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Measuring the Chromaticity from Head-Tail Oscillations

R. Bossart, J.P. Delahaye, A. Hofmann

Introduction

In modern particle colliders it would be useful to monitor the chromaticity during the energy ramping by a continuous measurement of the transverse oscillations of the head and of the tail of a single bunch of beam. The phase between these oscillations would allow to measure the chromaticity during an individual acceleration cycle without displacing the closed orbit from its standard position. In the following we describe a possible implementation of a chromaticity tracking method with a phase detector of the head-tail oscillations. The method has not yet been tested with beam, and it is possible that the measurement is complicated by other modes of beam oscillation or by limitations of the instrumentation.

Chromaticity Measurement from Momentum Change

The chromatictiy ξ of an accelerator is given by the ratio between the relative tune variation $\partial Q/Q$ and the relative momentum variation $\partial p/p$ of the beam: $\xi = \frac{\partial Q}{Q} \;/\; \frac{\partial p}{p} = Q'/Q$.

For the commissioning of large particle colliders like LEP or LHC, it is necessary to carefully adjust the tune Q and the chromaticity ξ of the machine during the energy ramping. There exist different methods to measure the tune Q and to correct the machine cycle. Normally the chromaticity ξ is evaluated from the tune difference between two identical machine cycles with an offset Δp in beam momentum. The accuracy of the difference measurement $Q' \simeq \Delta Q \cdot p/\Delta p$ is limited by the reproducibility of the machine tune between cycles and by the dynamic aperture of the machine, which allows only for a small momentum offset Δp from the nominal momentum.

Chromaticity Measurement from Head-Tail Oscillations

In order to measure the chromaticity without changing the machine cycle, the properties of the head-tail mode oscillation can be used. Frequency domain measurements, observing the shift of the global mode spectrum, are not very sensitive. A new method is proposed which takes advantage of the phase difference X between the betatron oscillation of the head and the tail given by [1]:

Ref. [1]: F. Sacherer, Transverse bunched-beam instability-theory, Proc. 9th Int. Conf. on High Energy Accelerators, Stanford 1974, p. 347

$$X = (Q'/\eta)\,\omega_0\,\tau\,(m+1), \text{ where}$$

η = $1/\gamma^2_T - 1/\gamma^2 \simeq 1/\alpha_p$ for lepton machines $\gamma \gg \gamma_T$

γ = mass ratio of particles to rest mass

γ_T = mass ratio at transition, $\gamma_T \simeq Q$

ω_0 = angular frequency of beam revolution

T = beam revolution period, $\omega_0 = 2\,\pi/T$

τ = bunch length from head to tail, for EPA τ = 3...8 ns (4 σ)

m = mode of head-tail oscillation; m = 0, 1, 2 ...

Since the phase shift is linear along the bunch, the chromaticity can be measured from the phase difference X between any particles separated by the time interval τ inside a bunch. The betatron oscillations of the beam are monitored by a wideband position pick-up which provides an intensity signal Σ and a position signal Δ, see fig. 1.

The measurement technique is similar to that of the Q-measurement of a single bunch. But instead of one sample, two samples of beam position are taken from the head and the tail of the bunch at every beam revolution. For a sufficient signal amplitude, the bunch signal Δ is sampled when the sum signal Σ reaches half of its peak amplitude (fig. 1). For the different head-tail oscillation modes, the head and the tail of the bunch execute betatron oscillations at the same frequency. Their amplitude and phase can be determined by a fast Fourier transform (FFT) of the measurement data. There exist commercial spectrum analyzers which provide directly the phase difference X between the two betatron oscillations of the head and tail of a bunch.

For a cosine shaped bunch of 4.5 ns length at the base, the samples of the head and tail taken at half of the peak intensity are distant by 3 ns. Two fast sampling heads operating with a relative time delay of 3 ns are used as analogue signal buffers for the ADC which must convert the sampled Δ-signal in less than a beam revolution period with a resolution $< 10^{-3}$.

The accuracy of the phase measurement depends on the aperture jitter of the sampling heads among each other and the time jitter between the sampling command and the beam. For an overall sampling jitter of 0.1 ns and a sampling distance of 3 ns, the phase angle between the two betatron oscillations can be measured with an accuracy of 0.05 radians.

Because of the low value $\eta = 0$ at the transition of the machine cycle, the head-tail phase will change rapidly during the energy ramping of hadron beams and will go through infinity at transition (fig. 2).

If the phase angle between the head and the tail oscillations is larger than 2 π, the time interval τ between the two sampling points can be reduced. Reducing the sampling distance τ from 3 ns to 0.3 ns allows to measure the integer N of the head-tail phase $\chi = \chi_0 + 2N\pi$ up to $N \lesssim 10$ provided that the sampling jitter is less than $\pm$ 0.1 ns. Because of the reduced accuracy, the zooming with reduced sampling distance serves only to determine the integer number N, whereas the phase measurement with a sampling distance of 3 ns provides the phase $-\pi < \chi_0 < \pi$ with an accuracy of about 0.05 radians over a large range $0 \leqslant \chi \leqslant 64\pi$. Moreover, the phase measurement by the FFT-technique is rather insensitive to amplitude variations of more or less damped head-tail oscillations.

For a correct phase measurement only a single mode of the head-tail oscillations must be excited, preferentially mode zero. This can be checked from the observation of the envelope of the Δ-signal (fig. 1). If the bunch is shorter than 1 ns, the analogue observation of the head-tail oscillations is no more possible with a fast oscilloscope. A fast sampling head (t_r = 20 ps) must be used to check if there is no oscillation node between the head and the tail of the bunch.

If the amplitude of the head-tail oscillation is sufficiently stable, the phase between the head and the tail can be measured with an analogue phase detector and a low pass filter. If a faster measurement technique is required, a digital signal processor can be used.

Beam Excitation

Before the chromaticity can be measured, it is necessay that the machine tune Q has been measured and adjusted for its nominal value, and that the closed orbit has been corrected. The chromaticity sextupoles should provide the right sign for the corrected chromaticity below and above the transition energy of the machine, otherwise the beam could suffer from head-tail instabilities.

The beam is stable if the head-tail oscillations have a positive chromatic frequency shift f_ξ[1]:

$$f_\xi = \frac{\chi}{2\pi\,\tau_b} = \frac{\xi}{\eta}\, Q \cdot f_{rev}\,(m+1) > 0,$$

hence $\xi > 0$ for $\gamma > \gamma_T$

$\xi < 0$ for $\gamma < \gamma_T$

In order to excite damped head-tail oscillations, similar excitation techniques can be used as for the tune measurements. Among the usual methods for exciting coherent betatron oscillations, the following excitation techniques should provide coherent head-tail oscillations:

1. The beam is deflected horizontally or vertically by a short kicker pulse, which excites only a single bunch.

2. The beam is deflected transversely by a RF sweep around a betatron frequency of the beam. For continuous excitation, the RF generator must stay exactly at the aliased frequency f_β of the betratron oscillation: $f_\beta = |Q \pm N|\, f_{rev}$, with N as integer number.

The second method has been employed at the electron positron accumulator EPA to excite head-tail oscillations by a frequency sweep and to monitor the beam response on a spectrum analyser with a tracking generator [2]. The measurement has been carried out at the lowest frequency f_β i.e. below the revolution frequency of the beam. Thereby the machine chromaticity has been trimmed to a low value slightly positive.

The chromatic frequency shift f_0 of the head tail mode zero (m = 0) in lepton machines ($\eta = \alpha_p$ for $\gamma >> \gamma_T$) is given by

$$f_0 = \xi\, Q\, f_{rev}/\alpha_p = Q'\, f_{rev}/\alpha_p$$

For a reasonably well tuned machine, the chromaticity should not exceed the limits $0 \leqslant \xi \leqslant 1$ at high energy $\gamma >> \gamma_T$. As an example, in EPA we have f_{rev} = 2.386 MHz, α_p = 0.033, Q_v = 4.38 and Q'_v = +0.78 providing a frequency shift

$$f_0 = Q \quad 72 \text{ MHz} = 56 \text{ MHz}$$

In order to measure the chromaticity over a range $0 < \xi < 5$, an excitation bandwidth of 1.6 GHz is required in EPA. As a deflector a directional coupler is proposed. Its length l for a bandwidth f_2 = 1.6 GHz (-4 dB) is defined by

$$l = \lambda_2/4 = c/4\, f_2 \quad \text{with} \quad c = 3 \quad 10^8 \text{ m/s}$$

$$l = 47 \text{ mm for } f_2 = 1.6 \text{ GHz}$$

The transverse beam deflection δ by a narrow stripline terminated by its characteristic impedance and mounted inside a circular cylinder is given by [3]

$$\delta = \frac{2\,U}{E/e} \cdot \frac{1}{r} \cdot \frac{\sin\Phi}{\Phi} \qquad \text{where}$$

Ref. [2]: D. Brandt, J.P. Delahaye. A. Hofmann, Transverse Mode Measurements with Positrons in EPA, LEP Note 585, Note PS/LP/87-35.

Ref. [3]: G. Lambertson, Dynamic devices: Pick-ups and kickers, AIP Conf. Proc. 153, ed. M. Month (1987) 1413.

U : deflection voltage of stripline

E : beam energy, $e = 1.6 \cdot 10^{-19}$ As

l : length of stripline

r : radius of circular vacuum chamber

ω : angular frequency of deflection voltage

Φ : $\omega l/c$

$\sin \Phi/\Phi$: transit time factor for ultrarelativistic beams $v \simeq c$.

It is expected that excitation of the head-tail mode is easy at energies sufficiently above transition $\gamma > 2\ \gamma_T$. However, at low energies the chromatic frequency changes rapidly during the energy ramping and gets very high at transition (fig. 2). Since not all particles go through transition at the same time, a large frequency spread arises at this moment and the beam can no more be excited [4]

The damping decrement of the coherent oscillation is proportional to the frequency spread of the beam oscillations. The larger the frequency spread, the higher the excitation strength must be. In proton machines, continuous excitation of transverse oscillations may lead to beam losses because of emittance growth by those protons oscillating exactly at the resonance frequency [5]. In lepton machines, the emittance growth by resonant particles is damped and overcome by the synchrotron light emission.

Ref. [4]: R. Bossart, L. Burnod, J. Gareyte, B. de Raad, V. Rossi, The damper for the transverse instabilities of the SPS. IEEE Trans. on Nuclear Science, Vol. NS-26 (1979) 3284.

[5] R. Bossart, R. Louwerse, J. Mourier and L. Vos, Excitation of continuous betatron oscillations for tune measurements, SPS/ABM/ME 85-04 (1985).

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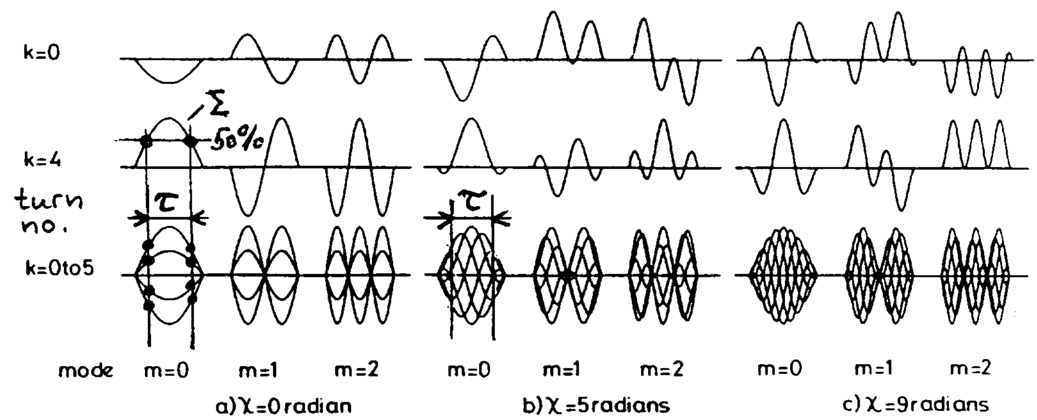

Fig. 1 Δ-signal of a single bunch on separate revolutions, and with six revolutions superimposed. Vertical axis is difference signal from position monitor, horizontal axis is time, and Q = 4.833. (from ref. 1)

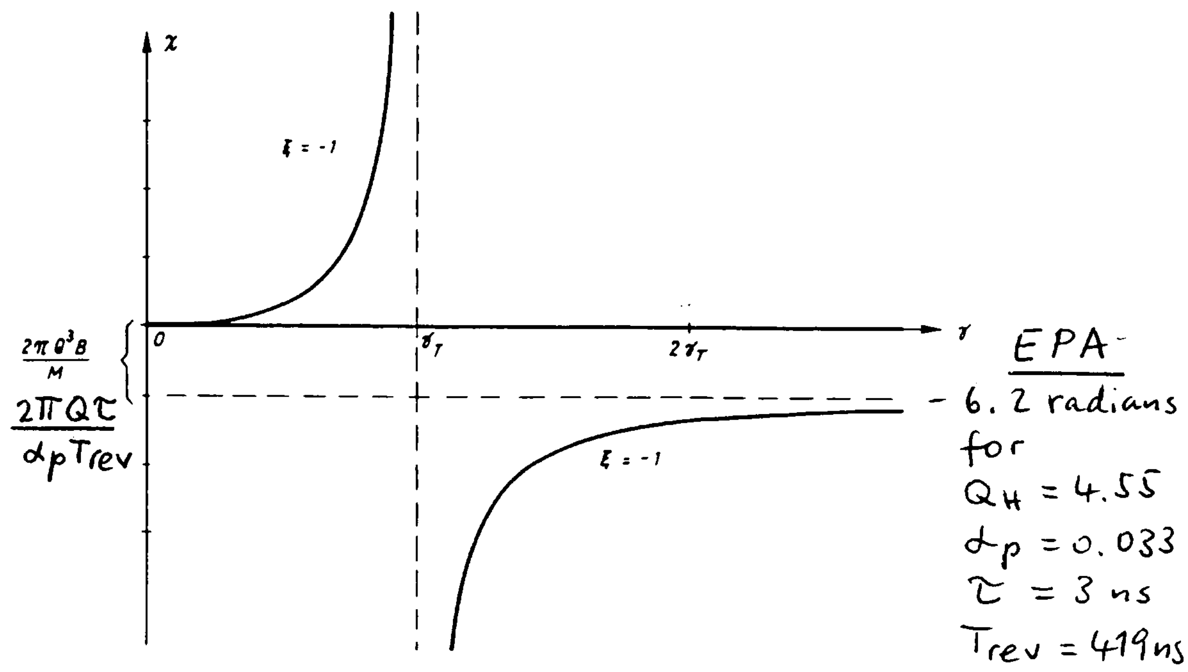

Fig. 2 Head-tail phase shift χ versus energy (from ref 1)