 11.02 | 8.37 | 9.58 | 1.87 | 355.1 | 251.1 | 803.8 | 568.5 | .34 | -20 | -12.2 |
| 25.70 | 5.66 | 21.87 | 6.23 | 7.01 | 2.63 | 452.4 | 319.9 | 738.0 | 521.9 | .36 | -33 | -13.2 |
| 26.45 | 5.75 | 21.89 | 6.23 | 7.01 | 2.63 | 452.1 | 319.7 | 737.6 | 521.7 | .36 | -33 | -13.2 |
| 26.80 | 7.17 | 20.95 | 6.36 | 7.17 | 2.58 | 445.1 | 314.8 | 742.1 | 524.8 | .36 | -33 | -13.1 |
| 29.70 | 11.91 | 11.63 | 7.99 | 9.02 | 1.92 | 374.2 | 264.7 | 811.2 | 573.7 | .35 | -21 | -12.4 |
| 32.20 | 13.63 | 5.28 | 10.02 | 11.31 | 1.29 | 311.4 | 220.2 | 920.8 | 651.2 | .33 | -20 | -11.6 |
| 33.00 | 13.76 | 4.74 | 10.02 | 11.32 | 1.23 | 312.1 | 220.7 | 948.4 | 670.7 | .33 | -30 | -11.7 |
| 35.50 | 12.67 | 6.63 | 7.97 | 9.03 | 1.45 | 382.3 | 270.3 | 954.0 | 674.7 | .39 | -35 | -12.7 |
| 38.40 | 8.25 | 10.54 | 6.33 | 7.18 | 1.83 | 462.2 | 326.9 | 916.2 | 648.0 | .44 | -39 | -13.5 |
| 38.74 | 7.84 | 10.98 | 6.20 | 7.03 | 1.87 | 470.1 | 332.4 | 912.6 | 645.4 | .44 | -26 | -13.6 |
| 39.10 | 7.65 | 11.22 | 6.13 | 6.95 | 1.89 | 474.4 | 335.5 | 910.8 | 644.1 | .44 | -27 | -13.7 |

Annex 2

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

95% HORIZONTAL EMITTANCE= 8.500 PI.mm.mrad

95% VERTICAL EMITTANCE = 3.400 PI.mm.mrad

RMS LONGITUDINAL SPREAD = 1.074 o/oo in DeltaP/P

NUMBER OF P-BARS........= 3.88E+11

TABLE 1 :

(FIRST HICCUP CONDITION)

$(3-Q_r) f_r = 1'377\,kHz$

$(3-Q_H) f_r = 1'350$

| Dist. (m) | BetaH (m) | BetaV (m) | Alfap (m) | RMS Half width (mm) | RMS Half height (mm) | Horizontal frequency H+ (Khz) | Horizontal frequency H2+ (Khz) | Vertical frequency H+ (Khz) | Vertical frequency H2+ (Khz) | Max Field Kv/m | Min Clrg Voltg (volts) | Beam Potential (volts) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONG STRAIGHT SECTION--------(0 DISPERSION) | | | | | | | | | | | | |
| 0.00 | 7.36 | 8.32 | 0.00 | 3.23 | 2.17 | 994.1 | 703.0 | 1212.2 | 857.3 | .91 | -55 | -20.5 |
| .38 | 7.54 | 8.13 | 0.00 | 3.27 | 2.15 | 986.8 | 697.9 | 1217.7 | 861.1 | .91 | -54 | -20.5 |
| .66 | 3.00 | 7.70 | 0.00 | 3.37 | 2.09 | 968.6 | 685.0 | 1229.7 | 869.7 | .91 | -73 | -20.5 |
| 2.38 | 10.16 | 6.77 | 0.00 | 3.80 | 1.96 | 888.6 | 628.4 | 1236.7 | 874.6 | .87 | -70 | -20.1 |
| 4.38 | 14.10 | 6.63 | 0.00 | 4.47 | 1.94 | 775.7 | 548.6 | 1177.9 | 833.0 | .78 | -62 | -19.3 |
| 6.14 | 13.69 | 7.53 | 0.00 | 5.15 | 2.07 | 681.5 | 481.9 | 1075.5 | 760.6 | .67 | -54 | -18.5 |
| 6.89 | 13.61 | 9.48 | 0.00 | 5.14 | 2.32 | 671.1 | 474.6 | 998.8 | 706.4 | .63 | -50 | -18.2 |
| BLG MAGNET-------- | | | | | | | | | | | | |
| 7.29 | 17.40 | 11.05 | 0.00 | 4.97 | 2.50 | 681.8 | 482.2 | 960.3 | 679.1 | .62 | -55 | -18.2 |
| 8.30 | 12.50 | 21.00 | .50 | 4.25 | 3.45 | 726.7 | 513.9 | 805.9 | 569.9 | .53 | -48 | -18.0 |
| 12.20 | 7.71 | 33.66 | 1.63 | 3.74 | 4.37 | 753.9 | 533.2 | 697.6 | 493.4 | .46 | -55 | -17.6 |
| END OF BLG MAGNET-------- | | | | | | | | | | | | |
| 12.66 | 7.23 | 34.80 | 1.91 | 3.80 | 4.44 | 741.7 | 524.5 | 686.2 | 485.3 | .44 | -53 | -17.5 |
| 13.40 | 7.10 | 33.91 | 2.54 | 4.19 | 4.39 | 693.5 | 490.5 | 677.4 | 479.1 | .43 | -51 | -17.3 |
| 19.20 | 15.87 | 11.52 | 8.14 | 10.02 | 2.56 | 370.0 | 261.7 | 732.5 | 518.1 | .34 | -20 | -14.5 |
| 20.00 | 15.81 | 11.02 | 8.37 | 10.23 | 2.50 | 363.9 | 257.4 | 736.2 | 520.6 | .34 | -20 | -14.4 |
| 25.70 | 5.66 | 21.87 | 6.23 | 7.37 | 3.52 | 463.9 | 328.1 | 670.7 | 474.3 | .35 | -32 | -15.6 |
| 26.45 | 5.75 | 21.89 | 6.23 | 7.37 | 3.52 | 463.4 | 327.7 | 670.3 | 474.0 | .35 | -32 | -15.5 |
| 26.80 | 7.17 | 20.95 | 6.36 | 7.54 | 3.45 | 456.4 | 322.7 | 674.9 | 477.3 | .35 | -32 | -15.5 |
| 29.70 | 11.91 | 11.63 | 7.99 | 9.52 | 2.57 | 387.3 | 273.9 | 745.5 | 527.2 | .36 | -21 | -14.8 |
| 32.20 | 13.63 | 5.28 | 10.02 | 11.93 | 1.73 | 325.4 | 230.1 | 854.3 | 604.2 | .35 | -21 | -13.9 |
| 33.00 | 13.76 | 4.74 | 10.02 | 11.94 | 1.64 | 326.3 | 230.8 | 880.3 | 622.6 | .36 | -32 | -14.0 |
| 35.50 | 12.67 | 6.63 | 7.97 | 9.55 | 1.94 | 396.4 | 280.3 | 879.8 | 622.2 | .41 | -37 | -15.2 |
| 38.40 | 3.25 | 10.54 | 6.33 | 7.61 | 2.45 | 474.7 | 335.7 | 837.6 | 592.3 | .44 | -40 | -16.1 |
| 38.74 | 7.84 | 10.98 | 6.20 | 7.45 | 2.50 | 482.6 | 341.3 | 833.7 | 589.6 | .45 | -27 | -16.2 |
| 39.10 | 7.65 | 11.22 | 6.13 | 7.36 | 2.52 | 486.8 | 344.3 | 831.7 | 588.2 | .45 | -27 | -16.2 |

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

95% HORIZONTAL EMITTANCE= 4.500 PI.mm.mrad

95% VERTICAL EMITTANCE = 2.200 PI.mm.mrad

RMS LONGITUDINAL SPREAD = 1.074 o/oo in DeltaP/P

NUMBER OF P-BARS........= 3.88E+11

TABLE 2 :

(SUBSEQUENT HICCUPS CONDITIONS)

V & H

| Dist. (m) | BetaH (m) | BetaV (m) | Alfap (m) | RMS Half width (mm) | RMS Half height (mm) | Horizontal frequency H+ (Khz) | Horizontal frequency H2+ (Khz) | Vertical frequency H+ (Khz) | Vertical frequency H2+ (Khz) | Max Field Kv/m | Min Clrg Voltg (volts) | Beam Potential (volts) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONG STRAIGHT SECTION--------(0 DISPERSION) | | | | | | | | | | | | |
| 0.00 | 7.36 | 8.32 | 0.00 | 2.35 | 1.75 | 1338.1 | 946.3 | 1552.0 | 1097.6 | 1.26 | -76 | -22.5 |
| .38 | 7.54 | 8.13 | 0.00 | 2.38 | 1.73 | 1328.7 | 939.7 | 1559.3 | 1102.8 | 1.26 | -76 | -22.5 |
| .66 | 3.00 | 7.70 | 0.00 | 2.45 | 1.68 | 1305.2 | 923.0 | 1575.8 | 1114.5 | 1.26 | -100 | -22.4 |
| 2.38 | 10.16 | 6.77 | 0.00 | 2.76 | 1.58 | 1199.9 | 848.6 | 1588.2 | 1123.2 | 1.21 | -97 | -22.1 |
| 4.38 | 14.10 | 6.63 | 0.00 | 3.25 | 1.56 | 1049.5 | 742.2 | 1515.6 | 1071.9 | 1.09 | -87 | -21.4 |
| 6.14 | 13.69 | 7.53 | 0.00 | 3.75 | 1.66 | 922.7 | 652.6 | 1385.1 | 979.6 | .96 | -77 | -20.5 |
| 6.89 | 13.61 | 9.48 | 0.00 | 3.74 | 1.87 | 907.5 | 641.8 | 1284.7 | 908.5 | .90 | -72 | -20.3 |
| BLG MAGNET-------- | | | | | | | | | | | | |
| 7.29 | 17.40 | 11.05 | 0.00 | 3.62 | 2.01 | 920.8 | 651.2 | 1233.6 | 872.4 | .88 | -80 | -20.2 |
| 8.30 | 12.50 | 21.00 | .50 | 3.11 | 2.78 | 970.7 | 686.5 | 1027.4 | 726.6 | .78 | -70 | -19.9 |
| 12.20 | 7.71 | 33.66 | 1.63 | 2.98 | 3.52 | 945.1 | 668.4 | 869.5 | 615.0 | .65 | -78 | -19.2 |
| END OF BLG MAGNET-------- | | | | | | | | | | | | |
| 12.66 | 7.23 | 34.80 | 1.91 | 3.11 | 3.57 | 912.2 | 645.1 | 850.1 | 601.2 | .62 | -75 | -19.0 |
| 13.40 | 7.10 | 33.91 | 2.54 | 3.57 | 3.53 | 824.4 | 583.0 | 829.7 | 586.8 | .58 | -70 | -18.6 |
| 19.20 | 15.87 | 11.52 | 8.14 | 9.44 | 2.06 | 398.7 | 281.9 | 854.2 | 604.1 | .40 | -24 | -15.2 |
| 20.00 | 15.81 | 11.02 | 8.37 | 9.67 | 2.01 | 390.9 | 276.4 | 857.0 | 606.1 | .40 | -24 | -15.1 |
| 25.70 | 5.66 | 21.87 | 6.23 | 7.06 | 2.83 | 497.2 | 351.6 | 784.6 | 554.9 | .43 | -39 | -16.2 |
| 26.45 | 5.75 | 21.89 | 6.23 | 7.06 | 2.84 | 496.9 | 351.4 | 784.2 | 554.6 | .43 | -39 | -16.2 |
| 26.80 | 7.17 | 20.95 | 6.36 | 7.22 | 2.77 | 489.3 | 346.0 | 789.2 | 558.1 | .43 | -39 | -16.2 |
| 29.70 | 11.91 | 11.63 | 7.99 | 9.09 | 2.07 | 412.5 | 291.7 | 865.1 | 611.8 | .42 | -25 | -15.4 |
| 32.20 | 13.63 | 5.28 | 10.02 | 11.40 | 1.39 | 344.1 | 243.3 | 984.4 | 696.2 | .40 | -24 | -14.4 |
| 33.00 | 13.76 | 4.74 | 10.02 | 11.40 | 1.32 | 345.0 | 244.0 | 1014.1 | 717.2 | .40 | -36 | -14.4 |
| 35.50 | 12.67 | 6.63 | 7.97 | 9.10 | 1.56 | 421.7 | 298.2 | 1018.5 | 720.3 | .47 | -42 | -15.7 |
| 38.40 | 3.25 | 10.54 | 6.33 | 7.24 | 1.97 | 508.7 | 359.7 | 976.0 | 690.2 | .52 | -47 | -16.7 |
| 38.74 | 7.84 | 10.98 | 6.20 | 7.09 | 2.01 | 517.3 | 365.8 | 972.0 | 687.4 | .53 | -32 | -16.8 |
| 39.10 | 7.65 | 11.22 | 6.13 | 7.01 | 2.03 | 521.9 | 369.1 | 969.9 | 685.9 | .53 | -32 | -16.9 |

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

## AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

95% HORIZONTAL EMITTANCE= 2.00E PI.mm.mrad

95% VERTICAL EMITTANCE = 2.00E PI.mm.mrad

RMS LONGITUDINAL SPREAD = 1.074 o/oo in DeltaP/P

NUMBER OF P-BARS........= 1.00E+12

| Dist. (m) | BetaH (m) | BetaV (m) | Alfap (m) | RMS Half width (mm) | RMS Half height (mm) | Horizontal frequency H+ (Khz) | Horizontal frequency H2+ (Khz) | Vertical frequency H+ (Khz) | Vertical frequency H2+ (Khz) | Max Field Kv/m | Min Clrg Voltg (volts) | Beam Potential (volts) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LONG STRAIGHT SECTION--------(0 DISPERSION)---------------------------- | | | | | | | | | | | | |
| 0.00 | 7.36 | 8.32 | 0.00 | 1.57 | 1.67 | 2962.1 | 2094.8 | 2872.7 | 2031.6 | 4.16 | -250 | -62 |
| .38 | 7.54 | 8.13 | 0.00 | 1.59 | 1.65 | 2944.3 | 2082.2 | 2889.4 | 2043.4 | 4.17 | -250 | -62 |
| .66 | 8.00 | 7.70 | 0.00 | 1.63 | 1.60 | 2899.5 | 2050.5 | 2927.3 | 2070.2 | 4.19 | -335 | -62 |
| 2.38 | 10.16 | 6.77 | 0.00 | 1.84 | 1.50 | 2687.0 | 1900.3 | 2974.0 | 2103.3 | 4.10 | -328 | -62 |
| 4.38 | 14.10 | 6.63 | 0.00 | 2.17 | 1.49 | 2367.6 | 1674.4 | 2859.1 | 2022.0 | 3.75 | -300 | -60 |
| 6.14 | 13.69 | 7.53 | 0.00 | 2.50 | 1.59 | 2088.2 | 1476.8 | 2621.1 | 1853.7 | 3.32 | -266 | -58 |
| 6.89 | 19.61 | 9.48 | 0.00 | 2.49 | 1.78 | 2043.9 | 1445.5 | 2419.4 | 1711.0 | 3.11 | -249 | -57 |
| BLG MAGNET---------------------------------------------------------- | | | | | | | | | | | | |
| 7.29 | 17.40 | 11.05 | 0.00 | 2.41 | 1.92 | 2064.3 | 1459.9 | 2312.4 | 1635.4 | 3.03 | -272 | -57 |
| 8.30 | 12.50 | 21.00 | .50 | 2.11 | 2.65 | 2103.3 | 1487.5 | 1878.6 | 1328.6 | 2.55 | -236 | -55 |
| 12.20 | 7.71 | 33.66 | 1.63 | 2.38 | 3.35 | 1808.3 | 1278.9 | 1522.1 | 1076.5 | 1.96 | -235 | -52 |
| END OF BLG MAGNET--------------------------------------------------- | | | | | | | | | | | | |
| 12.66 | 7.23 | 34.80 | 1.91 | 2.57 | 3.41 | 1699.6 | 1202.0 | 1476.9 | 1044.5 | 1.85 | -222 | -51 |
| 13.40 | 7.10 | 33.91 | 2.54 | 3.13 | 3.36 | 1478.1 | 1045.3 | 1426.4 | 1008.7 | 1.69 | -203 | -50 |
| 19.20 | 15.87 | 11.52 | 8.14 | 9.06 | 1.96 | 667.3 | 471.9 | 1434.4 | 1014.5 | 1.11 | -66 | -40 |
| 20.00 | 15.81 | 11.02 | 8.37 | 9.30 | 1.92 | 653.0 | 461.8 | 1437.8 | 1016.8 | 1.09 | -66 | -40 |
| 25.70 | 5.66 | 21.87 | 6.23 | 6.86 | 2.70 | 823.7 | 582.5 | 1312.2 | 928.0 | 1.17 | -106 | -42 |
| 26.45 | 5.75 | 21.89 | 6.23 | 6.86 | 2.70 | 823.4 | 582.3 | 1311.7 | 927.7 | 1.17 | -106 | -42 |
| 26.80 | 7.17 | 20.95 | 6.36 | 7.01 | 2.64 | 811.1 | 573.6 | 1320.1 | 933.6 | 1.17 | -105 | -42 |
| 29.70 | 11.91 | 11.63 | 7.99 | 8.81 | 1.97 | 684.1 | 483.8 | 1446.8 | 1023.2 | 1.13 | -68 | -40 |
| 32.20 | 18.63 | 5.28 | 10.02 | 11.05 | 1.33 | 570.2 | 403.3 | 1645.1 | 1163.5 | 1.07 | -64 | -38 |
| 33.00 | 18.76 | 4.74 | 10.02 | 11.05 | 1.26 | 571.8 | 404.4 | 1694.8 | 1198.6 | 1.09 | -98 | -38 |
| 35.50 | 12.67 | 6.63 | 7.97 | 8.81 | 1.49 | 700.5 | 495.4 | 1704.2 | 1205.2 | 1.28 | -115 | -41 |
| 38.40 | 8.25 | 10.54 | 6.33 | 7.00 | 1.88 | 846.0 | 598.3 | 1634.4 | 1155.9 | 1.42 | -128 | -44 |
| 38.74 | 7.84 | 10.98 | 6.20 | 6.85 | 1.91 | 860.2 | 608.3 | 1627.6 | 1151.1 | 1.43 | -86 | -44 |
| 39.10 | 7.65 | 11.22 | 6.13 | 6.78 | 1.94 | 867.9 | 613.8 | 1624.1 | 1148.6 | 1.44 | -86 | -44 |

AA ION BOUNCE FREQUENCIES (F=eE and zero neutralisation)

## - TABLE 5 -

AA nominal conditions

# ANNEX 1

## OBSERVATION OF ION ACCUMULATION AND CLEARING IN AA $\bar{p}$ STACKS

(Summary)

### 1. AA Clearing System and Clearing Current Measuring Electrode

The AA clearing system has been designed to give an average neutralization of $\eta = 2 \times 10^{-3}$ (Fig. 1), with special focus on smooth chambers to avoid neutralization pockets, and electrodes at edges of magnets where drift velocities are small. This resulted in 30 clearing electrodes divided in two sets: 12 beam position pick-ups polarized to -24 volts and 18 plates placed in chamber transitions. One of them, QDN 13, is used in connection with an electrometer and a power supply to monitor the clearing current collected over approximately half (~7 m) of one of the two long straight sections where the beam is smallest.

### 2. Ion Production Rate

(See also PS/AA/ME/Note 71).

At end of 1984, the average gauge pressure in the AA was ~$5 \times 10^{-11}$ Torr with 90% $H_2$ and 10% CO (traces of $CH_4$), i.e. molecular densities of

$$n_{H_2} = 4.24 \times 10^{12} \quad [m^{-3}]$$

$$n_{CO} = 4.20 \times 10^{11} \quad [m^{-3}] \ .$$

With the ionization cross-sections for $\bar{p}$ of 3.5 GeV/c (from Bethe's formula[7]):

$$\sigma_{H_2} = 2.1 \times 10^{-23} \quad [m^2]$$

$$\sigma_{CO} = 9.6 \times 10^{-23} \quad [m^2] \ ,$$

the ionization times are ($\tau_i = 1/\sigma_i nc$): $\tau_{H_2^+} = 37$ s

$$\tau_{CO^+} = 83 \text{ s} \ .$$

That is, an average of 26 s for our molecular composition. This fits well with QDN 13 clearing current observations.

Example with $N_{\bar{p}} = 1.9 \times 10^{11}$ (15.11.1984)

a) With only QDN 13 ON, one measures I = 1.28 $10^{-9}$ A (calculated with $\tau$ = 25 s, I = 1.21 $\times 10^{-9}$ A if all ions created in the machine end up onto QDN13 clearing plate).

b) With all clearing ON, one measures on QDN13 I = 3 $\times 10^{-11}$ A (the calculated current if QDN 13 clears 7 m -one half of SS 12 & 13 where the average pressure is $2 \times 10^{-11}$ Torr- is I = 2.2 $\times 10^{-11}$ A).

### 3. Linear Tune Shifts due to Ions

$$\Delta Q_{x,y} = r_p \frac{1}{\gamma} \frac{R}{Q_{x,y}} \frac{1}{\pi a(a+b)} \eta N_{\bar{p}}$$

For a cool stack of typically $1\pi.10^{-6}$ m.rad emittances and a longitudinal spread of +/- .003, this gives:

$$\Delta Q_{x,y} = 2.3 \times 10^{-14} \eta N_{\bar{p}}$$

Example

4th November 1984: $1.7 \times 10^{11}$ $\bar{p}$, $\varepsilon_{xy} \approx 1\pi \times 10^{-6}$ give a calculated $\Delta Q = 4 \times 10^{-3}\ \eta$. The measured $\Delta Q$ with all clearing off was $5 \times 10^{-3}$ vertically,
$4 \times 10^{-3}$ horizontally.

The fit is good and means that:

a) the beam can be fully neutralized with clearing off; that is, there is no significant absolute self clearing,

b) with clearing on, the neutralization is low, or certainly not more than a few percent.

4. Which Ion Species in Pockets? (we limit ourselves to double ions)

Let us consider 2 steps:

a) The pocket is initially empty (such as after a hiccup, for instance). At first, we neglect the primary multiple ionization (maybe a few percent for $H_2$ if mass spectrometer cracking patterns are extrapolable to high energy ionizing particles).

The production of simple ions is:

$$d_{i^+} = d_{\bar{p}} d_{m_i} \sigma_i c t - d_{i^{++}}$$

For double ions:

$$d_{i^{++}} = d_p d_{i^+} \sigma_i c t$$

(with a secondary ionization cross-section not too different from the primary one, and $d_{i^+}$, $d_{i^{++}}$, $d_{m_i}$, $d_{\bar{p}}$ respectively the simple ion, double ion, molecular and antiproton densities; t is the time).

Thus:

$$d_{i^+} = \frac{d_{\bar{p}} d_{m_i} \sigma_i c t}{1 + d_{\bar{p}} \sigma_i c t} \quad ; \qquad d_{i^{++}} = \frac{d_{\bar{p}}^2 d_{m_i} \sigma_i^2 c^2 t^2}{1 + d_{\bar{p}} \sigma_i c t}$$

giving the production rates:

$$Rp_{i^+} = \frac{d(d_{i^+})}{dt} = \frac{d_{\bar{p}} d_{m_i} \sigma_i c}{(1 + d_{\bar{p}} \sigma_i c t)^2} \quad ; \qquad Rp_{i^{++}} = \frac{d(d_{i^{++}})}{dt} = \frac{2 d_{\bar{p}}^2 d_{m_i} \sigma_i^2 c^2 t + d_{\bar{p}}^3 d_{m_i} \sigma_i^3 c^3 t^2}{(1 + d_{\bar{p}} \sigma_i c t)^2}$$

initially, the neutralization process is not linear; it becomes linear once the simple ion density $d_{i^+}$ exceeds the molecular density,

$$d_{\bar{p}}\ \sigma_i c\ t > 1$$

that is very early in the accumulation process if $d_{\bar{p}} > d_{m_i}$, which is our case.Then,

$$d_{i^+} \approx d_{m_i} ; \qquad d_i^{++} \approx d_{\bar{p}}\ d_{m_i} \sigma_i c\ t$$

$\eta$, the ratio of ion to beam charge is:

$$\eta = \frac{d_{i^+} + 2 d_{i^{++}}}{d_{\bar{p}}}$$

Numbers are, for typical AA hiccup conditions (Annex 2) of $p = 2 \cdot 10^{-11}$ Torr (pocket in long straight section 24/1 and 12/13) 90% $H_2$, 10% CO and RMS beam dimensions of ~2.5 mm, $N_{\bar{p}} = 2.5 \cdot 10^{11}$:

$$\eta(t) = 0.042(H_2^+) + 0.004(CO^+) + 0.053\ t\ (H^+) + 0.024\ t\ (CO^{++})$$

Full neutralization will be reached when $\eta \approx \eta_{H^+} + \eta_{CO^{++}} = 1$, that is in about 15 s. Thus a pocket will mainly charge with protons and multiply charged heavy ions, in a time shorter than the primary average ionization time of 25 s, due to the higher density of primary ions and $\bar{p}$'s than the molecular densities.

b) Once nearly full permitted neutralization is reached, heavy ions will gain the necessary escape energy by coulomb heating faster than light ones; the maximum proportion of heavy ions will be reached when their clearing rate equals their production rate. Then their proportion will decrease and over periods of seconds light ions will replace heavier ones and the final population will be mainly composed of protons.

Indeed, the clearing rate by coulomb heating is (see ISR Performance Report ISR/VA-OG/sm of 21.2.1978):

$$R_c = \frac{1}{\Delta V}\frac{dE}{dt} = \frac{2m_e c^2 r_e^2}{\Delta V}\,\frac{m_e Z}{2m_p}\,\frac{N_{\bar{p}} c}{\pi r_B^2 R}\,\ln(4.10^4\ Z^{-2/3})\ ,$$

i.e.

$$R_c \propto \frac{m_e}{2m_p} Z$$

with Z the atomic number, $m_e$ and $m_p$ the electron and proton mass, $r_e$ the radius of the electron, $r_B$ the beam radius and R the machine radius. $\Delta V$ is the escape energy, or the difference between the maximum possible beam potential for the fully neutralized pocket and the growing potential of the pocket in the process of accumulating ions.

To give numbers, for typical AA hiccup situation, with $\Delta V = 0.1$ V, $3.11 \cdot 10^{11}$ $\bar{p}$'s and $1\pi$ emittances, the coulomb clearing rates are:

| | | | |
|---|---|---|---|
| $H^+$ | $Z = 1$, | $R_c = 1.74 \cdot 10^{-2}\ s^{-1}$, | $\tau_c \approx 1$ min |
| $H_2^+$ | $Z = 2$, | $R_c = 3.35 \cdot 10^{-2}$, | $\tau_c \approx 30$ s |
| $CO^+$, $CO^{++}$ | $Z = 14$, | $R_c = 0.2$, | $\tau_c \approx 5$ s |
| $CO_2^+$ | $Z = 22$, | $R_c = 0.31$, | $\tau_c \approx 3$ s |

The process of accumulation for ion i can be expressed by:

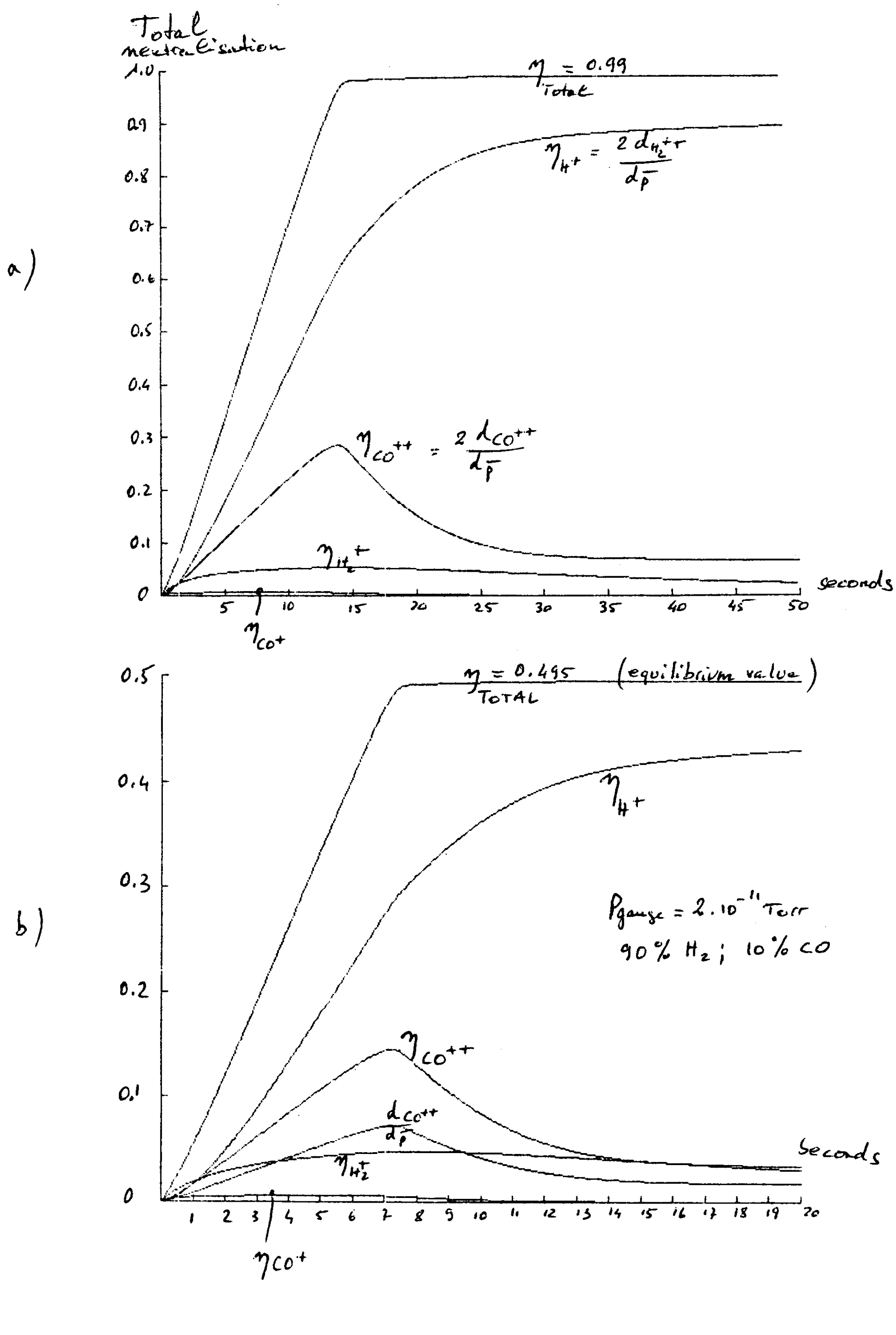
$$\frac{d(d_i)}{dt} = R_p^i - d_i R_c^i \quad \text{or} \quad \frac{d\eta_i}{dt} = \frac{R_p^i}{d_{\bar{p}}} - \eta_i R_c^i$$

where the production rate $R_p$ has been given above and the clearing rate $R_c^i \propto (Z_i/(\eta_p - \eta_T))$ with $\eta_p$ the maximum permissible neutralization given the depth of the pocket and $\eta_T = \sum \eta_i(t)$ the growing total neutralization. The process is highly non-linear, and for the numbers above, neglecting other ions than $H^+$, $H_2^+$, $CO^+$, $CO^{++}$ and taking into consideration the primary double ionization rate of $H_2$,

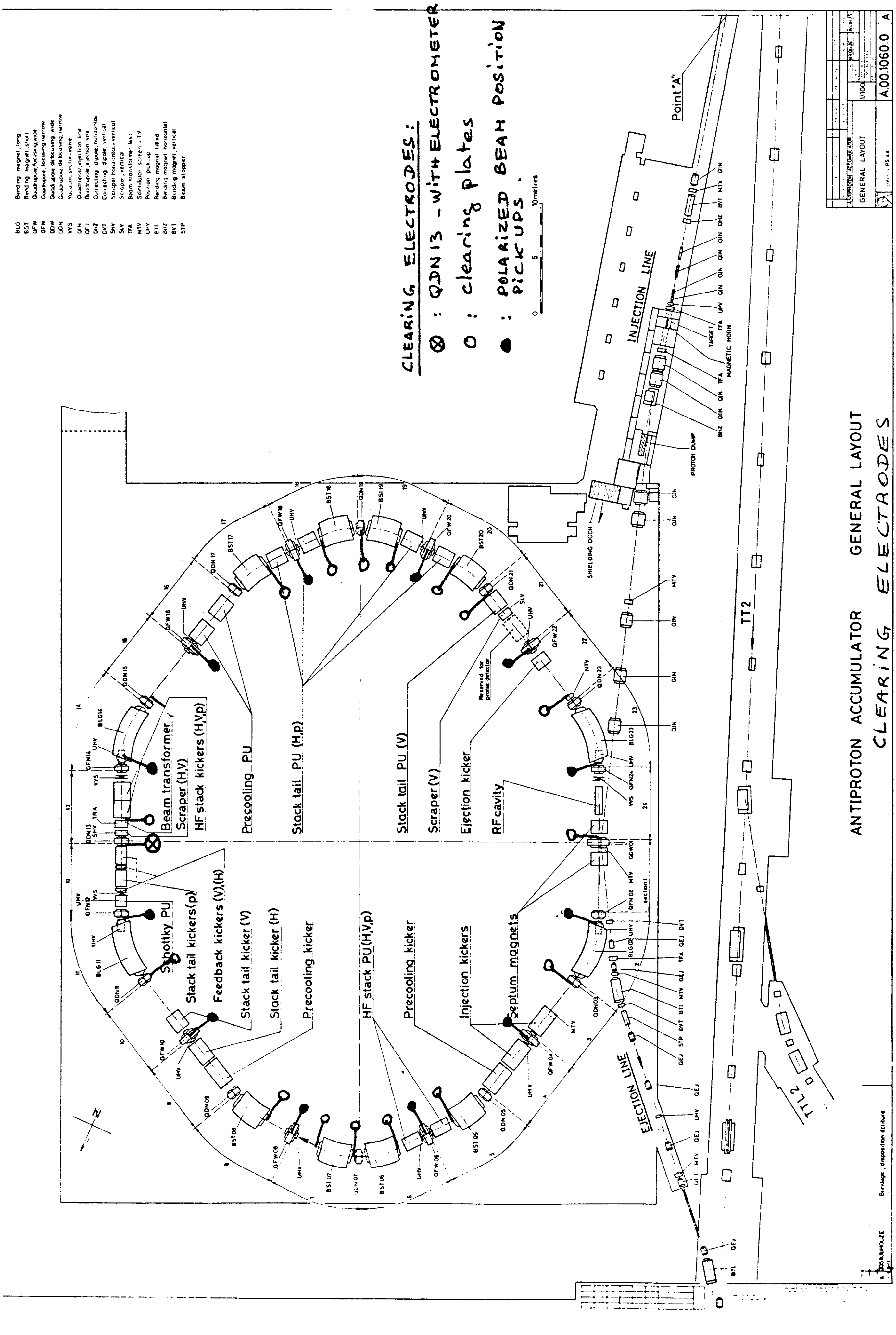
$$d_{H^+} = d_{\bar{p}}\, d_m\, \sigma_{H^+}\, ct\ ,$$

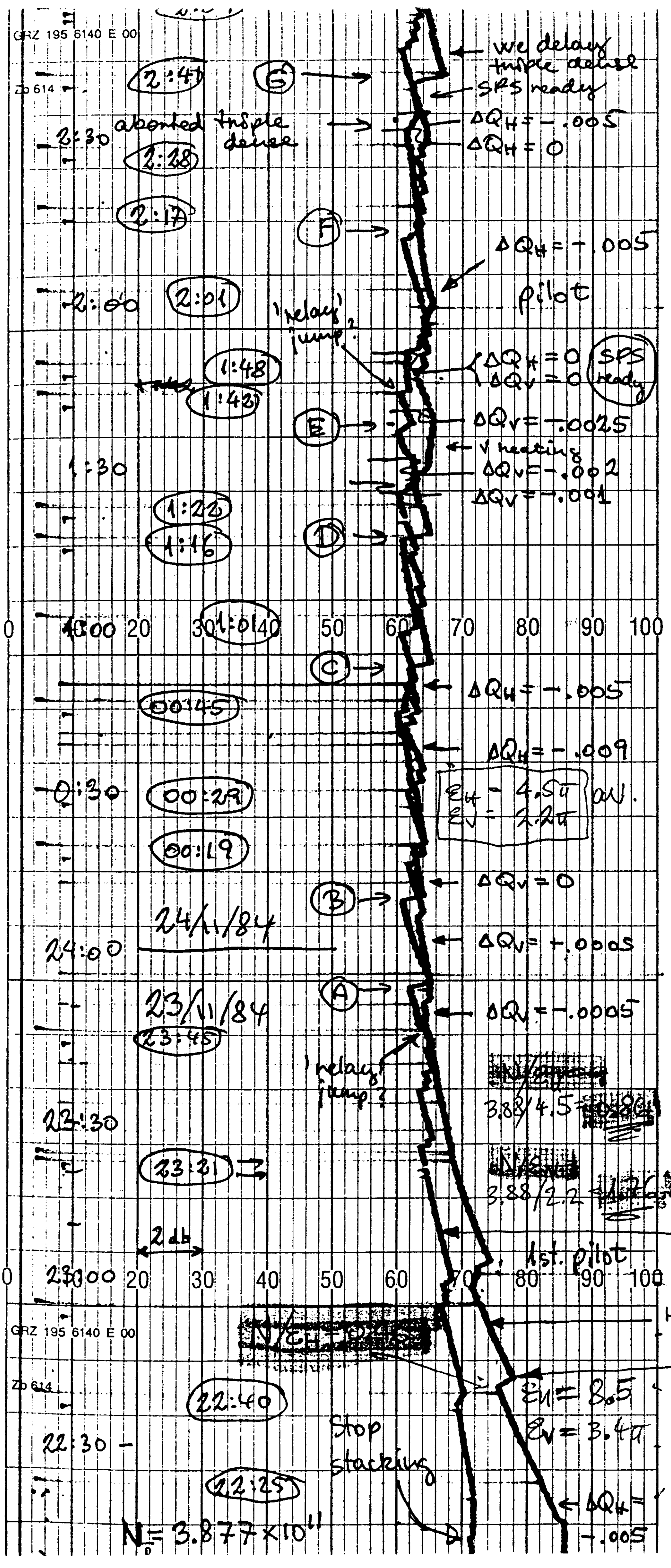
with $\sigma_{H^+} \approx 3\%$ of $\sigma_{H_2^+}$ (this is rendered necessary as $H^+$ being the most stable ion, $H_2^+$ will progressively be in small proportion at high level of neutralization and the direct production rate of $H^+$ will be of relative importance).

The graphs below give the population evolution as a function of time:a) for full neutralization ($\eta \approx 1$), b) for a pocket with 50% maximum permissible ionization.



_ FIGURE 1 _


ANTIPROTON ACCUMULATOR GENERAL LAYOUT
CLEARING ELECTRODES
CLEARING ELECTRODES:
⊗ : QDN13 - with ELECTROMETER
O : clearing plates
● : POLARIZED BEAM POSITION PICK UPS.
BLG Bending magnet, long
BST Bending magnet, short
QFW Quadrupole, focusing, wide
QFN Quadrupole, focusing, narrow
QDW Quadrupole, defocusing, wide
QDN Quadrupole, defocusing, narrow
VVS Vacuum, sector, valve
QIN Quadrupole, injection line
QEJ Quadrupole, ejection line
DHZ Correcting dipole, horizontal
DVT Correcting dipole, vertical
SHV Scraper horizontal + vertical
SLV Scraper, vertical
TFA Beam transformer, fast
MTV Scintillator screen + TV
UHV Position pick-up
BTI Bending magnet, tilted
BHZ Bending magnet, horizontal
BVT Bending magnet, vertical
STP Beam stopper
0 5 10 metres
Beam transformer
Scraper (H,V)
HF stack kickers (H,Vp)
Precooling PU
Stack tail PU (H,p)
Stack tail PU (V)
Scraper (V)
Ejection kicker
RF cavity
Schottky PU
Stack tail kickers (p)
Feedback kickers (V),(H)
Stack tail kicker (V)
Stack tail kicker (H)
Precooling kicker
HF stack PU (H,V,p)
Precooling kicker
Injection kickers
Septum magnets
INJECTION LINE
EJECTION LINE
TARGET
MAGNETIC HORN
PROTON DUMP
SHIELDING DOOR
Reserved for profile detector
Point "A"
TT2
TTL2
A.00.1060.0
A



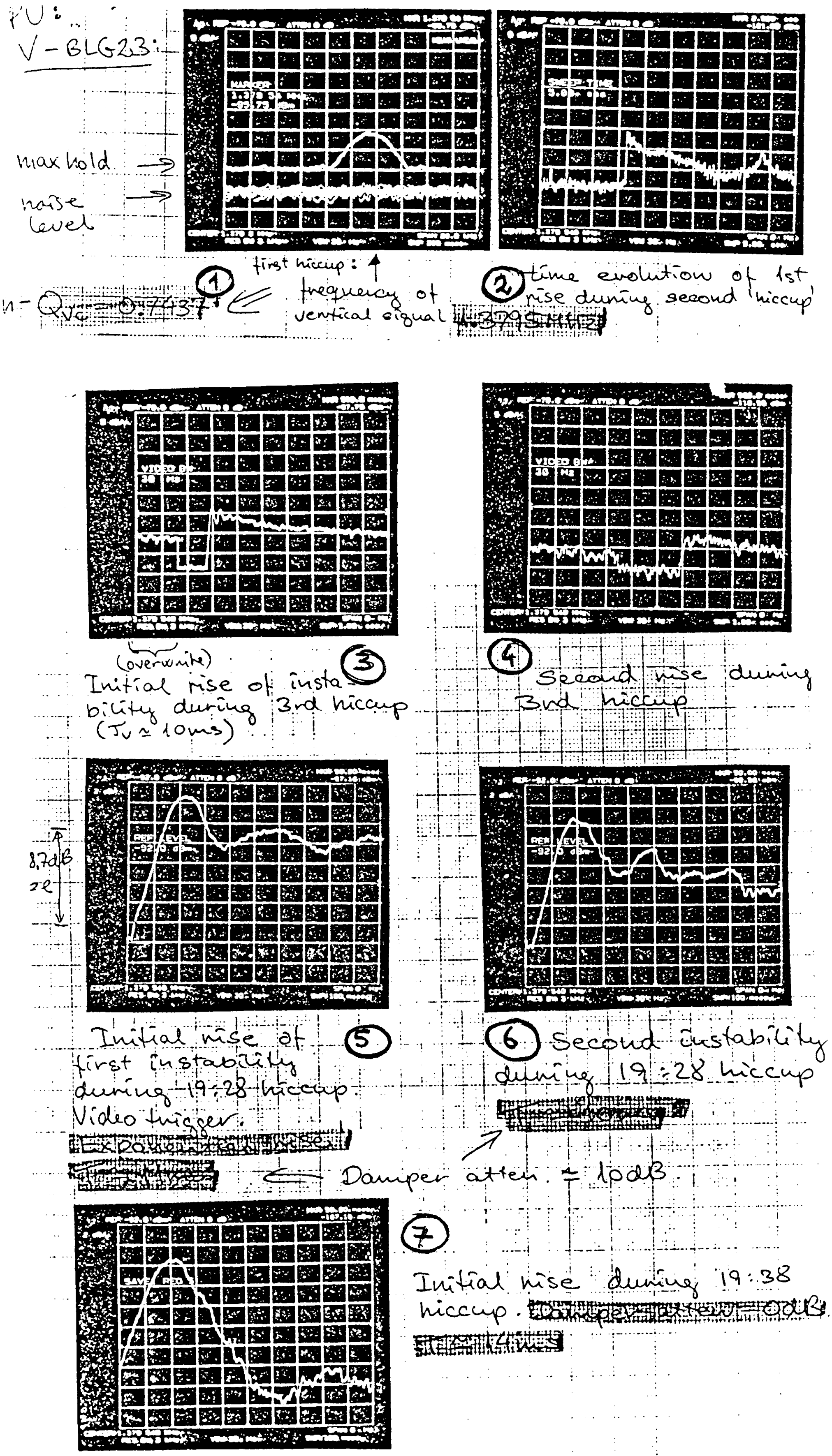

Time

First attempt to cool down a record stack of $3.877 \times 10^{11}$.
We use the new cooldown tunes:
$Q_H = 2.265$ $Q_V = 2.257$

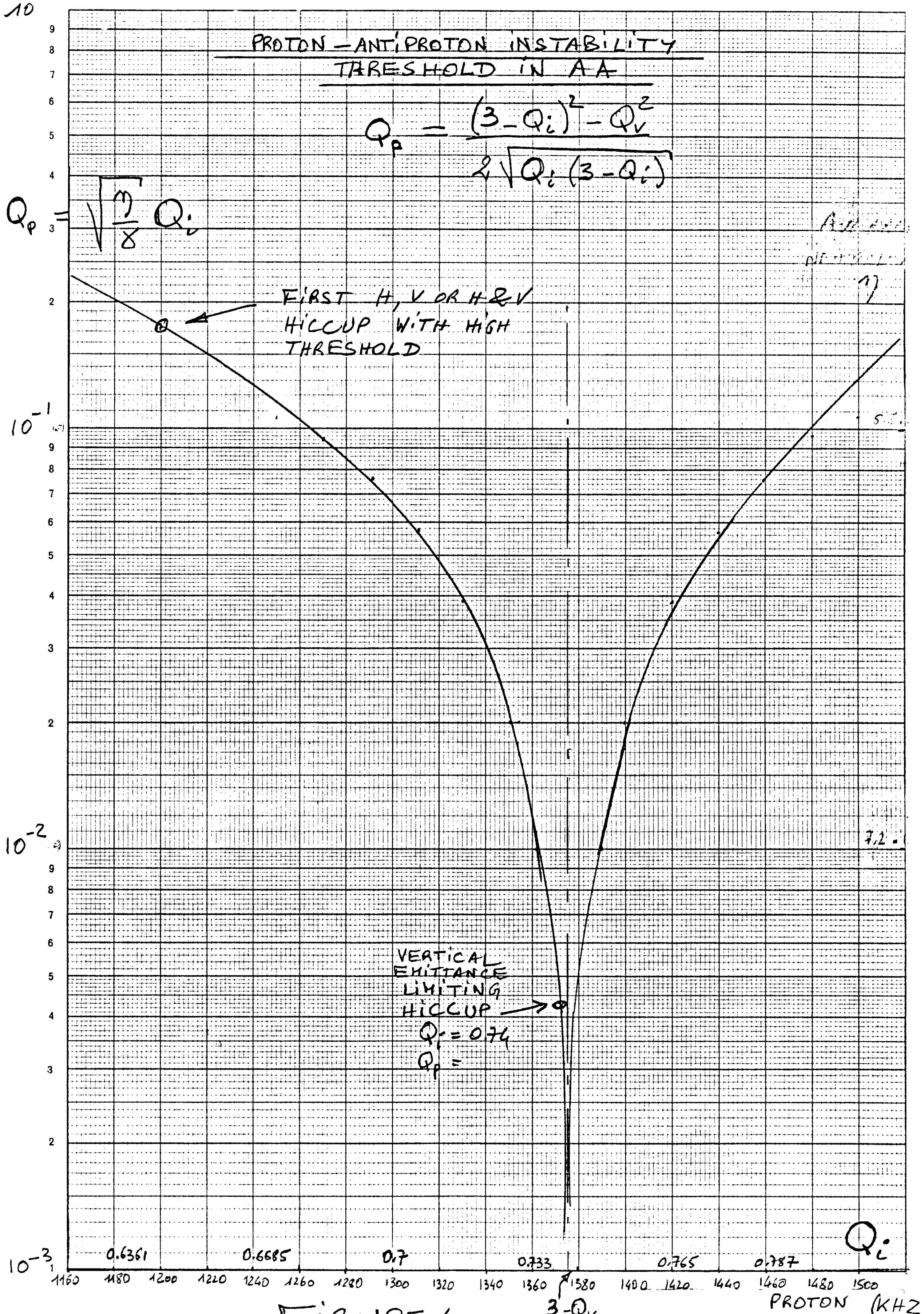
New problem:
~~Besides the usual~~ Vertical hic-ups we now also have horizontal hic-ups. These are somewhat more violent and really limits our cooldown in horiz. plane: Lowest meas. $\varepsilon_H = 2.9\pi$ at peak.

These bloody mini-phantoms!

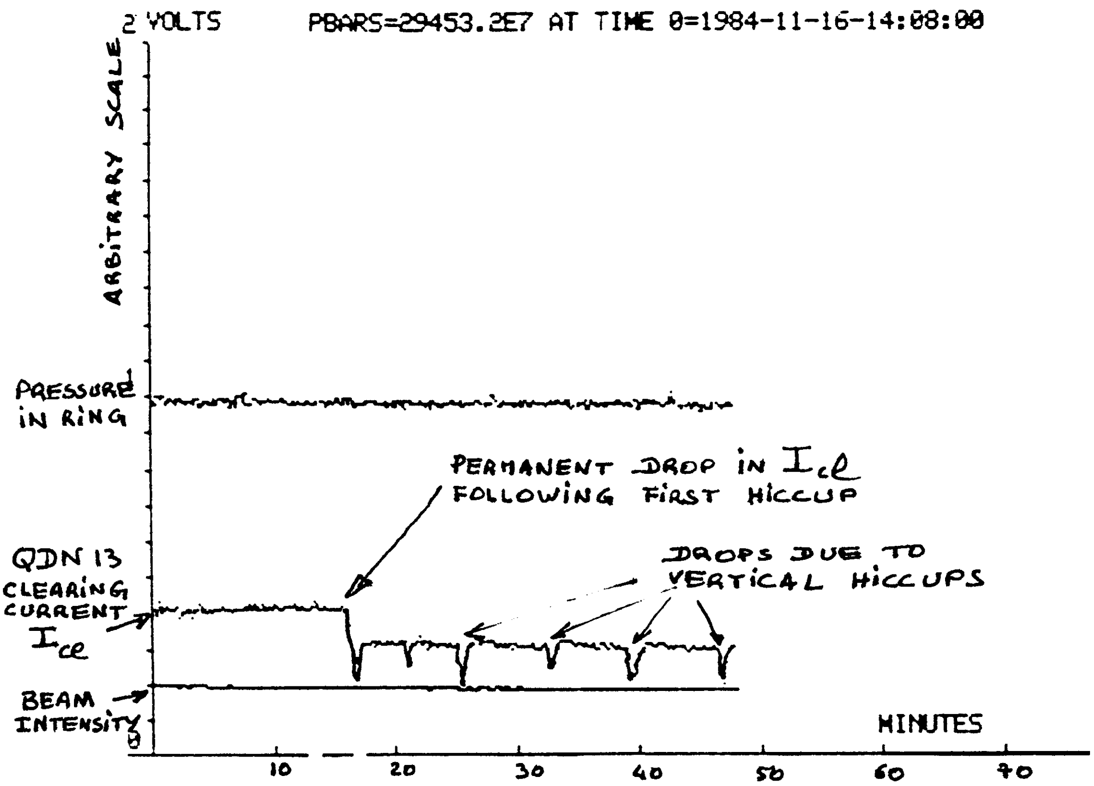
— FIGURE 2 —

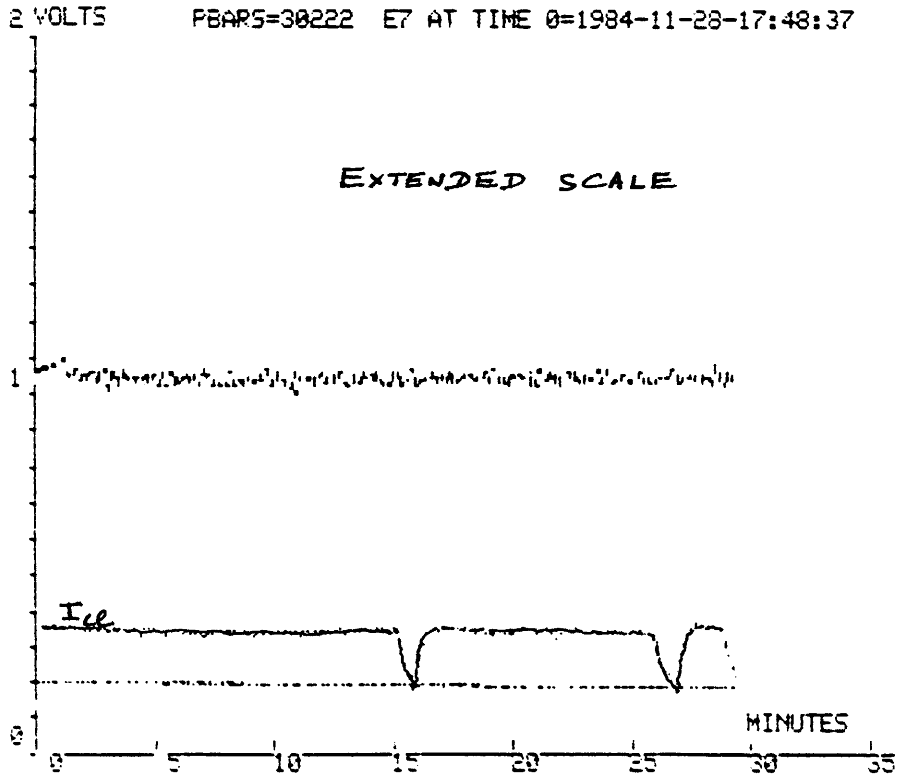

PU:
V-BLG23:
max hold
noise level
first hiccup:
1
frequency of vertical signal 4.3795 MHz
$n - Q_{vc} = 0.7437$
2 time evolution of 1st rise during second 'hiccup'
(overwrite)
3 Initial rise of instability during 3rd hiccup ($T_v \simeq 10$ ms)
4 Second rise during 3rd hiccup
8,7 dB
5 Initial rise of first instability during 19:28 hiccup. Video trigger.
[redacted]
6 Second instability during 19:28 hiccup
[redacted]
Damper atten. = 10 dB
7 Initial rise during 19:38 hiccup. [redacted]

- FIGURE 3 -


PROTON – ANTIPROTON INSTABILITY
THRESHOLD IN AA
$Q_p = \frac{(3-Q_i)^2 - Q_v^2}{2\sqrt{Q_i(3-Q_i)}}$
$Q_p = \sqrt{\frac{\eta}{8}}\, Q_i$
FIRST H, V OR H&V
HICCUP WITH HIGH
THRESHOLD
VERTICAL
EMITTANCE
LIMITING
HICCUP
$Q_i = 0.74$
$Q_p =$
$Q_i$
0.6361
0.6685
0.7
0.733
0.765
0.787
1160
1180
1200
1220
1240
1260
1280
1300
1320
1340
1360
1380
1400
1420
1440
1460
1480
1500
$10^{-1}$
$10^{-2}$
$10^{-3}$
10
$3 - Q_v$
$= 0.7425$
PROTON (KHZ)
FREQUENCY

– FIGURE 4 –


2 VOLTS
PBARS=29453.2E7 AT TIME 0=1984-11-16-14:08:00
ARBITRARY SCALE
PRESSURE IN RING
PERMANENT DROP IN $I_{cl}$ FOLLOWING FIRST HICCUP
QDN 13 CLEARING CURRENT $I_{cl}$
DROPS DUE TO VERTICAL HICCUPS
BEAM INTENSITY
0
MINUTES
10
20
30
40
50
60
70


2 VOLTS
PBARS=30222 E7 AT TIME 0=1984-11-28-17:48:37
EXTENDED SCALE
1
$I_{cl}$
0
MINUTES
0
5
10
15
20
25
30
35

– FIGURE 5 –