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SOME CONSIDERATIONS ON THE MAGNETS OF CIRCULAR ACCELERATORS

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The aim of the design of a new magnet is to achieve a high quality to cost ratio by limiting the number and cost of the magnets while keeping a large optical aperture.

THE FIELD STRENGTH AND MAGNET LENGTH

For proton machines, the stronger the field, the more compact is the accelerator; also the longer the magnets, the less space is lost in coil heads. However, if the magnets are too long they must either follow the curvature of the beam and be more difficult to build or if they are straight the "good field region" must be larger - which is also more expensive. The SPS magnet is a straight magnet, it reaches 2 Tesla and is 6 m long, the Sagitta is 6 mm.

For electron machines there are two different cases depending whether one wants to use or to reduce the effects of synchrotron radiation. Small accumulation rings (like EPA) which use the synchrotron radiation in order to increase the beam density are equipped with short, high field magnets (0.6 m - 1. 4 Tesla for EPA); large e^+e^- colliders (like LEP) reduce the energy loss due to synchrotron radiation by using very long (12 m) low field (0.13 Tesla) magnets.

THE GAP

The decision on the size of the gap of dipole magnets is usually the privilege of the project leader. It is one of the most expensive (or cheap) technical decisions of the project. The balance must be made between :

i) The cost of the magnet system goes approximately like the power 1.6 of the gap[1].

ii) The number of quadrupoles, because with more quadrupoles one can contain the beam in less space, and therefore reduce the gap.

iii) The size of the beam (+ safety margin) that the accelerator will accept.

Sometimes one matches the gap to the different beam sizes around the machine (39 and 52 mm in the SPS) at the cost of having two types of magnets.

THE END EFFECT

If the upstream and downstream faces of a dipole magnet are parallel the magnet is more simple to build but the beam comes at an angle in and out of the magnet. This gives an effect equivalent to a lens. Even if the end faces are perpendicular to the beam (wedge magnet), the curvature of the lines of forces of the magnetic field at the ends gives a focusing effect in the vertical plane[2]. In most cases the computation of end effects is a delicate task. Some of the most elaborate optic programs[3] have been modified in the last few years to take into account more and more details of the end effects[4]. Large machines are less sensitive to these effects, but in many cases the optical properties of the first beam injected in small machines came as a surprise to their designers. That is an area where 3 D programs which are becoming fast, accurate and user-friendly will be very useful[5].

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1 R. Billinge (private communication).

2 K.L. Brown, SLAC Report No. 75.
H.A. Enge, Review of scientific instruments Vol. 35 No. 3 p.278.

3 - F.C. Iselin, The MAD Program CERN-LEP-TH/85-15
- E.Keil et al. AGS CERN 75-13

4 T. Risselada, Une version d'AGS améliorée pour les petites machines, PS/PSR/Note 84-5.

5 R. Early, J. Cobb, Tosca Calculations and Measurements for the SLAC SLC Damping Ring Magnet, Vancouver Conference 1985.

The best situation is achieved when the optics can make use of the lenses corresponding to parallel end faces so that the magnet is both simple and efficient (like in EPA in spite of the large end face angle with respect to the beam : 11°).

SPECIFICATION OF THE TOLERANCES

Specifying the tolerances of a new magnet requires a lengthy procedure[6,7]. The table below lists the correspondence between the accelerator properties and the corresponding magnet parameters. As an example, the next paragraph shows a typical (and rough) calculation of one of these effects.

ACCELERATOR PROPERTIES	MAGNET PARAMETERS
- closed orbit deformations	- Dispersion of $\int Bdl$ over the dipoles - Tilt of the dipoles - Quadrupole magnetic center displacement.
- beam shape distortions (β functions)	- Dispersion of $\int Gdl$ over the quadrupoles - Tilt of the quadrupoles.
- Chromaticity (tune shift or spread)	- bad lattice design or systematic sextupole deviation in dipoles.
- Tune shift with betatron amplitude	- Systematic octupole or (which is less known) sextupole or higher order non linear terms.
- Non linear resonnances	- random non linear field distributions around the ring.
- dynamic aperture	- Sum of all above effects.

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6 G. Guignard, Tolerances for the magnetic elements of the LEP lattice.CERN-LEP-TH/83-38.

7 J.P. Delahaye, Qualité du champ magnétique requise dans l'aimant de courbure EPA. PS/LPI/Note 82-15.

Field strength and magnet length

In first approximation, a dipole magnet is characterised by the field integral $I = \int BD\ell$. If there are N equal dipoles around the ring each of them provides an angle $\alpha = \frac{2\pi}{N}$. If the rms dispersion of the integral is $I_{rms} = \lambda.I$ then the rms angular error is $\alpha_{rms} = \lambda \frac{2\pi}{N}$, the error in the trajectory of the particles in the ring due to this angle error is approximately given by a quantity called β expressed in meter of displacement by radian of error of deflecting angle. Since the displacement given by the random errors of the N magnets will only add in a random way the resulting amplitude x_{rms} of the distorted orbit will be approximately.

$$x_{rms} \simeq \beta.\lambda. \frac{2\pi}{N} . \sqrt{N}$$

for large machines. The quantity β varies (very approximately) like the square root of the size of the machine or of the number of magnets in the machine say

$$\beta \simeq \pi \sqrt{N} \quad \text{so that}$$

$$x_{rms} \simeq 20 \lambda$$

If one wants an orbit deviation along the machine (x rms) not larger than 2 mm then the rms error λ on the field integral should be of the order of 10^{-4} more or less independent of the machine.

For example in the SPS magnet this would mean a tolerance on the length of 0.6 mm and on the gap of 5 microns. The 0.6 mm can be achieved but not the 5 microns. The result is obtained by "cheating" with the random distribution of magnets - one selects the magnets before installation - and by installing correctors around the ring which also correct the effect of alignment errors of quadrupoles.

In the small machines - like EPA - the result is achieved by individually shimming each of the magnets on the magnetic measurement bench.

THE FIELD DISTORSIONS

The designer of a machine uses a model of the real magnet as an input into the computer programs which define the optics. One calls field distortions the difference between this model and the real magnet. Amongst accelerator specialists one knows a few examples where the model was wrong. With the increasing sophistication of modern optics it is more and more important to use an accurate model and to limit the field distortions. The quality of the model and the design of the magnet have been very much improved in the past 20 years.

At the beginning the model was limited to a hard edge approximation, end effects were treated as perturbation. The end effects where then included in the model as thin lenses. More recently one has developed programs which treat directly field tables coming from magnet calculations or magnetic measurements[8]. The field tables however present the inconvenience that they are not guaranteed to fulfil the Laplace equations that the real fields do fulfil. Some of the fundamental properties of the beam (like the Liouville theorem of emittances invariance) result from these properties of the real field. The development of algorithms matching the field tables to the Laplace equation will become more and more useful, as far as we know they do not exist yet.

Even if the design of the magnet and the representation of the model were perfect the various saturation effects and mechanical tolerances require an analysis of the effects of field distortions. The distortions have different effects when they are systematic - that is all magnets have the same defect - or random.

THE EFFECT OF SYSTEMATIC DISTORTIONS

Dipole or quadrupole systematic errors are of no importance in separated function machines where the absolute value of the field and gradient can be modified by adjusting the current in the power supply. This is one of the strongest arguments against combined function machine (which however are more compact).

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8 M. Bell, J.P. Delahaye, Particle Tracking in the EPA Bending Magnet. LEP/Note 495.

In small machines the detail of the field shape of the dipoles and quadrupoles can be individually adjusted on the magnetic measurement bench. The shimming of the AA quadrupoles with washers was the only way to achieve a good field quality over a very large aperture. It also allows a correction after measurement with the beam of the optical properties of the machine. It is interesting to note that for the EPA for example, the smallest shim of the 2 ton magnet weighs 3 grams.

On large machines this individual correction is not possible so that several prototypes must be built and measured before the series is launched. For high field magnets the window frame design is preferred because it reduces the distortions due to saturation.

THE EFFECT OF RANDOM FIELD DISTORTIONS

The stop bands

We have already seen, table I, the effect of random dipole errors and also the similar effect of random quadrupole errors which instead of producing orbit distortions modifies the beam shape around the machine. Systematic quadrupole errors only modify the "tune" of the machine that is the number of transverse betatronic oscillation per turn. If the random quadrupole errors are too large, at some values of the tune the particle oscillations enter in resonance with the perturbations of the quadrupole field around the machine and the beam becomes unstable. The range of tune where the resonance occurs is called the stop band. Similar effects occur for higher order terms in the series expension of the field : sextupoles, octupoles etc. The sensitivity of the beam extend far beyond what can be obtained by magnetic measurements; stop bands of the 15th order corresponding to 30 poles magnets have been observed.

Several attempts have been made to express these effects in terms of a simple tolerance, for example $\left(\frac{\Delta B}{B_{rms}}\right)$ at the edge of the good field region. One usually obtains tolerances in the range of $\left(\frac{\Delta B}{B_{rms}}\right) = 10^{-4}$, which is also (may be by chance) the limit of what can be reasonably measured.

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