PS/AA/RS/ip
May 4, 1982

AA Long Term Note 7

FOCUSSING TARGET

E. Jones, R. Sherwood

1. Introduction

Antiprotons produced inside a cylindrical wire target may be focussed by an azimuthal magnetic field, itself produced by passing a pulsed current along the wire. Two situations may be envisaged:

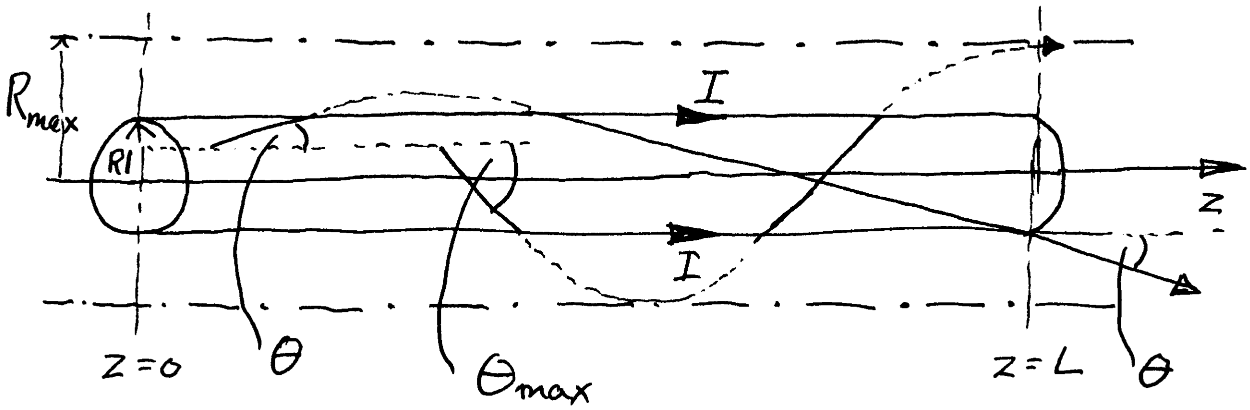
(i) The magnetic field does not penetrate into the conductor and focussing occurs outside the target, or

(ii) partial or total penetration of the field occurs so that some focussing is possible inside the target.

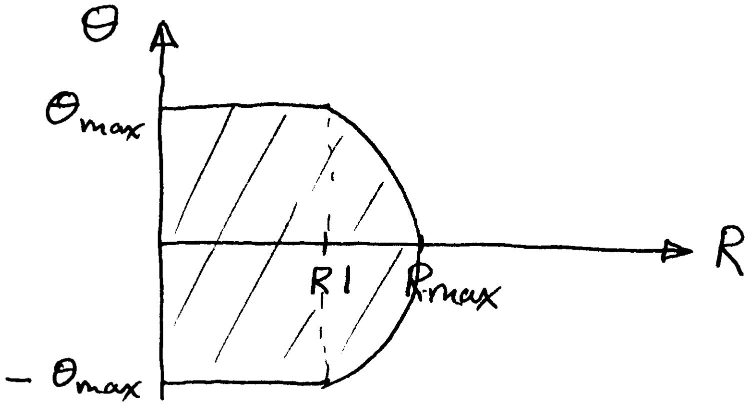
The first case is treated in some detail while the second has been treated by other authors[1,2]. Preliminary calculations show that if 200 (kA) can be pulsed through a 3 (mm) diameter target of reasonable length, a gain of a factor of three above the presently measured yield appears to be possible, in theory. In practice, technological problems may reduce this, but the calculations are sufficiently attractive to encourage some laboratory tests on pulsing large currents through thin wires.

2. Trajectories in the Focussing Target

If the current flows entirely along the surface of the target antiprotons which emerge into the magnetic field will have the following types of trajectory:



The motion of particles in a magnetic field which varies as R^{-1} is, for the sake of completion, given in Appendix I. It is obvious, from the diagram that if Coulomb scattering is ignored for the moment, all antiprotons produced at any point in z along the target are transported to z = L and will be contained within a phase space of roughly the following shape



where R_{max} and θ_{max} are related by

$$R_{max} = R1 \text{ EXP } (\theta^2_{max}/2A)$$

for $\theta_{max} \leq 0.2$ (radians).

A is a constant depending upon the total current I passing along the target and the momentum p of the antiprotons

$$A = \frac{\mu_o \mu q}{2\pi} \frac{I}{\gamma_{mo} \beta c} \qquad \text{(m.k.s.a units)}$$

or for $\mu = 1$ and I in (kA), p in (GeV/c)

$$A = 5.992 \times 10^{-5} \frac{I}{p}$$

The area of phase space occupied by acceptable antiprotons is:

a) independent of the length L (no Coulomb scattering)

b) and may be matched to the AA acceptance of $E_0 = \sqrt{E_V \cdot E_H} = 100\pi \times 10^{-6}$ (m.rads) by a linear lens. A short focal length is needed because of the largish values of θ_{max}. A linear horn appears to be the best choice. A lithium lens may also be appropriate but would have a smaller radius and larger divergence at its exit.

c) $R_{max}\theta_{max} = E_0/\pi$ if the phase space emittance is elliptical.

Fig. 1 shows some typical trajectories for a copper target of radius R1 = 1 (mm) and I = 200 (kA). It is seen that particles having production angles greater than about 15 (mrads) travel long trajectories which are, for the most part, outside the target and hence suffer little absorption and Coulomb scattering.

3. Yield Estimates

A pessimistic estimate of the yield of antiprotons per incident proton can be made by assuming that every antiproton produced at position z traverses the material of the target by at least the length (L-z). The yield may then be written (see Appendix II):

$$Y = (\text{constant})\Delta\Omega\ \Delta p\left[\text{EXP}(-L/\lambda_{\bar{p}}) - \text{EXP}(-L/\lambda_{ap})\right]\frac{\lambda_{\bar{p}}}{(\lambda_{\bar{p}} - \lambda_{ap})} ,$$

with $\Delta p/p = 1.5\%$; the momentum spread of acceptable antiprotons at 3.57 GeV/c,

$\Delta\Omega = \pi\ \theta^2_{max}$; the solid angle from which antiprotons can be accepted,

L = target length.

λ_{ap} and $\lambda_{\bar{p}}$ are the mean free paths for, respectively, the absorption of 26 GeV/c protons and the absorption of 3.57 GeV/c antiprotons in the target material.

The optimum target length L_{opt} is thus given by:

$$L_{opt} = \frac{\lambda_{ap}\lambda_{\bar{p}}}{(\lambda_{ap} - \lambda_{\bar{p}})}\ \text{LOG}[\lambda_{ap}/\lambda_{\bar{p}}] .$$

The table in Appendix II gives values of λ's and the material dependent factor, $F(L_{opt})$:

$$F(L_{opt}) = \frac{\lambda_{\bar{p}}}{(\lambda_{\bar{p}} - \lambda_{ap})} \left[EXP(-L_{opt}/\lambda_{\bar{p}}) - EXP(-L_{opt}/\lambda_{ap}) \right]$$

In the design report [3] the yield was estimated with a numerical "constant" which would result in

$$Y = 4.2 \times 10^{-3} \quad \theta^2_{max} \; F(L_{opt}) \qquad (\bar{p}\text{'s per p})$$

Recently, a review of the literature[4] indicated that the numerical constant which contains the value of the differential cross-section for $\bar{p}$ production, was optimistic by about a factor of two. In all that follows quantitative estimates of the yield are "corrected" by this factor of two. Thus, we take

$$\boxed{Y = 2.1 \times 10^{-3} \quad \theta^2_{max} \; F(L_{opt})}$$

Fig. 2 shows the yield estimates obtained with the above formula for a copper target passing various currents and for a 26 GeV/c proton beam of uniform cross-sectional density having the same radius as the target.

A more realistic estimate of the yield has been calculated assuming a proton beam having a gaussian cross-sectional density profile wherein 95% of the beam is to be found inside the target radii. Absorption in the target and Coulomb scattering in the horn is accounted for in this estimate and, as might be expected, gives the larger values for the yield shown in Fig. 3. The target length is again taken to be equal to the optimum estimated pessimistically, namely for copper $\sim$ 135 (mm). The real optimum length should be longer and this is illustrated in Fig. 4.

If the matching device is chosen to be a magnetic horn constructed out of 1 (mm) thick aluminium, the effect of Coulomb scattering in the walls of the horn results in a reduction of the yield estimates given in Figs. 3 and 4 by 15%.

Conclusions

If it is possible to construct a focussing target capable of sustaining the thermal shock of the 26 GeV/c proton beam while at the same time being subjected to the strains of containing a pulsed current of about 200 (kA), the yield of antiprotons presently available to the AA might be tripled.

The assumption that the current flows only on the surface is no doubt invalid. Some penetration of the current must occur. This will modify the results obtained here and may in fact give latter results for the focusing of the antiprotons but will lead to defocusing of the primary beam (see Appendix III). It may therefore be necessary to have stronger focusing of the protons before the target.

However, assuming, say, 1.5×10^{13} protons on the target, and furthermore an AA acceptance, as at present, of $80\pi \times 10^{-6}$ (m.rads) transversally and $\Delta p/p = 1.5\%$ longitudinally, then $\sim 2.5 \times 10^{7}$ $\bar{p}$'s might be injected into the machine per PS cycle. This assumes a copper target of diameter 3 (mm) as currently used and from which we at present obtain about 6×10^{6} $\bar{p}$'s per pulse of 10^{13} protons per PS cycle.

Acknowledgements

We gratefully acknowledge the contribution of C. Johnson and thank him for assembling the basic data on $\bar{p}$ production cross-sections from which the table in Appendix II was prepared. The "realistic" yield calculations are based on a computer programme kindly supplied by S. van der Meer.

References

1. Focussing Antiprotons Inside the Production Target.
   C. Rubbia, FNAL, $\bar{p}$ Source Note 12 (1979).
2. Study of Current-Carrying $\bar{p}$-Production Target.
   L.C. Teng, FNAL, $\bar{p}$-Source Note 14 (1979).
3. Design Study of a Proton-Antiproton Colliding Beam Facility.
   CERN/PS/AA 78-3 (1978).
4. J.V. Allaby, private communication.

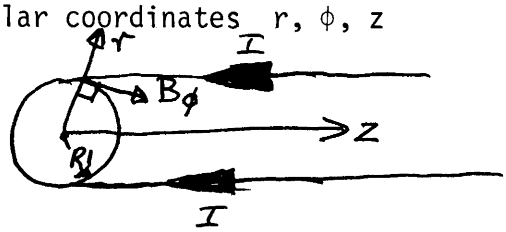
APPENDIX I

Charged Particle Motion in a Magnetic Field varying inversely with the Cylindrical Polar Coordinate r.

Assumptions:

i) current flowing into the direction of the z-axis, along the surface of a wire radius R1;

ii) cylindrical polar coordinates r, ϕ, z



iii) particles emerge into B_ϕ at angle θ (no ϕ motion) with velocities v_r and v_z, where

$$\tan\theta = \frac{v_r}{v_z} = \frac{\dot{r}}{\dot{z}} \; ; \; v_\phi = 0$$

iv) $B_z = B_r = 0$

and $\bar{B}_\phi = \bar{a}_\phi \frac{\mu \mu_0 I}{2\pi r}$ $\qquad r > R1$

v) Thus, Lorenz forces are for a charge q,

$$F_r \equiv \frac{d}{dt}(m\dot{r}) - mr\dot{\phi}^2 = q[-v_z B_\phi]$$

$$F_z \equiv \frac{d}{dt}(m\dot{z}) = q[v_r B_\phi]$$

$$rF_\phi \equiv \frac{d}{dt}(mr^2\dot{\phi}) = 0$$

vi) $\dot{r} \ll \dot{z} = v_z$; ie. $\theta \leq 0.2$ [radians]

and $\frac{d}{dt} = v_z \frac{d}{dz}$

vii) $\dot{\phi} = 0$; ie. no skew rays

The equations of motion then reduce to:

$$\frac{d^2 r}{dz^2} + \frac{A}{r}\left[1 + \left(\frac{dr}{dz}\right)^2\right] = 0 \qquad (1)$$

If m_0 is the rest mass of the particle and since $\dot{r} \ll v_z$ and with $\mu_r = 1$

$$A = \frac{q\mu_0 I}{2\pi\gamma m_0 \beta c} = \frac{q\mu_0 I}{2\pi p}$$

If p is the momentum of the particle in (GeV/c) and I is the current in (KA)

$$A = 5.9917 * 10^{-5} \frac{I[KA]}{P[GeV/c]}$$

Writing $\frac{dr}{dz} = r' = \theta$ and for $\theta \leq 0.2$, equation (1) is further simplified to

iiiv) $rr'' = -A$; $r > R1$

If at $r = R1$, $r' = R1' = \theta 1$, then

ix) $\frac{1}{2} r'^2 = \frac{1}{2} R1'^2 - A\ LOG(r/R1)$

x) $r' = [\theta 1^2 - 2A\ LOG(r/R1)]^{1/2}$

ix) $$\int_{z_1}^{z} dz = \int_{R1}^{r} \frac{dr}{[\theta 1^2 - 2A\,LOG(r/R1)]^{1/2}}$$

Furthermore, from the symmetry, the maximum value of r is reached when $\theta 1 = \theta_{max}$ and $r' = 0$ in equation x). Hence

$$R_{max} = R1\ EXP(\theta^2_{max}/2A) \quad \text{for } \theta_{max} \leq 0.2 \text{ (radians)}$$

In ix) put $\theta 1^2 - 2A\,LOG(r/R1) = A y^2$; $\theta 1, R1$ constants

or $$LOG(r/R1) = \theta 1^2/2A - y^2/2$$

$$r = R1\ EXP(\theta 1^2/2A - y^2/2)$$

and $$dr = -[R1\ EXP(\theta 1^2/2A) EXP(-y^2/2)]\, y\, dy$$

xii) $$z - z_1 = -(R1/A^{1/2}) EXP(\theta 1^2/2A) \int_{y_1}^{y} EXP(-y^2/2)\, dy$$

xiii) $$y_1 = (\theta 1^2/A)^{1/2}$$

$$y = [(\theta 1^2/A) - 2\,LOG(r/R1)]^{1/2}$$

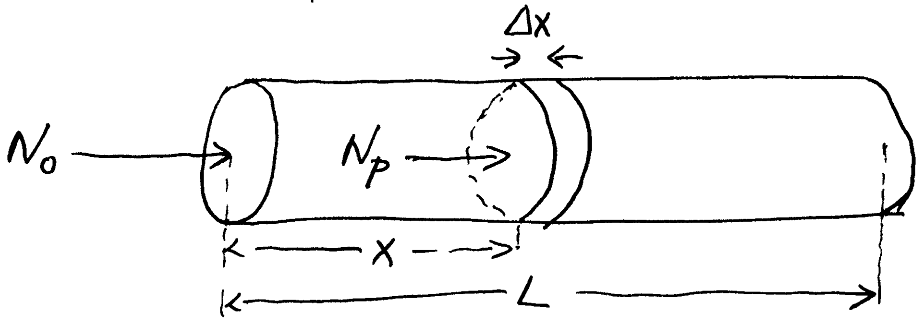
More detailed treatments are given in CERN yellow reports CERN 62-16 (S. van der Meer) and CERN 64-41 (E. Regenstreif).

A P P E N D I X II

Derivation of a limiting value for the Antiproton Yield

The following is a simple derivation in terms of the mean free paths of production λp by 26 GeV/c protons and absorption $\lambda\bar{p}$ of 3.5 GeV/c antiprotons.

The mean free paths are defined as follows:



(i) In a thick target (radius r >> length L), if the number of 26 GeV/c protons arriving at a distance x from the entrance plane is N_p, whereas N_o entered the target at x = 0, then

$$N_p = N_o \, EXP[-x/\lambda_{ap}]$$

λap is the mean free path for absorption of 26 GeV/c protons in the target material.

Likewise the number of 3.5 GeV/c antiprotons $\Delta N\bar{p}$ produced by these 26 GeV/c protons on traversing a further distance Δx at x is

$$\Delta N_{\bar{p}} = N_p \, \Delta x / \lambda_p$$

If the thick target length is L, these antiprotons, produced at x, have to traverse a distance (L - x) before emerging through the exit plane, thus the numbers which emerge (from x) are

$$\Delta N_{\bar{p}} \, EXP[-(L-x)/\lambda_{\bar{p}}]$$

Evidently, the total number emerging, having been produced in the straightforward manner (no secondaries, no cascades etc.), outlined above, may be written as

(i)

$$N_{\bar{p}} = \int_0^L \frac{N_0}{\lambda_p} EXP[-x/\lambda_{ap}] EXP[-(L-x)/\lambda_{\bar{p}}]\, dx$$

or

$$\frac{N_{\bar{p}}}{N_0} = \frac{EXP[-L/\lambda_{\bar{p}}]}{\lambda_p} \int_0^L EXP[-x(1/\lambda_{ap} - 1/\lambda_{\bar{p}})]\, dx$$

$$= \frac{\lambda_{ap}}{\lambda_p} \left[EXP[-L/\lambda_{\bar{p}}] - EXP[-L/\lambda_{ap}]\right] \frac{\lambda_{\bar{p}}}{(\lambda_{\bar{p}} - \lambda_{ap})}$$

$$= \frac{\lambda_{ap}}{\lambda_p} F(L)$$

In terms of cross sections for production and absorption,

$$\frac{\lambda_{ap}}{\lambda_p} = \frac{1}{\sigma_{ap}} \cdot \frac{\partial^2 \sigma_p}{\partial \Omega\, \partial p} \cdot \Delta\Omega\ \Delta p$$

with $\Delta\Omega$ the solid angle and Δp the momentum bite which contains the "acceptable" antiprotons.

From the design report[3)]

$$\frac{1}{\sigma_{ap}} \frac{\partial^2 \sigma_p}{\partial \Omega\, \partial p} = 24 \cdot 6 * 10^{-3}\ [ster^{-1} . GeV/c^{-1}]$$

and the yield of acceptable antiprotons is, pessimistically, choosing the optimum value for L:

$$Y = \frac{N_{\bar{p}}}{N_0} = \frac{1}{\sigma_{ap}} \cdot \frac{\partial^2 \sigma_p}{\partial \Omega\, \partial p} \cdot F(L_{opt}) \cdot \Delta\Omega \cdot \Delta p \qquad (a)$$

The estimate is pessimistic in the sense that the target is assumed to be thick and that there are no "secondary" antiprotons produced by mechanisms other than the direct interaction of a 26 GeV/c proton with the target nuclei.

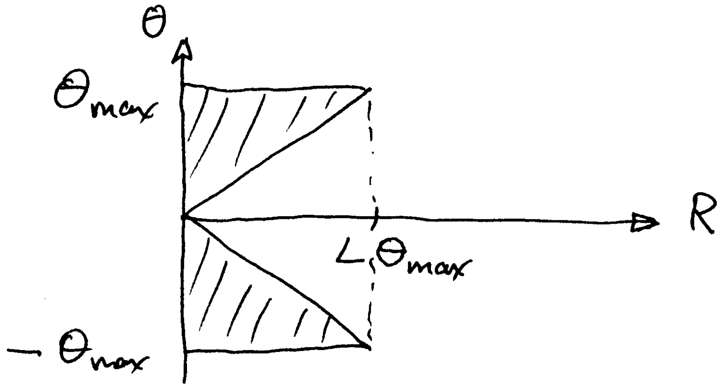
(ii) For a thin target, $r < L << \lambda_{\bar{p}}$, a similar analysis gives

$$Y = \frac{1}{\sigma_{ap}} \frac{\partial^2 \sigma_p}{\partial \Omega\, \partial p} \left[1 - EXP(-L/\lambda_{ap})\right] \Delta\Omega \cdot \Delta p \qquad (b)$$

In order to make numerical estimates, the design report values are assumed for $\Delta p = p \cdot 1.5 \cdot 10^{-2}$ (GeV/c)

$$\Delta\Omega = \pi\ \theta^2_{max}$$

If the target is neither "focussing" nor Coulomb scattering, the antiprotons emerge at its exit plane into the emittance shown cross-hatched below



If this emittance can be matched perfectly into the acceptance E_o (= $100\ \pi \times 10^{-6}$ (m.rad)) of the accumulator, then

$$E_o = 2\ L\theta^2_{max}$$

Thus, using the design report values in equation (a), with a <u>thick</u> target

$$Y = 4.2 \times 10^{-3}\ F(L_{opt})\ \theta^2_{max}$$

and for copper with L_{opt} = 13.3 (cm), $F(L_{opt})$ = 0.35 (see Table below)

$$Y = 1.47 \times 10^{-3}\ \theta^2_{max}$$

and $$\theta^2_{max} = \frac{E_o}{2\ L_{opt}} = \frac{100\pi * 10^{-6}}{2 * 0.133}$$

or $$\theta_{max} = 34.4\ [\text{mrads}]$$

and $\boxed{Y = 1.74 \times 10^{-6}}$ $\bar{p}$'s per proton on target.

A P P E N D I X   III

Defocusing of the Primary Proton Beam by the Focusing Target

Assumptions

(i) The primary beam is round with a uniformly distributed proton density.

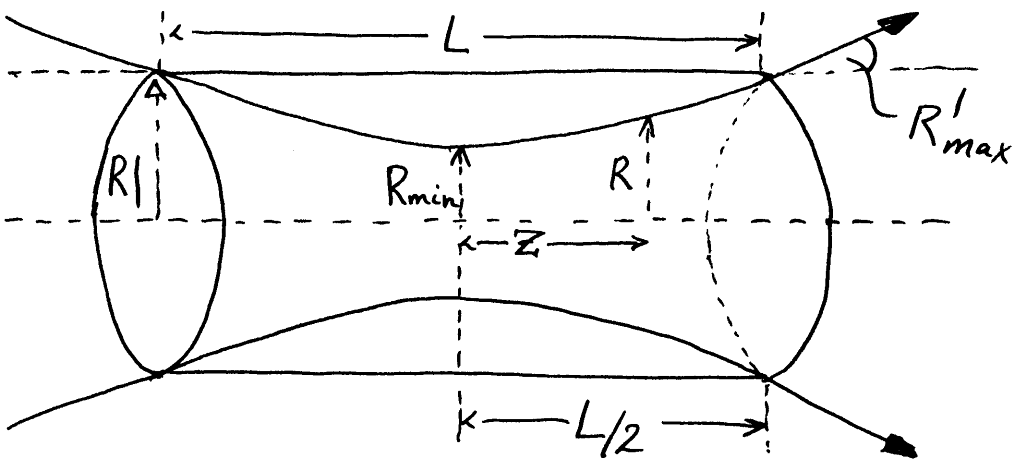
(ii) The radius of the proton beam is equal to the radius R1 of the target at the plane of entry and exit.

(iii) Assumption (ii) implies many things but most of all it assumes negligible coulomb scattering of protons in the target.

(iv) The current pulse I into the target penetrates completely and the current density is assumed to be uniformly constant and given by :

$$j = \frac{I}{2\pi R1^2} .$$

With the above assumptions the primary beam envelope should appear as :



The proton beam envelope trajectory is then simply described by :

$$R = R_{min} \cosh[\sqrt{K}\, Z]$$

$$\frac{dR}{dz} = R' = \sqrt{K}\, R_{min} \sinh[\sqrt{K}\, Z]$$

with :

From (b), as an upper limit with a thin target (no $\bar{p}$'s absorbed) and perfect matching,

$$Y = 4.2 \times 10^{-3} \left[1 - EXP(- L / \lambda_{ap})\right] \theta^2_{max}$$

$$= 4.2 \times 10^{-3} \left[1 - EXP(-L/\lambda_{ap})\right] \frac{E_0}{2L}$$

The optimum value for L occurs for $L/\lambda ap \sim 0$, when

$$Y = 4.2 \times 10^{-13} \frac{E_0}{2\lambda_{ap}}$$

or

$$\boxed{Y = 4.7 \times 10^{-6}} \quad \text{for } \lambda_{ap} = 0.139 \text{ (m) (copper)}$$

This optimum is of course unrealistic as it implies no limit on θ_{max}, except through E_0. In practice, at present, L = 0.11 (m), also in copper, which gives a more realistic value of the upper limit as

$$\boxed{Y = 3.3 \times 10^{-6}}$$

Furthermore, if the differential cross-section values are reduced by a factor of two, as would appear to be more "natural" and if, instead of

$$E_0 = 100\,\pi \times 10^{-6} \text{ (m.rad)},$$

a value of $80\,\pi \times 10^{-6}$ (m.rad) is used because this is nearer to the measured value of the AA acceptance, then the best realistic value for perfect matching and no absorption of $\bar{p}$'s in a copper target of length 0.11 (m) is

$$\boxed{Y = 1.32 \times 10^{-6}}$$

The best measured value of yield to date is about 57% of this value. Operationally it is more often 50% of the above.

The focussing system designed by S. van der Meer consists of a non-linear horn which has a "depth of focus" which is smaller than the total length of the target and is also able to divert $\bar{p}$'s into the machine acceptance, up to a θ_{max} of about 50 to 60 (mrads.). Hence the design report yield of $2.4 \cdot 10^{-6}$ with the optimistic cross-section data.

TABLE

Material	A	$\rho(20^oC)$ g cm^{-3}	λ_{ap} cm	$\lambda_{\bar{p}}$ cm	L_{opt}	λ_R	$\frac{\lambda_R}{L_{opt}}$	$F(L_{opt})$
Li	6.94	0.53	108.8	94.6	101	148	1.47	0.34
Be	9.01	1.85	35.6	30.0	32.6	34.7	1.06	0.34
C	12.01	2.27	34.6	28.4	31.3	27	0.86	0.33
Al	26.98	2.7	35.2	31.3	33.2	8.9	0.27	0.33
Cu	63.55	8.93	13.9	12.8	13.3	1.34	0.1	0.35
Sn	118.69	7.28	21.7	19.3	20.5	1.22	0.06	0.35
Pb	207.2	11.34	17.3	14.8	16.0	0.58	0.04	0.34
W	188.85	19.25	9.7	8.3	9.0	0.36	0.04	0.34

$\lambda = \frac{A}{N\rho\sigma}$; A = atomic weight (gms mole^{-1})

Np = No. of atoms per unit volume

N = 6.0225×10^{23} (mole^{-1})

The σ_{ap} are for 20 GeV/c protons and $\sigma_{\bar{p}}$ are for 3.5 GeV/c antiprotons obtained from the following references:

- Belletini et al. Nuclear Physics 79 (1966)
- Denisov et al. Nuclear Physics B61 (1973)

λ_R is the "radiation" length or the mean free path for Coulomb scattering.

$$K = \frac{\mu_0 q j}{2\gamma m_0 \beta c} ,$$

with

$$j = \frac{I}{\pi RI^2}$$

and γ, β the relativistic parameters of the primary proton beam.

For 26 GeV/c protons in the primary beam, the pulsed current I in the target equals to 150 (kA) and RI = 1.5 (mm).

$$\sqrt{K} = 12.4\ [m^{-1}]$$

and for

$$Z = L/2 \sim 0.05\ [m]$$

Then

$$R_{min} = 1.25\ [mm]$$

$$R_{max} = 8R_{min} \cong 10\ [mrad]$$

Thus even with complete current penetration into the target, it does not appear to be unreasonable to match the proton beam to the defocusing proportion of the pulsed target either by means of a short focal length ($\sim$1 metre) HORN or a lithium lens of modest proportion and currents.