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SOME ASPECTS OF WIDE-BAND CATHODE-FOLLOWER CIRCUITS

Summary

Wide-band cathode-followers, if designed on the basis of low frequency formulas, often yield unsatisfactory results if subjected to high frequency input signals.

This report sets out to show that the maximum excitation level for distortion-free operation at high frequencies is considerably smaller than that given by the low frequency formula.

Moreover, the input conductance may assume a very high, and even negative, value which may result in unstable operation of the circuit.

Finally, it is shown that the input capacitance of the circuit increases with rising frequency which prevents one distinct advantage of a cathode-follower, i.e. a small input capacitance, from materializing.

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1. <u>Low frequency excitation limit</u>

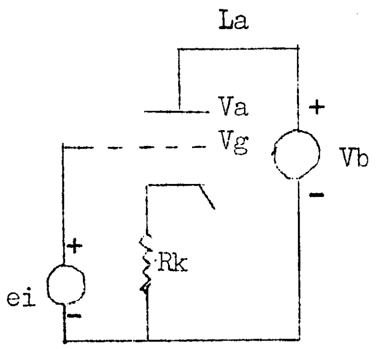

Fig. 1

From fig. 1 we can write the mesh equations :

$$Va + ia \cdot Rk = Vb \qquad (1)$$

$$Vg + ia \cdot Rk = ei \qquad (2)$$

If we now assume the tube to be linear, we can write the following general expression for the anode current :

$$ia \cdot Ri = \mu \cdot Vg + Va \qquad (3)$$

Replacing Vg and Va in (3) by the values for Vg and Va as expressed in (1) and (2), we find :

$$ia \cdot Ri = \mu(ei - ia \cdot Rk) + Vb - ia \cdot Rk \qquad (4)$$

The anode current consists of two components, one which is a D.C. current due to Vb and the other is a signal current due to ei.

The maximum current, which does not yet produce distortion **occurs when**

$Vg = 0$. This condition is met for $ei = ia \cdot Rk$. The value for the maximum current due to Vb is the D.C. current and from (4) we see that

$$ia_{max} = \frac{Vb}{Ri + Rk} \tag{5}$$

The current produced by the input signal ei, on the assumption that Vb is by-passed for any input signal frequency, is :

$$ia = \frac{\mu \cdot ei}{Ri + Rk(\mu+1)} \tag{6}$$

The limiting value of ei produces the current given in (5). Setting (5) and (6) equal, one obtains for ei_{ℓ} :

$$ei_{\ell} = \frac{Vb}{\mu} \left[\frac{Ri + Rk(\mu+1)}{Ri + Rk} \right] \tag{7}$$

This expression actually gives the whole grid voltage space. If, however, the circuit is designed so that the bias current through the tube is $\beta \cdot ia_{max}$, the expression (7) has to be multiplied by a factor $(1 - \beta)$, thus :

$$ei_{\ell} = \frac{Vb}{2\mu} \left[\frac{1 + \frac{Rk}{Ri} \cdot \mu}{1 + \frac{Rk}{Ri}} \right] \tag{9}$$

which is the well-known formula with which the input voltage limits for class A cathode followers are computed.

2. <u>High frequency excitation reduction</u>

The output voltage of the circuit of fig. 1 can be written as :

$$e_o = ia \cdot Rk = \frac{\mu \cdot Rk \cdot ei}{Ri + Rk(\mu+1)} \qquad (10)$$

and the gain of the circuit is

$$\frac{e_o}{ei} = \frac{\mu \cdot Rk}{Ri + Rk(\mu+1)} = G_o \qquad (11)$$

The grid-cathode voltage Vg from mesh equation 2 can with (11) be written as :

$$Vg = ei\,(1 - G_o) \qquad (12)$$

It will be shown that for high frequencies the gain becomes complex and can be written as :

$$\bar{G} = G_o \sqrt{1-\alpha_1} \cdot e^{j\theta_1} \qquad (13)$$

Under these conditions one has to substitute (13) for G_o in equation (12) . Under no circumstances must the grid-cathode voltage become larger than that value of Vg which exists for $ei = ei_\ell$, As this limiting value of Vg is the same for low and high frequencies, we can write :

$$Vg_\ell = ei_\ell\,(1 - G_o) = ei_{\omega\ell}\,(1 - G_o\sqrt{1-\alpha_1} \cdot e^{j\theta_1}) \qquad (14)$$

and from (14) we derive :

$$ei_{\omega\ell} = ei_{\ell} \frac{1 - G_o}{1 - G_o\sqrt{1-\alpha_1} \cdot e^{j\theta_1}} \qquad (15)$$

and the high frequency excitation reduction factor therefore is :

$$\frac{1 - G_o}{1 - G_o\sqrt{1-\alpha_1} \cdot e^{j\theta_1}} \qquad (16)$$

This reduction in excitation limit may be explained from a vector diagram. In fig. 2, no difference in phase exists between input and output signal, so fig. 2 is the vector representation of equation 12 :

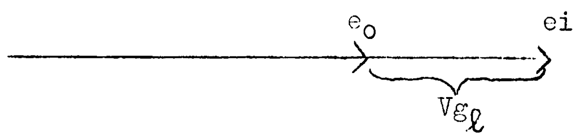

Fig. 2

If now the gain becomes smaller and shows a phase difference in comparison to the input signal, the vector representation is given in fig. 3 :

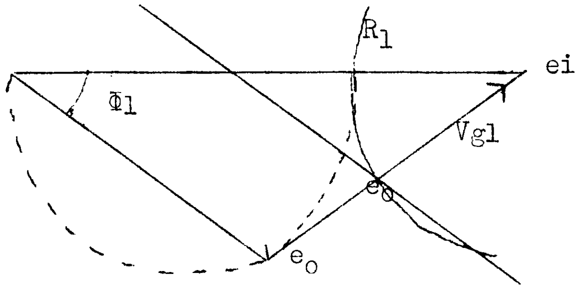

Fig. 3

and the vector Vg is now larger than the radius of the circle R_1. In order to bring Vg_ℓ to its maximum value the excitation has to be reduced until e_o lies on the circle R_1.

3.1 Derivation of cathode-follower matrix

The general circuit of a cathode-follower circuit (when the power supply is by-passed for all frequencies, and disregarding the grid leak resistor) is given in fig. 4 :

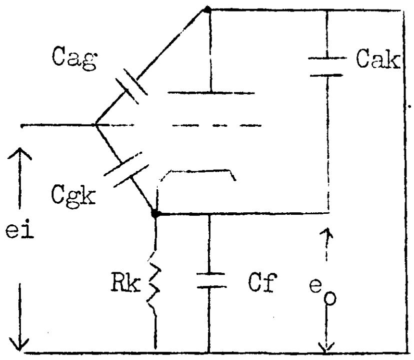

Fig. 4

To analyse the circuit of fig. 4, the circuit will be regarded as a four-terminal network, and the matrix method for the analysis of four-terminal networks is applied (ref. 1, 2, 3).

The circuit of fig. 2 is split into a passive and an active four-terminal network, see figs. 5 and 6,

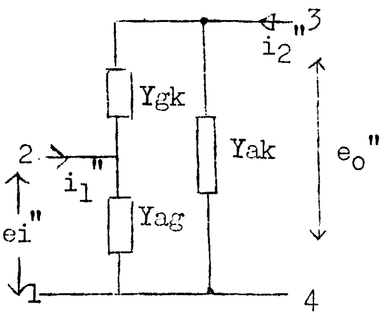

Fig. 5

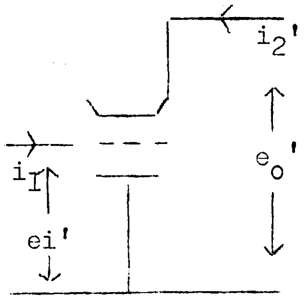

Fig. 6

where

$Yag = j\omega Cag$

$Ygk = j\omega Cgk$

$Yak = \frac{1}{Rk} + j\omega Cf + j\omega Cak + j\omega C_{stray}$

From figs 3 and 4 the following relationships are derived :

$$i_1 = i_1' + i_1'' \qquad (19)$$

$$i_2 = i_2' + i_2'' \qquad (20)$$

$$ei = ei' + ei'' \qquad (21)$$

$$e_o = e_o' + e_o'' \qquad (22)$$

$$i_1 = 0 \qquad \text{no grid current} \qquad (23)$$

$$i_2 = -\, gm.ei' + (gm + \frac{1}{Ri})\, e_o' \qquad (24)$$

The passive matrix from fig. 5 is :

$$\begin{Vmatrix} i_1'' \\ i_2'' \end{Vmatrix} = \begin{Vmatrix} Yag + Ygk & -\,Ygk \\ -\,Ygk & +\,(Ygk + Yak) \end{Vmatrix} \cdot \begin{Vmatrix} ei'' \\ e_o'' \end{Vmatrix} \qquad (M_1)$$

The equations (23) and (24) in matrix form :

$$\begin{Vmatrix} L_1' \\ L_2' \end{Vmatrix} = \begin{Vmatrix} 0 & 0 \\ -\,gm & +\,(gm + \frac{1}{Ri}) \end{Vmatrix} \cdot \begin{Vmatrix} ei' \\ e_o' \end{Vmatrix} \qquad (M_2)$$

The matrix for i_1, i_2 and ei, e_o is the sum of the matrices $M_1 + M_2$ giving as the complete matrix :

$$\begin{Vmatrix} L_1 \\ L_2 \end{Vmatrix} = \begin{Vmatrix} Ygk + Yag & - Ygk \\ -(gm + Ygk) & +(Ygk + Yak + gm + \frac{1}{Ri}) \end{Vmatrix} \cdot \begin{Vmatrix} ei \\ e_o \end{Vmatrix} \qquad (M\ 3)$$

We can now compare (M 3) with the admittance matrix :

$$\begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{Vmatrix} \cdot \begin{Vmatrix} ei \\ e_o \end{Vmatrix} \qquad (M\ 4)$$

From the admittance matrix we know that :

$$\frac{e_o}{ei} = - \frac{Y_{21}}{Y_{22}} \qquad (25)$$

$$Y_{im} = Y_{11} - \frac{Y_{12} \cdot Y_{21}}{Y_{22}} \qquad (26)$$

$$Y_{out} = Y_{22} \qquad (27)$$

By comparing (M 3) with (M 4) we get :

$$Y_{11} = Yag + Vgk \qquad (28)$$

$$Y_{12} = - Ygk \qquad (29)$$

$$Y_{21} = - (gm + Ygk) \qquad (30)$$

$$Y_{22} = Ygk + Yak + gm + \frac{1}{Ri} \qquad (31)$$

where

$$\left.\begin{aligned} Yag &= j\omega Cag \\ Ygk &= j\omega Cgk \\ Yak &= j\omega Cf + j\omega Cak + j\omega C_{stray} + \frac{1}{Rk} \\ &= j\omega C_o + \frac{1}{Rk} \end{aligned}\right) \qquad (32)$$

3.2 <u>Circuit Parameters</u>

Inserting equations 28 - 32 into 25, 26 and 27 gives the following :

$$\text{gain} = - \frac{Y_{21}}{Y_{22}} = \frac{j\omega Cgk + gm}{j\omega \, (Cgk+C_o) + gm + \frac{1}{Ri} + \frac{1}{Rk}} \qquad (33)$$

If in (33) we assume ω to be zero, the expression for the low frequency gain is

$$G_o = \frac{gm}{gm + \frac{1}{Ri} + \frac{1}{Rk}} = \frac{\mu \cdot Rk}{Ri + Rk\,(\mu+1)} \qquad (34)$$

Input admittance

$$Y_{in} = Y_{11} - \frac{Y_{12} \cdot Y_{21}}{Y_{22}} = j\omega\,(Cag + Cgk) - \frac{j\omega Cgk\,(gm + j\omega Cgk)}{j\omega(Cgk + C_o) + gm + \frac{1}{Ri} + \frac{1}{Rk}} \qquad (35)$$

If in (35) we divide the expression by $j\omega$ which we assume to be near zero, we get the capacitive component of the input admittance. So

$$C_{in} = Cag + Cgk - Cgk \cdot \frac{gm}{gm + \frac{1}{Ri} + \frac{1}{Rk}} \qquad (36)$$

Substitution of (34) in (36) gives the following for the input capacitance :

$$C_{in} = Cag + Cgk\,(1 - G_o) \qquad (37)$$

Output admittance :

$$Y_{22} = j\omega\,(Cgk + C_o) + gm + \frac{1}{Ri} + \frac{1}{Rk} \qquad (38)$$

Again we assume ω to be zero so that the output conductance becomes :

$$Y_{22} = gm + \frac{1}{Ri} + \frac{1}{Rk} \tag{39}$$

Generally one assumes $\frac{1}{Ri}$ and $\frac{1}{Rk}$ to be smaller than gm so that the output resistance of the cathode-follower is quoted to be :

$$Z_o = \frac{1}{Y_{22}} = \frac{1}{gm} \tag{40}$$

4.1 Magnitude of H.F. excitation reduction factor

It has already been shown that this reduction factor is given by the expression :

$$\frac{1 - G_o}{1 - G_o\sqrt{1-\alpha_1} \cdot e^{j\theta_1}} \tag{16}$$

From equation (33), which is the expression for the complex gain, we can write for the modulus :

$$|\bar{G}| = \left[\frac{\omega^2 Cgk^2 + gm^2}{\omega^2(Cgk+C_o)^2 + (gm+\frac{1}{Ri}+\frac{1}{Rk})^2}\right]^{1/2} \tag{35}$$

which can be rearranged to :

$$\left|\bar{G}\right| = G_o \left[1 - \frac{\omega^2\left\{(Cgk+C_o)^2 gm^2 - Cgk^2\left(gm+\frac{1}{Ri}+\frac{1}{Rk}\right)^2\right\}}{gm^2\left(gm+\frac{1}{Ri}+\frac{1}{Rk}\right)^2 + gm^2\omega^2(Cgk+C_o)^2}\right]^{1/2} = G_o(1-\alpha)^{1/2} \qquad (41)$$

The argument of the gain is :

$$\theta_1 = \theta a - \theta b \qquad (42)$$

where, again from (33)

$$\theta_a = \text{arc tg}\,\frac{\omega Cgk}{gm} \qquad (43)$$

$$\theta_b = \text{arc tg}\,\frac{\omega(Cgk+C_o)}{gm+\frac{1}{Ri}+\frac{1}{Rk}} \qquad (44)$$

and, via the trigonometric identity

$$\text{tg}\,(\theta_a-\theta_b) = \frac{\text{tg}\theta_a - \text{tg}\theta_b}{1+\text{tg}\theta_a\cdot\text{tg}\theta_b} \qquad (45)$$

we can write for the phase angle θ_1 :

$$\theta_1 = \text{arc} \cdot \text{tg} \frac{\omega Cgk(gm+\frac{1}{Ri}+\frac{1}{Rk}) - \omega gm(Cgk+C_o)}{gm(gm+\frac{1}{Ri}+\frac{1}{Rk}) + \omega^2 Cgk(Cgk+C_o)} \tag{46}$$

If in equation (41) we substitute the following expressions :

$$\omega_1 = \frac{gm}{Cgk} \tag{47}$$

$$\omega_2 = \frac{gm + \frac{1}{Ri} + \frac{1}{Rk}}{Cgk + C_o} \tag{48}$$

the equation (41) can be written as :

$$G_o \, (1-\alpha)^{1/2} = G_o \left[\frac{1 + \frac{\omega^2}{\omega_1^2}}{1 + \frac{\omega^2}{\omega_2^2}} \right]^{1/2} \tag{49}$$

Substitution of (47) and (48) in (46) gives :

$$\theta_1 = \text{arc tg} \frac{\omega(\omega_2 - \omega_1)}{\omega^2 + \omega_1 \cdot \omega_2} \tag{50}$$

With the expressions (49) and (50) we can now write for the complete expression for the excitation limit under class A conditions (where we write for $(1-G_o) = \frac{Ri + Rk}{Ri + Rk(\mu+1)}$) :

$$ei_{\ell(\omega)} = \frac{Vb}{2\mu} \cdot \frac{1}{1 - G_o \left[\frac{1+\frac{\omega^2}{\omega_1^2}}{1+\frac{\omega^2}{\omega_2^2}} \right]^{1/2} \cdot \cos \text{arc tg} \frac{(\omega_2-\omega_1)}{\omega^2+\omega_1\omega_2}} \qquad (51)$$

4.2 <u>Simplified expressions</u>

If we now assume that

$$Cgk < C_o \qquad (52)$$

and

$$gm > \frac{1}{Ri} + \frac{1}{Rk} \qquad (53)$$

it then follows that ω_2 comes much earlier than ω_1 , and if we take only ω_2 into consideration the expression (51) can be written as follows :

$$ei_{\ell(\omega)} = \frac{Vb}{2\mu} \cdot \frac{1}{1 - G_o \left(1+\frac{\omega^2}{\omega_2^2}\right)^{-1/2} \cos \text{arc tg} \frac{\omega}{\omega_2}} \qquad (54)$$

This expression is plotted against $\frac{\omega}{\omega_2}$ and multiplication factors of $\frac{Vb}{2\mu}$ with the low frequency gain G_o as a parameter in nomograph N I .

When the cathode-follower is used to drive a matched coaxial cable, we can assume that, if a high slope tube is used, the grid cathode capacitance and the output capacitance are roughly equal.

The expression (51) can then be written as :

$$ei_{\ell(\omega)} = \frac{Vb}{2\mu} \cdot \frac{1}{1 - G_o\left[1+\left(\frac{\omega}{2\omega_2}\right)^2\right]^{1/2} \cdot \left(1+\frac{\omega^2}{\omega_2^2}\right)^{-1/2} \cos \operatorname{arc} \operatorname{tg}\left(\frac{\frac{\omega}{\omega_2}}{1+\frac{\omega^2}{2\omega_2^2}}\right)} \qquad (55)$$

This expression is plotted, in the same way as (54) , in nomograph N II

5.1 Input admittance of cathode-follower

In equation 35, the input admittance has been given as :

$$Y_{in} = j\omega\,(Cag+Cgk) - \frac{j\omega\, Cgk\,(gm+j\omega Cgk)}{j\omega(Cgk+C_o) + gm + \frac{1}{Ri} + \frac{1}{Rk}} \qquad (35)$$

If we split (35) into its real and imaginary component we get :

$$\text{Real Part :} \quad \frac{\omega^2\, Cgk\left[Cgk\,(gm+\frac{1}{R}) - gm\,(Cgk+C_o)\right]}{(gm+\frac{1}{R})^2 + \omega^2(Cgk+C_o)^2} \qquad (57)$$

Imaginary part :

$$\omega(Cag+Cgk) - \frac{\omega^2 Cgk \left[gm\ (gm+\frac{1}{R}) + \omega Cgk\ (Cgk+C_o)\right]}{(gm+\frac{1}{R})^2 + \omega^2\ (Cgk+C_o)^2} \tag{58}$$

where both in (57) and (58)

$$\frac{1}{R} = \frac{1}{Ri} + \frac{1}{Rk} \tag{59}$$

5.2 <u>Input conductance</u>

The input conductance of the circuit is given by the real part of the input admittance (57). Examination shows that for large values of gm, practically any value of C_o will make this input conductance negative.

As in general the grid circuit of the cathode-follower has a grid leak resistance to earth (Rg), the input admittance of the complete circuit with grid leak resistance, on the assumption that $gm > \frac{1}{R}$ (53), is :

$$Y_{in} = \frac{1}{Rg} - \frac{\omega^2 Cgk \cdot gm^2 \cdot C_o}{(gm^2)^2 + \omega^2 (Cgk+C_o)^2} \tag{60}$$

and the input conductance will go through zero for the ω of :

$$\omega_3 = \frac{1}{\sqrt{\frac{Rg \cdot Cgk \cdot C_o}{gm} - \left(\frac{Cgk+C_o}{gm}\right)^2}} \tag{61}$$

5.3 Input capacitance

The input capacitance is given by the imaginary part of the input admittance (58) divided by ω.

$$C_{in} = Cag + Cgk - \frac{\omega\, Cgk\left[gm^2 + \omega(Cgk+C_o)\, Cgk\right]}{gm^2 + \omega^2(Cgk+C_o)^2} \qquad (63)$$

which, for those frequencies where $\omega\, Cgk(Cgk+C_o) > gm^2$ and $\omega^2(Cgk+C_o)^2 > gm^2$, may be written as :

$$C_{in} = Cag + Cgk\,(1 - \frac{Cgk}{Cgk + C_o}) \qquad (64)$$

and it becomes clear that for large values of C_o the input capacitance of the cathode-follower assumes the value

$$C_{in} \approx Cag + Cgk \qquad (65)$$

6. Special conditions

On the foregoing pages it has been shown that the excitation limit is rather dependent on the value of the output capacitance.

If, however, the designer of a cathode-follower circuit adds some lumped capacitance between the grid and the cathode to make Cgk equal to the output capacitance C_o and if, on the other hand, he chooses the cathode-load so low that the admittance of cathode resistance and internal resistance equal the slope, a very special condition is established.

Consequently, we can write :

$$Cgk = C_o \quad (66)$$

$$gm = \frac{1}{Ri} + \frac{1}{Rk} \quad (67)$$

$$\omega_1 = \frac{gm}{Cgk} \quad (47)$$

$$\omega_2 = \frac{2\ gm}{2\ Cgk} = \omega_1 \quad (68)$$

Thus, it can easily be shown that the gain of the circuit is constant and 0,5 (up to rather high frequencies). This means that no excitation limitation factor for high frequency operation exists and that the excitation limit is given by equation (9). Under these conditions the input conductance of the tube is always zero, and so the input resistance of the circuit is only determined by the grid leak and the stopper resistance. The input capacitance of the the circuit under these conditions is :

$$C_{in} = Cag + \frac{1}{2}\, Cgk\ . \quad (69)$$

Acknowledgments

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K. Gase

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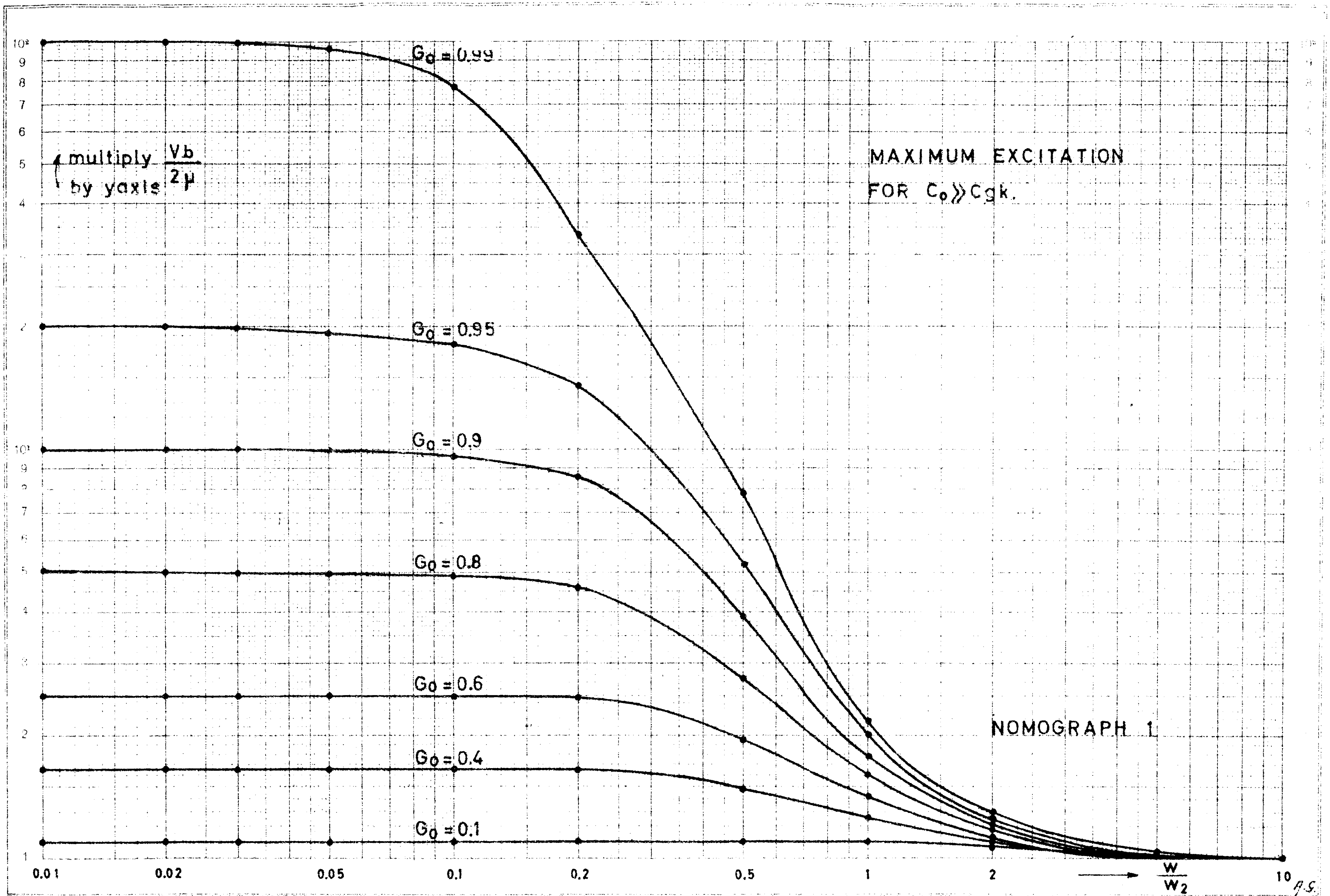

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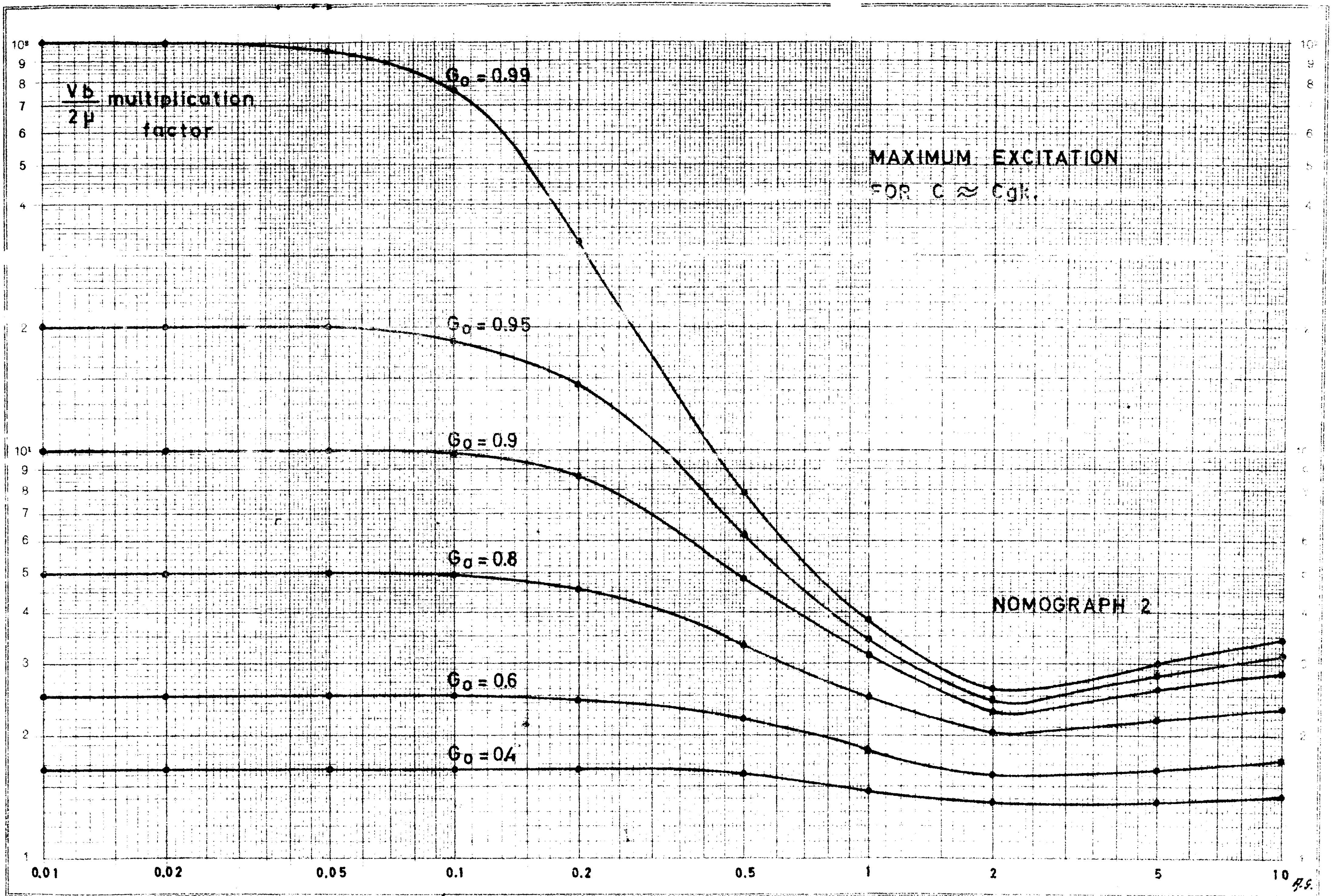

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