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Resistive-wall Instabilities in the Antiproton Accumulator

by

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The chromaticity of the AA focussing system is proclaimed to be zero. Transverse resistive wall instabilities will therefore need to be actively damped. The bandwidth of the active-damper is dictated by the ratio of the estimated transverse coupling impedance $Z_\perp$ of all machine components to $|Z_\perp^{th}|$ obtained from the theoretical stability criterion[1]. Estimates of both these quantities are given below and yield a bandwidth of at most 81 MHz.

Longitudinal resistive wall instabilities are not treated in great detail because Z_{11} is kept to well below the instability limit by insistence on a "smoothed" vacuum chamber and an RF cavity having a $|\frac{Z_{11}^{th}}{n}| \ll 840\Omega$.

1. Transverse Resistive Wall Instabilities

The instability criterion is used in the form given by G. Guignard[2], namely:

$$|Z_\perp|_{\substack{V\\H}} \leq kF \frac{\beta^3\gamma}{\bar{\beta}_{\substack{V\\H}} I} \left| (n-Q_{\substack{V\\H}})\eta - \frac{\partial Q_{\substack{V\\H}}}{\partial(\frac{\Delta p}{p})} \right| \frac{\Delta p}{p}$$

with $k = 2.9476 \times 10^9$ [volts]

F = a form factor depending upon the aspect ratio of the beam.

The chromaticity $Q' = \frac{\partial Q}{\partial(\frac{\Delta p}{p})}$ is put equal to zero.

A glossary of terms and definitions is given in Appendix I. Values of the parameters used are taken from the 7th Edition of the AA parameter list, and are summarized in Appendix II. The mks system of units is used throughout the following.

A cooled stack is then expected to be unstable for n=3 if $Z_\perp$ exceeds about 6.3×10^5 $[\Omega\ m^{-1}]$ and a cooled bunched beam circulating on the ejection orbit if $Z_\perp$ exceeds 6.9×10^5 $[\Omega\ m^{-1}]$.

2. Definitions of Transverse Impedance

2a Introduction

Generally [1)]

$$Z_\perp = \frac{j}{\beta \xi I} \int_0^{2\pi R} \frac{\langle F_\perp \rangle ds}{q} \qquad ; \ j = \sqrt{-1}$$

ξ = coherent amplitude of the transverse oscillation of the beam such that ξI is the beam's "perturbed" dipole moment which induces electromagnetic fields $\bar{E}$ and $\bar{B}$. The latter yield the Lorentz force $\bar{F}$ on a charge q where

$$\bar{F} = F_{11} + F_\perp = q(\bar{E} + \bar{v}_\wedge \bar{B})$$

The time dependance of the transverse force $F_\perp$ is of the form $e^{j\omega t}$, and $\langle F_\perp \rangle$ is the average over the beam cross-section. In all that follows it is assumed that $\lambda = \frac{2\pi c}{\omega}$ is large compared with the characteristic dimensions of the beam impeding devices.

The early theoretical papers on this subject [3,4] used the parameters U and V to describe the instability. These parameters are related to the impedance by

$$Z_{\perp} = -iZ_o \left(\frac{q}{r_o}\right) \frac{Q\gamma}{I} \left[U + (1+i)V\right] \quad \left[\Omega\ m^{-1}\right]$$

where $i = -j$ and $Z_o = \mu_o c$ is the "free-space" impedance in ohms.

In terms of the beam-induced electromagnetic fields

$$U + (1+i)\ V = \frac{1}{Z_o} \left(\frac{r_o}{q}\right) \frac{I}{Q\gamma} \frac{1}{\beta\xi I} \int_o^{2\pi R} \langle(\bar{E} + \bar{v}_\wedge \bar{B})\rangle_\perp ds$$

Physically (U+V) is the frequency shift $\Delta\omega$ produced by the transverse force $\langle F_\perp \rangle$ and V is the growth rate e^{Vt} of the instability.

If the transverse impedance is decomposed in a manner analogous to the normal circuit concepts of resistance, inductance and capacitance (remembering the units of transverse impedance are(Ωm^{-1}); such that

$$Z_\perp = R_\perp + j\left(\omega L_\perp - \frac{1}{\omega C_\perp}\right)$$

one can identify

$$V = \frac{R_\perp}{Z_o} \left[\left(\frac{r_o}{q}\right) \frac{I}{Q\gamma}\right] \quad ; \quad U + V = \frac{\left(\omega L_\perp - \frac{1}{\omega C_\perp}\right)}{Z_o} \left[\left(\frac{r_o}{q}\right) \quad \frac{I}{Q\gamma}\right]$$

This formula is used to estimate the transverse resistive wall impedance of the stainless steel vacuum chamber. The radius of the beam is

$$b_{v,H} = \sqrt{\frac{E_{v,H}\beta_{v,H}}{\pi}}$$

and the half-aperture of the vacuum chamber is 'a'. The thickness of the vacuum chamber wall is always greater than the skin depth $\delta(\sim 0.5$ mm at $n=3)$ and $\omega = 2\pi(n-Q)f_R$ where f_R is the revolution frequency. L is the length of vacuum chamber encountered by the beam during one revolution.

For example, when β_v is a minimum and, typically, $b \sim 2\times10^{-3}$ [m], $a \sim 5\times10^{-2}$ [m], for n = 3,

$$\frac{Z_\perp}{L} = 240 + j\left[240 - 1.1 \times 10^6\right] \quad \left[\Omega m^{-2}\right]$$

Thus, in the AA ring vacuum chamber, the transverse impedance is essentially capacitive, dominated by the $\frac{1}{b^2}$ term.

This is due to the incoherent space charge Q-shift term in the equation of motion for a single particle subjected to the forcing terms of a beam, betatron oscillating coherently far from the walls (a>>b) of a hypothetically perfectly conducting vacuum chamber.

A. Poncet has used the AA beam envelope computer programme BEPO[6] to calculate the capacitive part of the transverse impedance of the ring for <u>a cooled stack</u> and finds

$$Z_{\perp V} = -\,j\,3.24 \times 10^7 \quad \left[\Omega m^{-1}\right]$$

$$Z_{\perp H} = -\,j\,1.08 \times 10^7 \quad \left[\Omega m^{-1}\right]$$

A cooled beam circulating on the injection orbit prior to being ejected is expected to suffer similar but marginally larger impediments. However, since the $|Z_\perp^{th}|$ leading to instability of the soon to be ejected beam, is also slightly larger, it is considered to be sufficient to estimate the counter measures needed for a cooled stack and assume that such measures are equally valid for cooled beams on all orbits.

The growth rate of an undamped instability, estimated from the real part of $Z_\perp$ is given by

$$V \simeq 20 \quad \left[s^{-1}\right]$$

i.e. an e-folding time of about 50 mseconds.

2c. <u>Transverse impedance of a thick ferrite cylinder surrounding the beam</u>[4)]

$$\frac{Z_\perp}{L} = \frac{1}{2\pi}\left\{\frac{2Z_o}{a^2}\left(\frac{\tan\delta_\varepsilon}{\beta^2\varepsilon} + \frac{\tan\delta_\mu}{\mu}\right) + j\left[\frac{2Z_o}{a^2}\left(\frac{\tan\delta_\varepsilon}{\beta^2\varepsilon} + \frac{\tan\delta_\mu}{\mu}\right) - \frac{Z_o}{\beta^2\gamma^2}\left(\frac{1}{b^2} - \frac{1}{a^2}\right)\right]\right\} \quad \left[\Omega m^{-2}\right]$$

The capacitive term is identical with that given in 2b above and is not estimated here. The resistive and inductive terms are estimated to be at most, in the region of the injection kickers ($a \sim 5\times10^{-2}$ [m]):

$$\frac{Z_\perp}{L} = 10 + j\ 10 \quad \left[\Omega\, m^{-2}\right]$$

and the ejection kickers ($a \sim 10^{-3}$ [m])

$$\frac{Z_\perp}{L} = 2.9 \times 10^4 + j\ 2.9\times10^4 \quad \left[\Omega m^{-2}\right]$$

where for the ferrite used (Phillips 8c11) at 100 KHz

$$\varepsilon \simeq 13 \quad \text{and} \quad \frac{\tan\delta_\mu}{\mu} \simeq 2.5 \times 10^{-5}$$

At n ∿ 3 and frequencies in the megaherz range, we assume in order to arrive at the estimate given above for $Z_\perp$,

$$\frac{\tan\delta_\varepsilon}{\beta^2\varepsilon} + \frac{\tan\delta_\mu}{\mu} \simeq 2.5 \times 10^{-4}$$

2d. Transverse impedance of a thick ceramic cylinder surrounding the beam.[4)]

$$\frac{Z_\perp}{L} = \frac{1}{2\pi}\left\{\frac{2Z_o}{a^2}\,\frac{\varepsilon\tan\delta_\varepsilon}{\beta^2(\varepsilon+1)^2} + j\left[\frac{2Z_o}{a^2}\,\frac{\varepsilon\tan\delta_\varepsilon}{\beta^2(\varepsilon+1)^2} - \frac{Z_o}{\beta^2\gamma^2}\left(\frac{1}{b^2} - \frac{\gamma^2}{a^2}\,\frac{(\varepsilon-1)}{(\varepsilon+1)}\right)\right]\right\}$$

In spite of the γ^2 term in the "capacitive" part of the expression, the capacitive impedance is hardly different from that of 2b.

The AA ring contains about four metres of ceramic vacuum chamber with $\varepsilon = 9.5$ and $\tan\delta_\varepsilon \simeq 10^{-3}$ yielding

$$\frac{Z_\perp}{L} = 4.5 + j.4.5 \qquad \left[\Omega m^{-2}\right]$$

2e. Transverse impedance of a very thin metalic film on a ceramic cylinder.[4)]

It is usual to "metalize" ceramic vacuum chambers in accelerators so that no stray charge may collect on the walls and subsequently perturb the circulation of a beam. The ceramic vacuum chambers in the AA ring are part of the stack cooling stations which are to have a bandwidth of up to 500 MHz. The cooling kickers are outside the ceramic so that signals have to traverse both ceramic and metalic film. The thickness of the film has then to be much less than the skin depth at 500 MHz and the resistance per unit length of the metal deposited has to be much larger than the impedance per unit length along the beam of the kickers. The latter condition is much more stringent than the former and dictates thicknesses d, of metals such as Nichrome of the order of 1Å. The transverse impedance of such very thin metal film is given by [4),]

$$\frac{Z_\perp}{L} = \left[\frac{1}{2\pi}\,\frac{2(n-Q)}{\beta R a d \mu \sigma} - j\,\frac{Z_o}{\beta^2\gamma^2}\left(\frac{1}{b^2} - \frac{\gamma^2}{a^2}\right)\right] \qquad \left[\Omega m^{-2}\right]$$

With μ=1 and σ (nichrome) = 10^6 $\left[\Omega^{-1}\ m^{-1}\right]$, and estimating only the resistive term,

$$\frac{R_\perp}{L} = \frac{0.27(n-Q)\times 10^{-6}}{d} = 0.27 \times 10^4 (n-Q) \qquad \left[\Omega m^{-2}\right] \quad \text{for } d = 10^{-10}\ [m]$$

Thus for roughly four metres of such a vacuum chamber

$$R_\perp \simeq 10^4 \quad \left[\Omega m^{-1}\right] \quad \text{for} \quad n-Q \sim 1$$

and

$$R_\perp \simeq 10^6 \quad \left[\Omega m^{-1}\right] \quad \text{for} \quad n-Q \sim 100$$

This leads to a growth rate of an unstable beam at high mode numbers some twenty-five times larger than the rest of the stainless steel vacuum chamber - namely an e-folding time of 2 mseconds.

Fortunately, the $R_\perp$ term will only dominate the instability criterion if $d \leq 10^{-12} [m]$, at which thickness the film can hardly be called metalic.

2f. Transverse impedence of ferrite Cooling Pick-Ups (C.P.U.) or Kickers

Appendix III contains a primitive analysis of a simplified model of a cooling pick-up. Using such an anlaysis it is apparent that

$$\frac{Z_\perp}{Z_{11}} = \frac{S\ 2c}{a^2\ \omega}$$

as was pointed out earlier by Sacherer where S=1 for a resistive smooth vacuum chamber in the long wavelength limit.

It is fairly straightforward to measure Z_{11} for a single C.P.U. This has been done by L. Thorndahl and G. Carron who found

$$Z_{11} = j\omega L_{11} \qquad \text{(wholly inductive)}$$

with

$$L_{11} = 40 \, [\mathrm{nH}] \qquad \text{(measured)}$$

The simple analysis given in Appendix III yields a theoretical value of

$$L_{11} = 30 \, [\mathrm{nH}] \qquad \text{(theoretical)}$$

The form factor S obtained from the same analysis is approximately unity.

However, using the measured value for L_{11} and the Sacherer relation, one finds

$$Z_{\perp} = j \frac{2c}{a^2} L_{11} \qquad [\Omega m^{-1}]$$

or

$$\frac{Z_{\perp}}{L} = j \; 5 \times 10^5 \qquad [\Omega m^{-2}]$$

2g. Transverse impedance of plates such as pick-ups and clearing electrodes

A typical set of "plates" in the AA ring are about 0.5 [m] wide and 0.1 [m] apart. Such plates are separated from the wall of the vacuum chamber by gaps which are usually not less than 0.005 [m] . Such sets of plates have not yet been measured but an estimate for Z_{11} may be obtained from [2]

$$\frac{Z_{11}}{n} = j \frac{L}{2\pi R} \beta \frac{Z_o}{2\pi} \frac{W}{a_p} \log \frac{a}{a_p}$$

with w = plate width and a_p = half height of the plate

or $$\frac{Z_{11}}{nL} = j\ 0.2 \quad [\Omega m^{-1}]$$

Again using the Sacherer relation with S ∿ 1, yields

$$\frac{Z_\perp}{L} = j\ 4 \times 10^3 \quad [\Omega m^{-2}]$$

3. Transverse Impedence - Summary

For perturbing frequencies in the megahertz range

	Length [m]	$Z_\perp$ (total) $[\Omega m^{-1}]$	
Vacuum chamber/beam	157	$3.8\times10^4 - j\ 3.2\times10^7$	vertical
		" $- j\ 1.1\times10^7$	horizontal
Injection kickers	∿ 10	$10^2 + j\ 10^2$	
Ejection kicker	∿ 0.5	$1.5\times10^4 + j\ 1.5\times10^4$	
Metallized ceramic	∿ 4	10^4	
Cooling system (ferrite)	∿ 10	$+ j\ 5\times10^6$	
Plates (P.U.'s etc.)	∿ 25	$+ j\ 10^5$	

It is not obvious to the author if the inductive impedance (positive sign) can be relied upon to compensate for the capacitive impedance (negative sign). However, if this is so then the total expected impedance is

$$Z_{\perp V} \simeq 6.3 \times 10^4 - j\ 2.7 \times 10^7 \quad [\Omega m^{-1}]$$

$$Z_{\perp H} \simeq 6.3 \times 10^4 - j\ 0.6 \times 10^7 \quad [\Omega m^{-1}]$$

if it is not so, then the magnitude of the reactive part of the transverse impedance becomes

$$Z_{\perp V} = 3.7 \times 10^7 \quad [\Omega m^{-1}]$$

$$Z_{\perp H} = 1.6 \times 10^7 \quad [\Omega m^{-1}]$$

The instability criterion for a cooled stack is given by

$$\left| Z_\perp^{th} \right|_{\substack{V\\H}} \simeq 9 \times 10^5 (n - Q_{\substack{V\\H}})$$

Thus all modes are unstable up to (for $Q_V \sim Q_H$)

$$n-Q = \frac{3.7x10^7}{9x10^5} = 41.1 \qquad \text{vertically}$$

and $$n-Q = \frac{1.6x10^7}{9x10^5} = 17.8 \qquad \text{horizontally}$$

or $$n-Q = \frac{2.7x10^7}{9x10^5} = 30 \qquad \text{vertically}$$

and $$n-Q = \frac{0.6x10^7}{9x10^5} = 6.7 \qquad \text{horizontally}$$

The bandwidth of a damping system suitable for the AA ring is, pessimistically, around 81 MHz vertically and 38 MHz horizontally or optimistically 60 MHz vertically and 17 MHz horizontally.

The growth times of the instabilities are expected to lie between 2 and 10 mseconds. The total real Q-shift $\left(= \frac{U+V}{2\pi f_R} \right)$ is about $2x10^{-5}$.

4. Longitudinal Resistive Wall Instabilities [2)]

The instability criterion is used in the form [7)]

$$\left| \frac{Z_{11}}{n}^{th} \right| \leq k'F' \frac{\beta^2 \gamma |n|}{I} \left(\frac{\Delta p}{p} \right)^2 \qquad [\Omega]$$

with $k' = 0{,}938256 \times 10^{9}$ [V]

F' = a form factor depending upon the aspect ratio of the beam.

The form factor F' lies between 0.5 and unity for reasonable density distribution of the beam in longitudinal phase space.

Thus again using the values for the parameters given in Appendix II, for a cool stack one finds that

$$3268 \leq \left|\frac{Z_{11}^{th}}{n}\right| \leq 5889 \qquad [\Omega]$$

for $0.555 \leq F' \leq 1$

and for the cooled bunched beam on the injection orbit with a bunching factor of around 3:

$$840 \leq \left|\frac{Z_{11}^{th}}{n}\right| \leq 1520 \qquad [\Omega]$$

4a. Definition of Longitudinal Impedance[2]) for Smooth Vacuum Chambers

The longitudinal impedance of a resistive wall is given by

$$\frac{Z_{11}}{nL} = \frac{1+j}{2\pi a\sqrt{2}}\sqrt{\frac{Z_o \beta}{Rn\sigma}} \qquad \left[\Omega m^{-1}\right]$$

Thus for $L = 2\pi R = 157$ [m], $n=3$, $a \sim 5\mathrm{x}10^{-2}$ [m]

$$\frac{Z_{11}}{n} = 0.9(1+j) \qquad [\Omega]$$

For a perfectly conducting wall ("negative mass instability")

$$\frac{Z_{11}}{nL} = -\frac{j}{4\pi R}\,\frac{Z_o}{\beta\gamma^2}\left[1 + 2\log\left(\frac{a}{b}\right)\right] \qquad \left[\Omega m^{-1}\right]$$

and for $\frac{a}{b} \sim 10$ on average in the AA ring,

$$\frac{Z_{11}}{n} = -j\ 78 \qquad [\Omega]$$

4b. Longitudinal Impedance of Simple Cylindrical Tanks

$$Z_{11} = 72\lambda\bar{Q}\,\frac{L}{a^2}\left[\frac{\sin(\pi L/\lambda)}{\pi L/\lambda}\right]^2$$

where λ is wavelength of the lowest mode (TM_{010}) and $\bar{Q}$ is the quality factor.

$$\lambda = 2.61a$$

$$\bar{Q} \simeq \frac{1.2\,Z_o}{(1+\frac{a}{L})}\sqrt{\frac{\lambda}{\pi}\cdot\frac{\sigma}{\mu Z_o}}$$

Thus for the large ($a=0.55[m]$) stainless steel tanks of the AA ring

$\lambda = 1.4355\ [m]$; $f = 208.986$ MHz

and $\bar{Q} \simeq 10^4$ when "empty".

Therefore, $Z_{11} = 1.5\text{x}10^5\ [\Omega]$ whereas $L = 2.5\ [m]$

Since $n = \frac{f}{f_R} = 113$

then $\frac{Z_{11}}{n} = 1.33\text{x}10^3\ [\Omega]$

The small ($a=0.36[m]$) tanks will have similar impedances at $f \simeq 303$ MHz when "empty".

However, such tanks are never empty, being filled with devices consisting either of ferrite or flat plates. The $\bar{Q}$ values are then expected to be reduced by at least an order of magnitude and likewise the impedance.

Care will have to be used to ensure that any otherwise empty tanks will have flat plates introduced to smooth-out the vacuum chambers as seen by the beam.

There are more than ten such tanks installed around the ring. Their resonance frequencies lie between 150 to 500 MHz the frequency band in which we "cool". There could be a problem.

4c. Longitudinal Impedance of Cross Section Variations

There are between 40 to 50 "transistion" regions in the AA ring where the large tanks meet-up with the flat rectangular vacuum chambers of the magnet sections. These transition regions contain large diameter ($\phi \sim 0.6$ and $0.7[m]$)bellows.

An approximate value of the longitudinal impedance of such a cross section variation is given by

$$\frac{Z_{11}}{n} = j \frac{L}{2\pi R} \beta Z_o \log \left(1 + \frac{\tau}{a}\right)$$

where a is the half height of the magnet vacuum chamber and τ is the radius of the transition region. The ratio $\frac{\tau}{a}$ is generally $\lesssim 7$, thus

$$\frac{Z_{11}}{nL} = j\ 5 \qquad \left[\Omega m^{-1}\right]$$

Wherever possible such transition regions have been kept "smooth" by means of clearing electrode plates so that L is kept to a minimum - probably not more than a few metres. Thus

$$\frac{Z_{11}}{n} \leq j\ 20 \qquad [\Omega]$$

4d. Longitudinal Impedence of Flat Plates

The formula given in Section 2g holds, namely

$$\frac{Z_{11}}{n} = j \frac{L}{2\pi R} \beta \frac{Z_o}{2\pi} \frac{W}{a_p} \log \frac{a}{a_p}$$

and $$\frac{Z_{11}}{n} = j\,0.2 \qquad [\Omega m^{-1}]$$

and for $L \leq 25\ [m]$; $\frac{Z_{11}}{n} \leq j\,5 \qquad [\Omega]$

4e. Longitudinal Impedance of Ferrite Cooling Pick-Ups and Kickers

From the measurements commented upon in section 2f above, each pick-up has the value

$$\frac{Z_{11}}{n} = j\,2\pi f_R \mathcal{L}_{11} \qquad [\Omega]$$

$$= j\,0.46 \qquad [\Omega]$$

or $$\frac{Z_{11}}{nL} = j\,23 \qquad [\Omega m^{-1}]$$

And for $$L \sim 10 \qquad [m]$$

$$\frac{Z_{11}}{n} \simeq j\,230 \qquad [\Omega]$$

5. Longitudinal Impedance - Summary

Excluding large tanks and the RF cavities, the longitudinal impedance of the A.A. ring and equipment is around

$$\frac{Z_{11}}{n} \lesssim 330 \quad [\Omega]$$

It is presumed that the tanks, damped by equipment contained therein, should not contribute much more than about another 100 ohms.

Since the instability limit is around 840 $[\Omega]$ at worse, the RF cavity must have an impedance which is also much less than 840 $[\Omega]$.
The large tanks must be "smoothed" by means of horizontally mounted plates.

References

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4. B. Zotter, CERN 69-15. (1969).

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6. D. Blechschmidt and A. Poncet, PS/AA/Memo.78-12 (1978)

7. E. Keil and W. Schnell, ISR-TH-RF/69-48 (1969)

Appendix I

Glossary of terms

β	ratio of particle velocity to that of light
$\beta_{V \atop H}$	beam envelope function [metres]
γ	ratio of total energy to rest of energy of particle
γ_t	value of γ at transition energy
η	$= \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}$
I	circulating beam current [amps]
$Q_{V \atop H}$	betatron-wave number
R	mean radius of machine [metres]
$\bar{\beta}_{V \atop H}$	$= \frac{R}{Q_{V \atop H}}$ = average value of beam envelope function
q	$= 1.602 \times 10^{-19}$ [Coulombs]
r_o	= classical proton radius = 1.535×10^{-18} [metres]
Z_o	= impedance of free space = $\mu_o c$ = 377 [ohms]
μ_o	= permeability of free space = $4\pi \times 10^{-7}$ [Henrys metres^{-1}]
c	$= 3 \times 10^8$ [metres sec $^{-1}$]
a	= half aperture of vacuum chamber [metres]
b	= half-height (or radius) of a beam [metres]
L	= length along beam [metres]

$E_{\substack{V\\H}}$	= beam emittance [metre radians]
$b_{\substack{V\\H}}$	$= \sqrt{\dfrac{E_{\substack{V\\H}} \beta_{\substack{V\\H}}}{\pi}}$ [metres]
ε_o	= permittivity of free space $\simeq \dfrac{1}{36\pi} \times 10^{-9}$ [Farad metres^{-1}]
μ	defined as $\dfrac{B}{\mu_o H}$
ε	defined as $\dfrac{D}{\varepsilon_o E}$
δ	skin depth = $\dfrac{1}{\sqrt{\pi f \mu \mu_o \sigma}}$ [metres]
f	frequency = $\dfrac{\omega}{2\pi}$
σ	conductivity = 7.69×10^5 [ohms^{-1} metre^{-1}] for stainless steel
$\dfrac{\tan\delta_\varepsilon}{\varepsilon}$	loss factor for dielectric
$\dfrac{\tan\delta_\mu}{\mu}$	loss factor for ferrite
n	mode number
$\bar{Q}$	quality factor
$\dfrac{\Delta p}{p}$	full momentum spread at half-height
f_R	revolution frequency

Appendix II

List of parameter values used above for cooled beams:

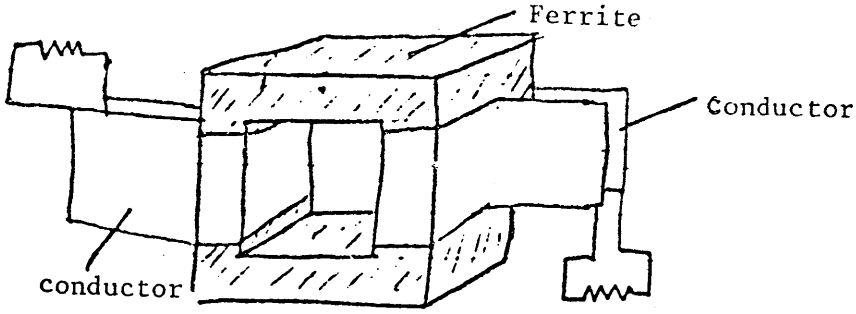
	Ejection Orbit	Stacking Orbit
β	0.9672	0.9639
γ	3.9395	3.7576
γ_t	2.317	2.574
$\bar{\beta}_V$ [m]	11	10.9
η	-0.122	-0.08
Q_V ($\simeq Q_H$)	2.269	2.27
I [amps]	0.0148	0.178
$\frac{\Delta p}{p}$	4×10^{-4}	2×10^{-3}
Q'	0	0
f_R	1.846 MHz	1.855 MHz

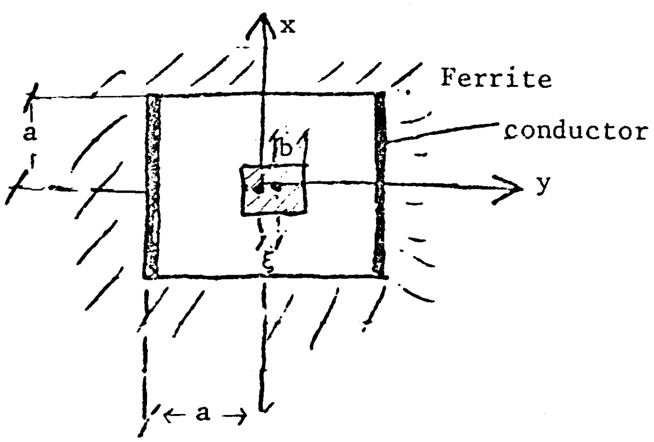
In the bunched beam on the ejection orbit, assume

$$I_{peak} \simeq 3I$$

Appendix III

Transverse Impedance of a Ferrite Cooling Pick-Up or Kicker





Ferrite is considered ideal magnet and conductors are perfect so that boundary conditions are

$$E_z = E_x = 0 \quad \text{at} \quad y = \pm a$$

$$H_y = 0 \quad \text{at} \quad x = \pm a$$

$$b \ll a$$

From the symmetry, with oscillation of the beam as a whole only in y-direction, at frequency ω and amplitude ξ, the self-fields can be described in terms of E_z and B_x.

The charge density inside the beam is assumed uniform so that charge distribution in time may be written

$$\rho(x,y - \xi e^{j\omega t}) = \rho o H - \xi e^{j\omega t} \frac{\partial\rho}{\partial y} \quad .....$$

where H = Helmholtz function = 1 for $-b \leq y \leq b$ and $-b \leq x \leq b$

so that

$$\frac{\partial\rho}{\partial y} = \rho_o\delta(y+b) - \rho_o\delta(y-b)$$

The beam current density, flowing in the z direction is thus

$$J_z = \rho_o v_z H - \rho_o v_z \xi e^{j\omega t}\left[\delta(y+b) - \delta(y-b)\right]$$

Thus in this approximation the time varying self fields result from two currents, one at y = b, and the other at y = -b flowing in opposite directions with the same time variation of $e^{j\omega t}$. In this approximation one may put B_x and E_z independant of both x and z.

The value of E_z may then be found by solving

$$\frac{\partial^2 E_z}{\partial y^2} + \mu\mu_o\varepsilon\varepsilon_o\omega^2 E_z = \mu\mu_o \frac{\partial J_z}{\partial t} = j\mu\mu_o\omega J_z(t)$$

and the time varying current $J_z(t)$ can be expanded in a Fourier series so that

$$\frac{\partial^2 E_z}{\partial y^2} + \Omega_o^2 E_z = K\sum_{n=1}^{\infty} \sin\frac{n\pi b}{a}\sin\frac{n\pi y}{a} \qquad (1)$$

with

$$\Omega_o^2 = \mu\mu_o\varepsilon\varepsilon_o\omega^2 = \mu\varepsilon\frac{\omega^2}{c^2}$$

and

$$K = 2j\mu\mu_o\frac{\omega\rho_o\xi v_z}{a}$$

$$B_x = \frac{j}{\omega}\frac{\partial E_z}{\partial y}$$

$B_y = 0$; $E_y = 0$ everywhere inside the pick-up

$E_z = 0$ at $y = \pm a$

Thus, since

$$\langle F_\perp \rangle = \langle F_y \rangle = \frac{q}{4b^2}\int_{-b}^{b} dx \int_{-b}^{b} dy\, v_z B_x \qquad (2)$$

and for $E_z = K\sum_{n=1}^{\infty}\dfrac{\sin\frac{n\pi b}{a}\sin\frac{n\pi y}{a}}{\Omega_o^2 - \left(\frac{n\pi}{a}\right)^2}$ from equation (1), substitution

in equation (2) yields

$$\frac{Z_\perp}{L} = j\,\frac{\mu\mu_o c}{2ab^3}\sum_{n=1}^{\infty}\frac{\sin^2\frac{n\pi b}{a}}{\left(\frac{n\pi}{a}\right)^2 - \Omega_o^2}$$

Since $\mu_o c = Z_o$ and $\mu \sim 1$ inside the pick-up, and further, in the long wavelength limit $\frac{n\pi}{a} >> \Omega_o$, one can write

$$\frac{Z_\perp}{L} = j \frac{Z_o}{2ab} \left[1 + \text{terms of order zero}\right] \tag{3}$$

Longitudinal Impedance of a Ferrite Cooling Pick-up or Kicker

In this case, we can write, with a similar model and assumptions as above:

$$J_z = \rho_o v_z e^{j\omega t} \left[\frac{b}{a} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} \sin\frac{n\pi b}{a} \cos\frac{n\pi y}{a}\right]$$

and obtain

$$\frac{Z_{11}}{L} = j\, Z_o \frac{\omega}{c} \frac{a}{b} \left[\frac{1}{8} + \frac{1}{\pi^2} + \text{terms of order zero}\right]$$

$$\simeq j\, Z_o \frac{\omega}{c}\, 0.23 \left(\frac{a}{b}\right) \tag{4}$$

Thus from (3) and (4)

$$\boxed{\frac{Z_\perp}{Z_{11}} \simeq \frac{1}{0.92} \cdot \frac{2c}{a^2} \frac{1}{\omega}}$$

which is the Sacherer formula with $S = \frac{1}{0.92} \sim 1$