PS/ML - Technical Note 89-06
AA/aa - December 20th, 1989

Structural analysis of a portal frame subjected to a uniform temperature thermal load and a horizontal vacuum force with the MATHCAD ® programme

Alberto Alberici, PS/ML

1 - Abstract

A number of particle accelerators ask for a bake-out of the vacuum envelope in order to achieve a reduced residual gas pressure inside the vacuum chamber.

Thermal expansion, consequently, becomes a problem in the design of the mechanical hardware whenever certain heated objects cannot freely expand: the link between supports and equipment may introduce very high and undesirable stresses.

This technical note describes a programme, running under MATHCAD®, meant to determine internal forces in a portal frame subjected to uniform temperature thermal load and a horizontal vacuum force. Such a portal describes closely the behaviour of most conventional bakeable vacuum chambers. This facility is available at CERN, on the computers of the local area network of the PS division.

2 - Introduction

2.1 - Definition of a vacuum chamber

The expression "vacuum chamber" defines those elements of a particle accelerator within which circulates the particle beam. The function of a vacuum chamber is to define a volume inside which, by means of an appropriate pumping system, a very low residual gas pressure can be reached. This volume must be separated from the external world, usually at atmospheric pressure, and must be leak-tight.

A vacuum chamber normally consists of a metallic tube, distinguished by various possible cross-section shapes and dimensions and provided with: flanges to connect with other similar elements, expansion bellows to allow installation adjustments and thermal expansion compensation, inlets to install instrumentation and vacuum pumps. The position of the chamber in space is defined by a supporting system and alignment device. To get a general idea about how a vacuum chamber looks like, see attached drawing ps_CLI-000-1085-2.

2.2 - Forces acting on a vacuum chamber

A vacuum chamber can be subjected to:

- circumferential load due to the difference between atmospheric pressure on external surfaces and extremely low pressure inside the vacuum envelope;
- constraint reaction forces;
- vacuum forces, generally along the axis of the chamber, due to cross-section variations of the vacuum pipe, or to blank-flanges;
- forces due to closed vacuum valves, supported on the chamber;
- its own weight
- weight of equipment directly fixed on the chamber (e.g. pumps, gauges, and so on);
- axial forces due to the Bourdon effect.

2.3 - Mechanical effect of bake-out on vacuum chambers and ideal design solutions

In certain particle accelerators, vacuum chambers must be heated (bake-out process) in order to obtain the lowest possible residual gas pressure (Ultra High Vacuum). Usually this operation is performed by means of a bake-out jacket heating the chamber (up to about 350C for stainless steel envelopes, around 150C for aluminium pipes). As a consequence, the vacuum chamber expands.

The construction of the vacuum chamber should allow free thermal expansion. This can be achieved, for instance, by considering the vacuum chamber as a beam whose displacements are blocked at one end, but not the rotation, while at the other end the beam is simply supported, as shown in figure 1.

2.4 - Possible variants from ideal design solution

However, this ideal solution is not always possible and both ends of the structure are often rigidly connected to a solid base. If this is the case the thermal expansion can be controlled by releasing one or more constraints during the bake-out process, providing in this way all necessary degrees of freedom to absorb heat induced movements. The original static situation is restored when the structure returns to room temperature.

Nevertheless, in case of misuse, the structure should be designed flexible enough to resist the supplementary stresses induced by the bake-out. This variant is usually a statically indeterminate structure, meaning that constraints are more than the strictly necessary ones.

A typical example of such a structure can be seen in figure 2. It represents a sort of portal where a vacuum chamber (horizontal beam) is rigidly linked by means of two supports (vertical columns) to a common solid base.

This note briefly introduces a method for calculating reaction forces and distribution of internal forces in such a structure when the vacuum chamber is heated, by baking and horizontally loaded with a vacuum force, neglecting all other forces. Then, it will describe how reaction forces can be calculated using a file running with the programme MATHCAD®.

3 - Analysis

3.1 - Statement of problem

Imagine a portal, like the one shown in figure 2.

The horizontal beam BC represents a vacuum chamber. BC is rigidly connected to a solid base AD, by means of the two supporting columns AB and CD. Elements AB, AD and CD, all stay at room temperature.

The vacuum chamber is subjected to a horizontal force F due to the vacuum envelope cross-section variations. It is also uniformly heated, DELTA T being the temperature increase. Effects of circumferential load due to atmospheric pressure on the vacuum envelope, mainly interesting for the determination of cross-section parameters, weight of the structure and rigidly connected equipment, often, but not always negligible, are not considered. The Bourdon effect is not considered either, since the vacuum chamber is straight.

Constraint reactions and resulting stresses in the strucure are to be determined. The hypothesis of linear elastic behaviour and small deflection is considered (this is the region of material behaviour where we must stay).

3.2 - Solution Method

The structure, as defined above, represents a two-dimensional problem and is statically indeterminate. The structure, in fact, has, in a plane, three degrees of freedom, whilst in the considered situation six constraints exist: a bending moment, a horizontal and a vertical force for each of the two built-in ends.

Therefore the three general equations for static equilibrium of forces and moments in a plane are not sufficient: there would be three mathematical conditions to find six physical unknowns. Elasticity of the structure must then be taken in consideration.

Solving elasticity problems, it is often advantegeous to use the Principle of Virtual Work, the most general and highest level tool to study statics and dynamics of rigid and elastic structures. The Priciple of Virtual Work affirms that:

necessary and sufficient conditions to maintain any material system in equilibrium is that the sum of work produced by all the forces acting on it be zero for every set of small and possible virtual displacements (i.e. compatible with the system geometry and boundary conditions).

The most important applications of the principle in the Theory of Elasticity are the following:

- in a statically determinate structure: to determine constraint reactions, to study the mechanical load of a generic cross-section of the structure and to calculate the structure's deformation
- in a statically indeterminate structure: to determine redundant constraint reactions, to study the mechanical load of a generic cross-section of the structure and to calculate the structure deformation.

The Principal of Virtual Work allows the definition of three linear algebraic equations derived from three integral conditions. Solving the three equations supplies the three required unknowns.

4 - Solving through the programme

4.1 - What are we looking for

Let us consider the portal of figure 2. The built-in ends are equivalent to the mechanical effects (reaction forces and moments) shown in figure 3.

Among the different possibilities, the reaction effects at point D have been chosen, considering their directions positive as shown.

The programme supplies, following indication of geometry of portal and boundary conditions, the value of reaction forces at point D.

4.2 - Input data

The following imput data are required:

GEOMETRY

- H, height of left column AB, measured in mm;
- A_1, cross-section area of left column AB, measured in mm^2;
- J_1, area second moment of inertia of left column AB, measured in mm^4;
- t1, shear factor of left column AB cross-section;

- L, length of horizontal beam BC, measured in mm;
- A_2, cross-section area of horizontal beam BC, measured in mm^2;
- J_2, area second moment of inertia of horizontal beam BC, measured in mm^4;
- t2, shear factor of horizontal beam BC cross-section;

- K, height of right column CD, measured in mm;
- A_3, cross-section area of right column CD, measured in mm^2;
- J_3, area second moment of inertia of right column CD, measured in mm^4;
- t3, shear factor of left column CD cross-section;

BOUNDARY CONDITIONS

- deltaT, bake-out temperature variation on horizontal beam BC, constant along length and cross-section of vacuum chamber, measured in degree Kelvin;
- F, horizontal vacuum force, measured in N;

MATERIAL

- E, Young's modulus, for the structure' s material, measured in N/mm^2;
- G, shear modulus. for the structure' s material, measured in N/mm^2;
- alfaT, coefficient of linear expansion for the material of the horizontal beam, measured in K^{-1}.

4.3 - How to use the programme

The programme runs under a MATHCAD® environment. You will enter MATHCAD®, e. g. by using the CERN-PS division Local Area Network. First of all you will need a copy of the programme which you will eventually back-up on your computer. An original on 3.5" disk and another one on 5.25" is available with Pierre Bourquin, CERN PS division, building 10-2-002, telephone 3085.

It will be briefly recalled how to enter MATHCAD®:

- go to a PC station you know being connectable to the PS-LAN and switch it on
- put your disk in a disk drive (A or B)
- enter the OFFICE NETWORK as a user, if you are registered or as a GUEST (no password required)
- when the main menu is in front type the ESC key and answer YES by return: you will go back to DOS
- type NMCAD and return.

You have just entered MATHCAD®. Now you will load the file with the programme:

- type F5 key: in the white line at the top you will be asked *file to load:*
- type *disk-drive* (A or B):BAKEOUT.

You have just entered the programme. With help of arrows you will move to input data, those ones described earlier (GEOMETRY, BOUNDARY CONDITIONS, MATERIAL), and you modify them. Pay attention and be consistent with units!!!

Once you have entered your input:

- type the ESC key: in the white line at the top you will be asked *COMMAND:*
- type GOTO 89.

You are now in front of the results. You can read MD, RHD, RVD with their units. Signs are coherent with directions indicated in figure 3.

4.4 - Example of calculation

We will consider an example of bakeable vacuum chamber effectively calculated with this programme: drawing TANK, ps_CLI-000-1085-2. The data we can retrieve from the drawing are:

- H = 227mm
- A_1 = 300mm^2
- J_1 = 225mm^4
- t1 = 1.2 (rectangular cross-section)
- L = 140mm
- A_2 = 478mm^2
- J_2 = 616090mm^4
- t2 = 1.111 (circular cross-section)
- K = 227mm
- A_3 = 2000mm^2
- J_3 = 66667mm^4
- t3 = 1.2 (rectangular cross-section)
- deltaT = 350K
- F = 1147N
- E = 206000N/mm^2
- G = 80000N/mm^2
- alfaT = 0.000176K^{-1}.

The results will appear as from picture xxx.

Once solved, the results permit to calculate, through fundamental equilibrium equations, all the reaction forces and consequently trace diagrams for internal actions (bending moments, shear and tensile forces).

5 - Acknowledgements

I would like to acknowledge very warmly Pierre Bourquin, head of the PS/ML design office, who gave strong evidence for the usefulness of this simple but indispensable tool for the design of several accelerator components, and Pier Luigi Riboni, PS/ML group leader and Balazs Szeless, of SPS/EMA group for their kind help in structuring this paper, and finally Leslie Petty, of PS/ML design office for the consultancy related to his mother tongue.

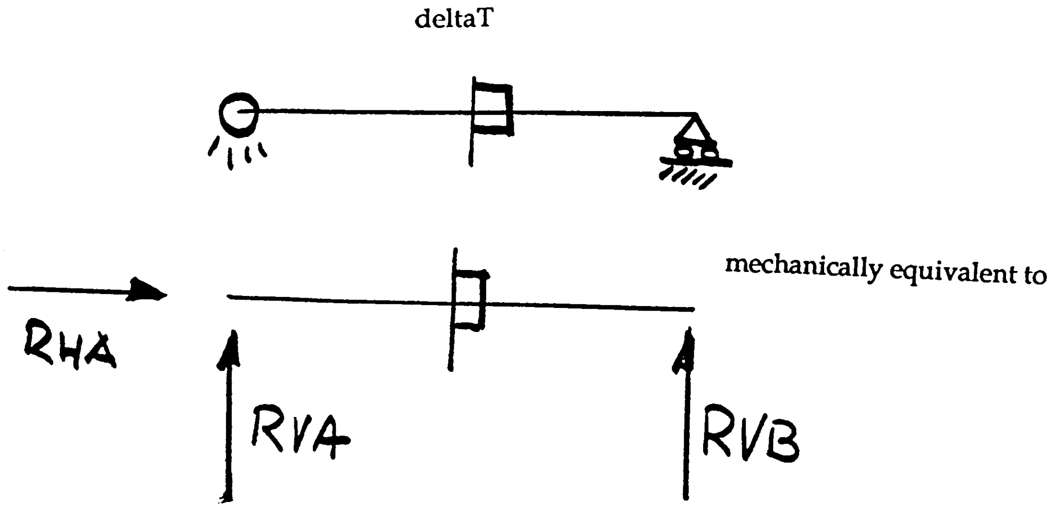


figure 1

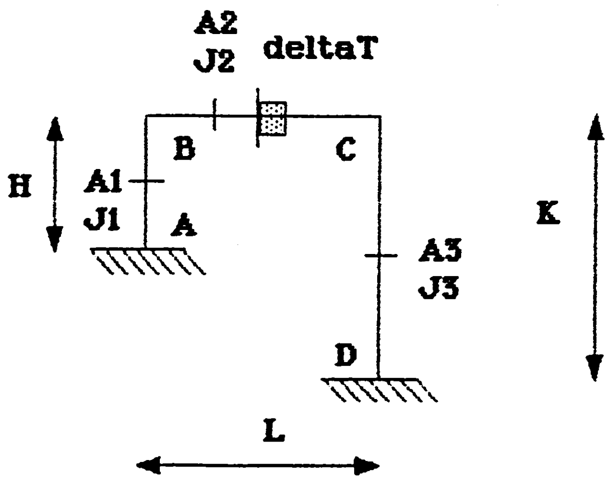


figure 2

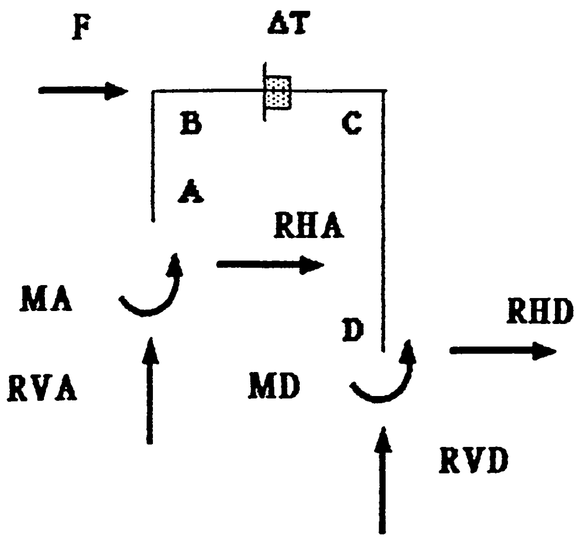
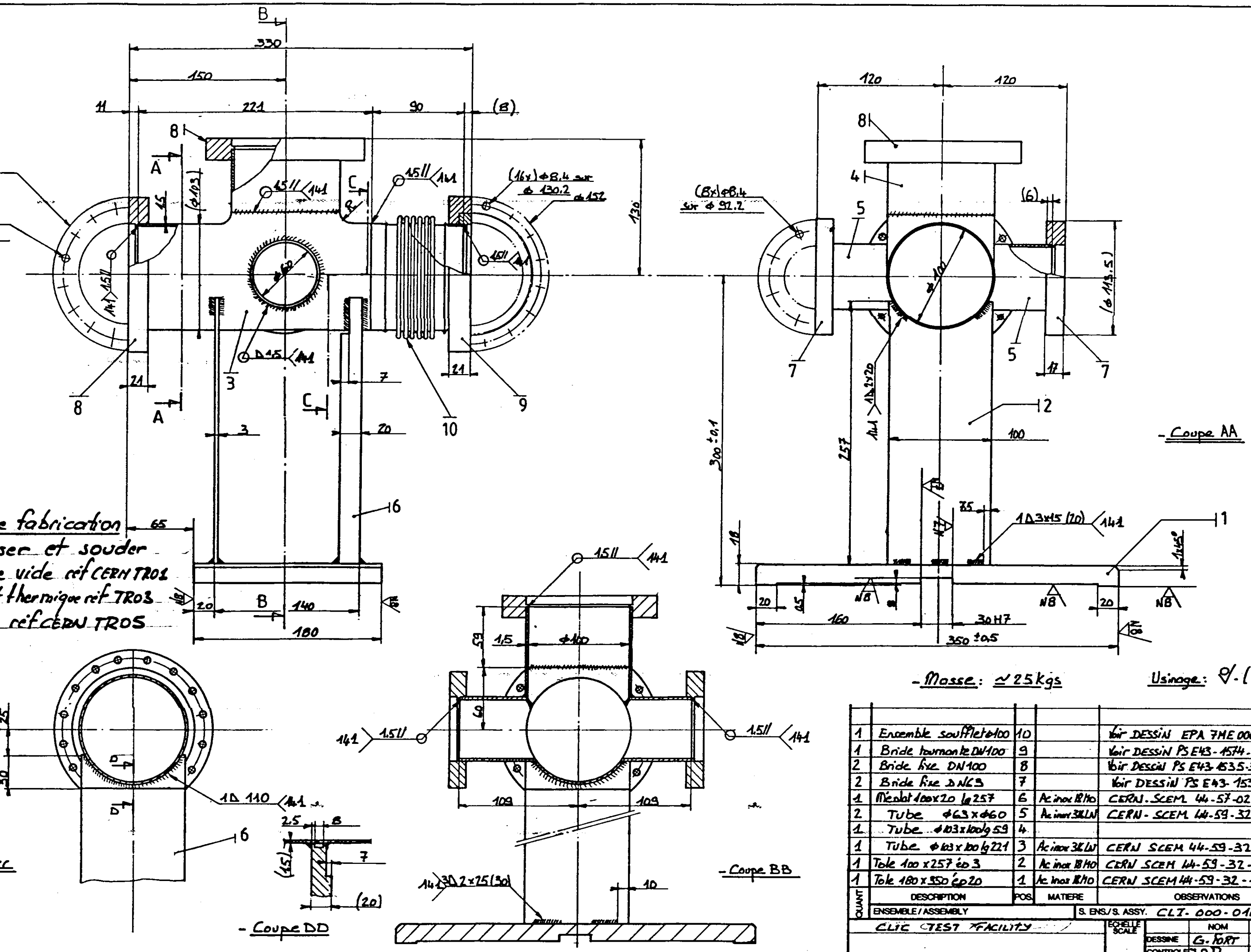


figure 3



Procédé de fabrication

1. Dégraisser et souder
2. Nettoyage vide réf CERN TR01
3. Traitement thermique réf TR03
4. Test vide réf CERN TR05

Masse : ≃ 25 kgs

QUANT	DESCRIPTION	POS.	MATIERE	OBSERVATIONS
1	Ensemble soufflet ⌀100	10		Voir DESSIN EPA 7ME 000 0361 4
1	Bride tournante DN100	9		Voir DESSIN PS E43-1574-3 pos 1
2	Bride fixe DN100	8		Voir DESSIN PS E43-1535-3 pos 2
2	Bride fixe DN63	7		Voir DESSIN PS E43-1531-3 pos 2
1	Méplat 100x20 lg 257	6	Ac inox 18/10	CERN-SCEM 44-57-02-182-6
2	Tube ⌀63x⌀60	5	Ac inox 316LN	CERN-SCEM 44-59-32-515-B
2	Tube ⌀103x100 lg 59	4		
1	Tube ⌀103x100 lg 221	3	Ac inox 316LN	CERN SCEM 44-59-32-515-8
1	Tole 100x257 ép 3	2	Ac inox 18/10	CERN SCEM 44-59-32-030-4
1	Tole 180x350 ép 20	1	Ac inox 18/10	CERN SCEM 44-59-32-120-3

ENSEMBLE / ASSEMBLY: CLIC TEST FACILITY
S. ENS./S. ASSY.: CLI-000-010-2

-TANK-

ECHELLE SCALE		NOM	DATE
1:2.5	DESSINE	G. Fort	1-89
	CONTROLE	[illegible]	2.89
	VU		
	REMPLACE		

ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE
EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
GENEVE

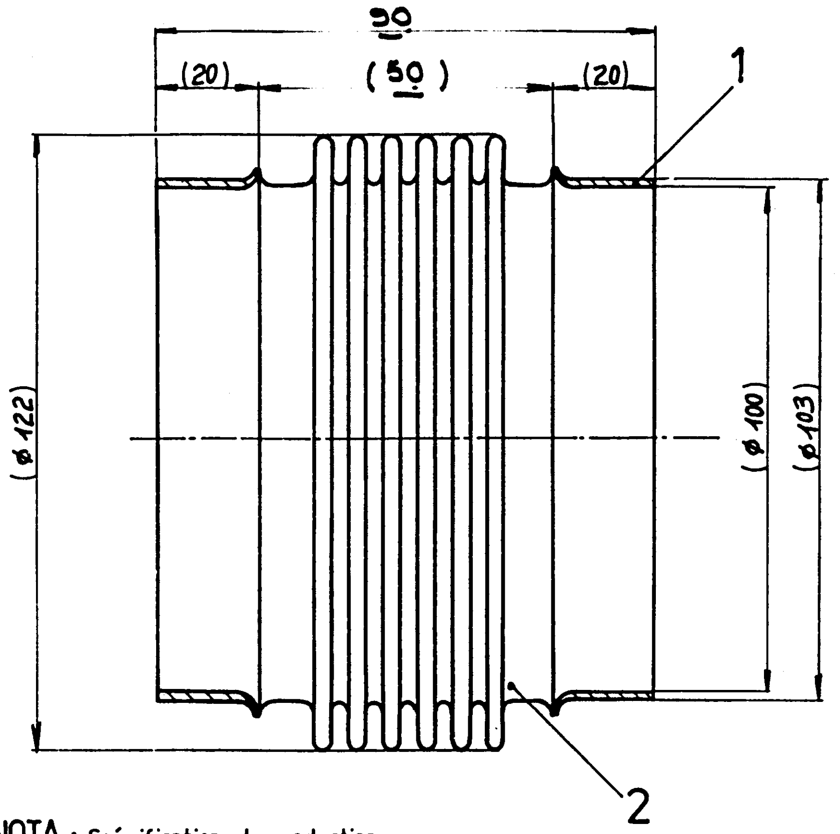
PS-CLI-000-1085 INDICE 2

INDICE	DATE	NOM	ZONE	MODIFICATION

DESSIN, RUGOSITE, TOLERANCES SELON NORMES ISO
DRAWING, RUGOSITY, TOLERANCES ACCORDING TO ISO STANDARDS

Projection européenne
First angle projection

INDICE	DATE	NOM	MODIFICATION



NOTA : Spécification de production

Spécification de traitement thermique

Spécification de nettoyage et de test au vide voir PS/ML/Spec 84/13

Masse ≃ 0,45 Kg

QUANT.	DESCRIPTION	POS.	MATIERE	OBSERVATIONS
1	Soufflet	2		EPA-7ME-0000-3019-4
2	Pièce de transition	1		EPA-7ME-0000-3018-4

ENSEMBLE / ASSEMBLY — S. ENS./S. ASSY. EPA-7ME-0000-0311-3

EPA

ENSEMBLE SOUFFLET ϕ100

ECHELLE / SCALE: 1:1

	NOM	DATE
DESSINE	ACROTECNA	85-02-05
CONTROLE	DUPLAN	85-02-19
VU	T[illegible]	85.03.11
REMPLACE		

ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE
EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
GENEVE

EPA7ME0000361 — INDICE 4

DESSIN, RUGOSITE, TOLERANCES SELON NORMES ISO
DRAWINGS, RUGOSITY, TOLERANCES ACCORDING TO ISO STANDARDS

Projection européenne
First angle projection

TOLERANCES GENERALES

DIMENSION	< 6	> 6	> 30	> 120	> 315	> 1000	> 2000
USINAGE DEGROSSIR / ROUGH MACHINING	± 0.2	± 0.5	± 0.8	± 1.2	± 2	± 3	± 4
MECANO SOUDURE / WELDED STRUCTURE	± 0.5	± 1	± 2	± 3	± 5	± 7	± 10

STRUCTURAL ANALYSIS OF A NON-SYMMETRIC BAKED-OUT FRAME WITH BUILT-IN ENDS, SUBJECTED TO A UNIFORM TEMPERATURE THERMAL LOAD AND A VACUUM FORCE ON THE HORIZONTAL BAR

GEOMETRY

Left column

- height [mm] — $H := 227$
- cross section area [mm^2] — $A1 := 300$
- area second moment of inertia [mm^4] — $J1 := 225$
- shear factor — $\tau 1 := 1.2$

Horizontal beam

- height [mm] — $L := 140$
- cross section area [mm^2] — $A2 := 478$
- area second moment of inertia [mm^4] — $J2 := 616090$
- shear factor — $\tau 2 := 1.111$

Right column

- height [mm] — $K := 227$
- cross section area [mm^2] — $A3 := 2000$
- area second moment of inertia [mm^4] — $J3 := 66667$
- shear factor — $\tau 3 := 1.2$

EXTERNAL LOADS

- bake-out temperature variation [K] — $\delta T := 350$
- horizontal vacuum force [N] — $F := 1147$

MATERIAL

AISI 304 L; AISI 316 LN

- modulus of elasticity [N/mm^2] — $E := 206000$
- shear modulus [N/mm^2] — $G := 80000$
- coefficient of linear expansion [K^-1] — $\alpha_T := 17.6 \cdot 10^{-6}$

DO NOT WRITE BEYOND THIS POINT - READ ONLY!!!!!!!!!!!

GO ON

DO NOT WRITE BEYOND THIS POINT - READ ONLY!!!!!!!!!!!

GO ON

DO NOT WRITE BEYOND THIS POINT - READ ONLY!!!!!!!!!!!

GO ON

RESULTS

DO NOT WRITE ~~BEYOND~~ THIS POINT - READ ONLY!!!!!!!!!!!

$$M^{-1} \cdot B = \begin{bmatrix} 1.404 \cdot 10^{5} \\ -1.183 \cdot 10^{3} \\ 886.455 \end{bmatrix}$$

N * mm	=	MD
N	=	RHD
N	=	RVD