From quantum foam to graviton condensation: the Zel'dovich route

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Abstract – Based on a previous ansatz by Zel'dovich for the gravitational energy of virtual particle-antiparticle pairs, supplemented with the Holographic Principle, we estimate the vacuum energy in a fairly reasonable agreement with the experimental values of the Cosmological Constant. We further highlight a connection between Wheeler's quantum foam and graviton condensation, as contemplated in the quantum N-portrait paradigm, and show that such connection also leads to a satisfactory prediction of the value of the cosmological constant. The above results suggest that the "unnaturally" small value of the cosmological constant may find a quite "natural" explanation once the nonlocal perspective of the large N-portrait gravitational condensation is endorsed.

Introduction. – In a previous Letter, these authors argued that the Zel'dovich picture of gravitational energy as due to the ceaseless generation and annihilation of particle-antiparticle pairs, once combined with the holographic principle, provides a natural and quantitative account of the exceedingly small value of the cosmological constant. More precisely, it was shown that the combination of Zel'dovich picture with the holographic principle predicts a cosmological constant of the order of $\Lambda(l)l_{\rm P}^2 \sim (l_{\rm P}/l)^2$, $l_{\rm P}$ being the Planck length and l the infrared scale. By taking l of the order of the size of the universe, this delivers $\Lambda l_{\rm P}^2 \sim 10^{-124}$ in close agreement with the observed value 10^{-122}.

In this paper, we push the idea further ahead by showing that the Zel'dovich picture also permits to draw a consistent connection between a revised version of Wheeler's quantum foam [1, 2] and the large N-portrait framework, according to which the cosmological constant is associated with the Bose-condensation of a gas of ultrasoft gravitons permeating the entire Universe [3].

In passing, we note that the connection between Wheeler's quantum foam and the large N-portrait via the Zel'dovich-holographic scenario, provides a very "natural" explanation for the allegedly "innatural" smallness of the cosmological constant. Such smallness is indeed due to the nonlocality of the gravitational excitations which, although generated at the Planck scale, do nonetheless persist and extend across the entire size of the Universe. In this respect, the alleged "unnaturalness" of the cosmological constant appears as an artifact of local quantum field theories, as opposed to the nonlocal N-portrait framework, along the lines envisaged in [4].

Wheeler's quantum foam. – Back in 1955, J. A. Wheeler argued that on account of the uncertainty principles, at the shortest scales of the order of the Planck length spacetime itself should fluctuate as a sort of quantum foam [5]. Even though experimental evidence of such quantum foam remains elusive, the notion of a fluctuating quantum spacetime has attracted considerable interest ever since. Wheeler's simplest claim for the possible existence of the quantum foam could be summarized as follows [1, 2].

Let us consider a cubic region of space of side l and volume l^3. The smallest quantum gravitational excitation fitting within such a volume is a graviton of wavelength l, with energy $\hbar c/l$, implying an energy density given by

$$\rho c^2 \sim \frac{\hbar c}{l^4}\,. \quad (1)$$

On the other hand, as it is known, from classical General

Relativity, a small metric fluctuation, δg, within this cube features an effective energy density:

$$\rho c^2 \sim \frac{c^4}{G}\left(\frac{\delta g}{l}\right)^2, \tag{2}$$

Equating Eqs. (1) and (2) and considering the definition of the Planck length, $l_\mathrm{P}^2 = \hbar G/c^3$, leads to

$$\delta g(l) \sim \frac{l_\mathrm{P}}{l}, \tag{3}$$

showing that the amplitude of metric fluctuations scales inversely with their size and become order one at the Planck scale. Incidentally, we note that the inverse size dependency is a definite signature of singularity of the continuum limit (Einstein's gravitation), since the gradient $\delta g(l)/l$ diverges like $1/l^2$ in the limit $l \to 0$.

During the past years, different models attempting to understand the emergence of the quantum foam have been proposed [6]. Arguably, two of them, among others, became significant candidates: the holographic model [7–10] and the random walk model [11, 12].

The holographic model became known as such given its consistency with the Holographic Principle [13, 14]. Roughly summarized, it considers the operational resolution limits for the measurement of spacetime (e.g. length and/or time intervals), to derive the well-known expression $\delta l \geq l^{1/3} l_\mathrm{P}^{2/3}$ for the length fluctuations.

The random walk model considers the operational definition of the minimal distance, l, in a fuzzy spacetime affected by quantum fluctuations. These fluctuations are primarily characterized by their root-mean-square deviation $\sigma_d = \langle(\delta l)^2\rangle^{1/2}$, the simplest proposal being $\sigma_d < l$. The model postulates the well-known expression $\delta l \geq l^{1/2} l_\mathrm{P}^{1/2}$ for the length fluctuations. It is beyond the scope of this article to offer a detailed review of both models, but an excellent account of them can be found in the aforementioned references.

Despite their phenomenological plausibility, both models are ruled out by recent experimental observations, which also set stringent limits to any phenomenological model considering the fuzziness of the spacetime at very small distances [15, 16]. A few important remarks are in order, especially in relation to the holographic model. The first point is that ruling the holographic model out does not necessarily imply the demise of the Holographic Principle [15]. Thus, a few questions remain: what is the origin of the metric fluctuations at the Planck scale? Can they be understood without invoking operational considerations related to the measurement of spacetime? Can the Holographic Principle still play a fundamental role in accounting for such fluctuations?

The second remark concerns the origin of dark energy and its implications for the small experimental value of the Cosmological Constant. The holographic model plausibly opened the possibility to relate holography and dark energy (holographic dark energy). Its demise thus, prompts out additional questions. How could dark energy be explained in relation to the Holographic Principle without invoking the holographic model? And with it, how might it be related to the experimental value of the Cosmological Constant?

In this article, we address the above questions by revisiting a previous ansatz due to Zel'dovich and connecting it with the Holographic Principle.

The Vacuum Catastrophe. – The so-called Vacuum Catastrophe refers to the astronomical discrepancy between the observed energy density of the vacuum and the one predicted by quantum field theory. It has been termed as the "worst prediction in physics." The predicted density is $\sim 10^{120}$ vs. $\sim 10^{-9}\ J/m^3$ estimated by current measurements [17–22]. Ignoring numerical factors, the theoretical vacuum energy density, ρ_v, is expressed as

$$\rho_v c^2 \sim \hbar\omega \frac{c^3}{\omega^3}, \tag{4}$$

where ω is the radiation frequency of the vacuum modes and the other symbols are standard. Let us assume, in connection with Wheeler, vacuum modes with a frequency $\omega \sim c/l$ associated with gravitons. The expression (9) rewrites as

$$\rho_v c^2 \sim \frac{\hbar c}{l^4}, \tag{5}$$

which is identical to Eq. (1). Therefore, applying the same heuristic argument as Wheeler, the expression (5) leads to the same metric fluctuations at the Planck scale given by (3). In the following section we discuss further how these metric fluctuations might be due to the gravitational energy of virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state.

Zel'dovich's ansatz and the Holographic Principle. – Recently, the authors have revisited Zel'dovich's ansatz to provide an explanation of the current value of the Cosmological Constant, see [23]. Here, a brief reminder is made for the sake of consistency. Zel'dovich argued that, since the bare zero-point energy is unobservable, the observable contribution to the vacuum energy density, $\rho_v c^2$, is given by the gravitational energy of virtual particle-antiparticle pairs ceaselessly generated and annihilated in the vacuum state [24, 25]. Therefore,

$$\rho_v c^2 \sim \frac{G m^2(l)}{l}\frac{1}{l^3}. \tag{6}$$

In the expression above, also according to Zel'dovich, the vacuum contains excitations with an effective density $m(l)/l^3$. Additionally, by considering the Compton's expression for the wavelength, the effective mass of the particles at scale l is taken as $m(l) \sim \hbar/(cl)$. This leads to an energy density

$$\rho_v c^2 \sim \frac{G\hbar^2}{c^2 l^6}. \tag{7}$$

Equating Eqs. (2) and (7), leads to

$$\delta g \sim \left(\frac{l_{\rm P}}{l}\right)^2 . \tag{8}$$

This shows a steepest (more singular) inverse size dependence than predicted by Wheeler, though still leading to metric fluctuations of order one at the Planck scale.

Connection of the vacuum energy with the cosmological constant. The connection of this ansatz with the Cosmological Constant goes as follows [20]. Let us define a local Cosmological Constant as

$$\Lambda(l) \sim \frac{G}{c^2}\rho_v(l) . \tag{9}$$

By considering Eqs. (6) and (7), one readily obtains

$$\Lambda(l) l_{\rm P}^2 \sim \left(\frac{l_{\rm P}}{l}\right)^6 . \tag{10}$$

Reasoning further, the steep $1/l^6$ dependence implies that $\Lambda(l)$ is largely dominated by the chosen UV cutoff, $l_{\rm UV}$. This suggests to rewrite Eq. (10) as

$$\Lambda l_{\rm P}^2 \sim \left(\frac{l_{\rm P}}{l_{\rm UV}}\right)^6 . \tag{11}$$

By considering the Holographic Principle in the form of $l_{\rm UV} = l^{1/3} l_{\rm P}^{2/3}$, one obtains:

$$\Lambda l_{\rm P}^2 \sim \left(\frac{l_{\rm P}}{l}\right)^2 , \tag{12}$$

and further

$$\Lambda \sim \frac{1}{l^2} . \tag{13}$$

Taking l as the current radius of the Universe, gives a value of $\Lambda \sim 10^{-54}\, m^{-2}$, fairly comparable, given the approximate nature of the assumptions taken, to the experimental value $\Lambda \sim 10^{-52}\, m^{-2}$ [26, 27]. An attempt to explain why this value provides such a remarkable approximation is made in the following section, also in relation with the questions formulated before. The expression obtained for the Cosmological Constant is put into relation with postulated theories involving graviton condensation.

Connections with graviton condensation theories. – Einstein's General Relativity (GR) is a classical theory of gravity. From the quantum point of view, it propagates a unique weakly coupled quantum particle with zero mass and spin-2. Recently, the so-called black hole quantum N-portrait paradigm, postulates a quantum self-coupling of gravitons consistently defined as [3, 28–33]

$$\alpha_g = \left(\frac{l_{\rm P}}{l}\right)^2 , \tag{14}$$

where l is a classical wavelength at low energies.

The idea behind this approach is that Einstein gravity, viewed as a quantum field theory, becomes self-complete as it prevents from probing distances shorter than the Planck length, $l_{\rm P}$, by producing a large occupation number N of very long wavelength $l \gg l_{\rm P}$ to any high-energy scattering.

The merit behind the black hole's quantum N-portrait is that it offers an explanation of black hole properties, such as thermality and Bekenstein's entropy, which was believed to be impossible within the existing framework of Einsteinian gravity, no matter whether classical or quantum. The explanation of the Bekenstein's entropy also establishes a link with the Holographic Principle. In the context of this article, it is worth noticing that the expression (12) directly determines the postulated quantum self-coupling of gravitons and its relationship to the Cosmological Constant:

$$\Lambda \sim \frac{1}{l_{\rm P}^2}\left(\frac{l}{l_{\rm P}}\right)^2 \sim \frac{\alpha_g}{l_{\rm P}^2} , \tag{15}$$

which, multiplying both sides by $l_{\rm P}^2$, becomes exactly Eq. (14), thereby delivering the postulated quantum self-coupling of gravitons:

$$\Lambda l_{\rm P}^2 \sim \left(\frac{l_{\rm P}}{l}\right)^2 \sim \alpha_g . \tag{16}$$

The above expression portrays the gravitons as the "sinews" of gravity, i.e., coherent structures transversally confined within a squarelet of area $l_{\rm P}^2$ on the surface of a sphere of radius l, and fully delocalized along the longitudinal direction, basically connecting different points on the sphere along its diameter of size l (nonlocality).

Consistently with this picture, the graviton counting is then $N l_{\rm P}^2 l \sim l^3$, namely $N \sim (l/l_{\rm P})^2$.

So far, we have shown that the Vacuum Catastrophe leads to a similar expression for the metric fluctuations at the Planck scale as those postulated by Wheeler, under the assumption of vacuum modes associated with gravitons.

These fluctuations have been linked to gravitational interactions between virtual particle-antiparticle pairs through the proposed connections between Zel'dovich's ansatz and the Holographic Principle, without invoking any operational limitations associated to the measurement of spacetime. Furthermore, the approach taken so far seems to reasonably agree with the experimental values of the Cosmological Constant and links up naturally to the quantum self-coupling of gravitons postulated by the quantum N-portrait hypothesis.

A question remains, though, as to why one should choose l comparable to the Universe radius. As explained elsewhere [34], considering the possibility for dark energy to be associated with "quanta of gravity", plausibly implies that these quanta should be naturally delocalized across the full extent of the Hubble sphere, which in our case is the Universe radius. An interesting side remark concerns the large occupation number N mentioned above.

According to the quantum N-portrait hypothesis, N is given by

$$N = \frac{1}{\alpha_g}\,, \tag{17}$$

By taking again $l_{\rm P} \sim 10^{-35}\,m$ and $l \sim 10^{27}\,m$ gives $N \sim 10^{120}$, corresponding to a very large occupation number N of a very long wavelength compared to the Planck one and coinciding with the order of magnitude difference between the measured value of the Cosmological Constant and the one obtained theoretically.

Another relevant remark is as follows. As mentioned before, while gravitons are assumed to be massless, they can still carry energy, like other gauge quanta, such as photons and gluons, do. The relation of the possible mass of a graviton, m_G, and the Cosmological Constant has been postulated elsewhere as [35–38]:

$$m_G \sim \frac{\hbar}{c}\Lambda^{1/2}\,. \tag{18}$$

Considering the expression for Λ obtained previously in (16), we obtain:

$$m_G \sim \frac{\hbar}{cl}\,, \tag{19}$$

which again, by taking l as the current radius of the Universe, gives a value of $m_G \sim 10^{-69}\,Kg$, in agreement with estimates obtained elsewhere of $m \sim 10^{-68}\,Kg$ (or $10^{-32}\,eV$) [35–38].

Expression (19) has been postulated in the context of a modified nonlocal theory of gravity, specifically, the quantum corrected Raychaudhuri equation, in which geodesics are replaced by quantum Bohmian trajectories. Our estimate does not need any such modifications, nor does it assume any non-locality through Bohmian trajectories. Instead, it naturally aligns with the proposed quantum N-portrait paradigm, in which "quanta of gravity"are naturally delocalized across the entire Hubble distance [34].

Towards Dark Energy as a Bose-Einstein Graviton Condensate. – As postulated by the quantum N-portrait, the strength of graviton-graviton interaction is measured by a dimensionless coupling constant given by expression (14), which could be interpreted as the relativistic generalization of the Newtonian attraction potential among two gravitons, written as [28]:

$$V(l) = \alpha_g\,\frac{\hbar c}{l}\,. \tag{20}$$

Multiplying expression (6) by l^3, directly gives

$$V(l) = \rho_v c^2 l^3 \sim \frac{G\hbar^2}{c^2 l^3} = \hbar c\left(\frac{l_{\rm P}}{l}\right)^2\frac{1}{l} = \alpha_g\,\frac{\hbar c}{l}\,, \tag{21}$$

which is exactly expression (20). It is noticeable that, while expression (20) has been estimated for the case of black holes, a similar one is obtained in the context of this paper in relation to dark energy and therefore the cosmological constant.

In passing, we note that this also reminds electrostatic interactions between relativistic electrons in graphene, which write $V_{el}(l) = \frac{e^2}{l} = \alpha_f\frac{\hbar c}{l}$, where $\alpha_f = \frac{e^2}{\hbar v_f}$ is the graphene fine-structure constant, $v_f \sim c/100$ being the Fermi speed of the electron excitations. Indeed, holographic analogies have been invoked in the context of electronic transport in graphene [39], which, due to the strong electronic coupling, $\alpha_f \sim 1$, can often be treated by classical hydrodynamic analogies [40].

An equally interesting similarity with the quantum N-portrait concerns the large occupation number N, which indicates the extent to which quantum states are filled up by the excitations of a quantum-mechanical system consisting of many identical gravitons sharing the same quantum state.

According to the quantum N-portrait hypothesis, N, is given by

$$N = \frac{1}{\alpha_g}\,, \tag{22}$$

By taking again $l_{\rm P} \sim 10^{-35}\,m$ and $l \sim 10^{27}\,m$ gives $N \sim 10^{120}$, corresponding to a large occupation number N of very long wavelength compared to the Planck one. As interestingly noted elsewhere, this shows that the contribution of the vacuum energy density is strongly suppressed by large value of N [41].

This could explain the small value of the Cosmological Constant, as well as the reasonable agreement we obtained with its experimental value, since the suppression of Λ by the number of gravitons would be of the order $1/N \sim 10^{-120}$.

The above considerations seem to indicate that, in line with the quantum N-portrait, the Zel'dovich scenario also involves graviton condensates with large occupation numbers. As this number increases, fueled by the ceaseless production of particle-antiparticle pairs in the quantum foam, collective effects become dominant to the point of triggering coalescence and self-condensation. Such scenario provides a physical realization of the inverse cascade envisaged in [4] on purely speculative grounds.

It is worth referring to G. Dvali and C. Gomez for an explanation [28]. As these authors point out, it is useful to imagine localizing as many gravitons as possible within a space region of size l, in our case the Universe radius. In other words, trying to form a Bose-Einstein graviton condensate of characteristic wavelength l by gradually increasing their occupation number N. When N is small, the graviton interaction is negligible, but as N increases, individual gravitons feel a stronger and stronger binding potential and at a critical occupation number, a self-sustained condensate forms. The quantum N-portrait predicts a critical occupation number given precisely by $N_c = 1/\alpha_g$, which in our case leads to

$$N_c = \frac{1}{\Lambda l_{\rm P}^2} \sim \left(\frac{l}{l_{\rm P}}\right)^2\,. \tag{23}$$

The critical occupation number also indicates that the graviton condensate is maximally packed. Following the quantum N-portrait paradigm, the spectrum of fluctuations is determined by the Bogoliubov-De Gennes equation. The energy gap to the first Bogoliubov level, ϵ_1, is then given by

$$\epsilon_1 = \frac{1}{N^{1/2}} \frac{\hbar c}{l} \,, \tag{24}$$

which, for the case of the present paper, delivers

$$\epsilon_1 = \hbar c \frac{l_{\rm P}}{l^2} \,. \tag{25}$$

Considering that the Planck energy is $E_{\rm P} = \hbar / t_{\rm P}$, being $t_{\rm P}$ the Planck time, expression (25) leads to:

$$\epsilon_1 = E_{\rm P} \left(\frac{l_{\rm P}}{l} \right)^2 \,. \tag{26}$$

It is also interesting to notice that expression (23) resembles the Bekenstein bound, thereby connecting with the Holographic Principle and consequently to a holographic nature of dark energy [42].

Conclusions. – Since hypothesized by Wheeler, quantum foam has generated a rich stream of physics speculations related to the nature of fluctuations of spacetime at the fundamental level. Recent experimental observations, though, indicate that previous models, based on plausible assumptions on the operational definition of length fluctuations at the Planck scale, should nevertheless be ruled out. The present paper offers an alternative angle to explain the origin of the so-called quantum foam and its potential relation to dark energy and the resulting experimental value of the Cosmological Constant.

The starting point in section 2, has been the assumption of gravitons with frequency by $\omega \sim c/l$ which lead to quantum foam metric fluctuations, as suggested by Wheeler when directly inserted into the expression of the vacuum energy density. In section 3, following Zel'dovich's ansatz, supplemented with the Holographic Principle, we have argued that such postulated gravitons are the mediators of the gravitational interaction between virtual particle-antiparticle pairs, continually generated and annihilated in the vacuum state. Furthermore, by considering the vacuum energy density, a satisfactory estimate of the experimental values of the Cosmological Constant has been obtained, upon assuming l as the Universe radius.

In section 4, the assumption of such gravitons has been cross-checked with the quantum N-portrait paradigm, which predicts a graviton coupling for the case of black holes. It has been argued that the predicted coupling is also plausibly applicable to our case, related to the Cosmological Constant estimate obtained in section 3.

In section 5, our approach has been cross-checked against the mass of gravitons postulated by a modified theory of gravity based on a quantum corrected Raychaudhuri equation, in which geodesics are replaced by nonlocal Bohmian trajectories. We have argued that the postulated mass could be obtained by following the Zel'dovich approach with no need to invoke any gravity modifications. Moreover, in such an approach, non-locality emerges in the form of "quanta of gravity"delocalized across the entire Hubble distance, thus accounting for the match with the experimental value of the Cosmological Constant when taking l as the Universe radius.

Finally, we have suggested that the approach discussed in this paper is conducive to a Bose-Einstein graviton gas condensate, characterized by very large occupation numbers, which would explain the small value of the Cosmological Constant as due to the suppression by the large number of gravitons, of the order $1/N \sim 10^{-120}$.

In summary, it seems plausible to consider the Zel'dovich's ansatz supplemented with the Holographic Principle, to explain the origin of quantum foam, dark energy and the small experimental value of the Cosmological Constant.

It remains of course for experimental observations to validate or disprove the picture presented here. At the moment, its main merit is simplicity, as it only necessitates two basic assumptions. First, as proposed by Zel'dovich, that quantum vacuum fluctuations are linked to gravitational interactions between virtual particle-antiparticle pairs. Second, the existence of a UV cut-off dictated by the Holographic Principle.

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