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The Fermi Interaction in β-Decay

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Consider an arbitrary mixture of the five linearly independent invariants in β-theory

Invariant	Coupling constant		Nuclear matrix element Allowed transitions		Selection rules		
Scalar	g_1	g_F	$\lvert\int 1\rvert$	F	$\Delta I = 0$	no	
Vector	g_2		$\lvert\int 1\rvert$		$\Delta I = 0$	no	
Tensor	g_3	g_{GT}	$\lvert\int \vec{\sigma}\rvert$	GT	$\Delta I = \begin{cases}0\\1\end{cases}$	no	no $0 \to 0$
Pseudovector	g_4		$\lvert\int \vec{\sigma}\rvert$		$\Delta I = \begin{cases}0\\1\end{cases}$	no	no $0 \to 0$
Pseudoscalar	g_5		$\lvert\int \gamma_5\rvert$		$I = 0$	yes	

Then, the β-spectrum for allowed transitions $\Delta I = \begin{cases}0\\1\end{cases}$ no is given by[1]

$$P_{\pm}(E) \sim F(Z,E)pE(E^{max}-E)^2\Big[(g_1^2+g_2^2)\left|\int 1\right|^2+(g_3^2+g_4^2)\left|\int\vec{\sigma}\right|^2$$

$$\mp(2\gamma/E)\Big(g_1g_2\left|\int 1\right|^2+g_3g_4\left|\int\vec{\sigma}\right|^2\Big)$$

$$= F(Z,E)pE(E^{max}-E)^2\Big[(g_1^2+g_2^2)(1\mp b_F/E)\left|\int 1\right|^2$$

$$+(g_3^2+g_4^2)(1\mp b_{GT}/E)\left|\int\vec{\sigma}\right|^2\Big]$$

Cross terms.

Since $|\int 1|$ transforms as Y_0 under rotations of space and $|\int \vec{\sigma}|$ as Y_1, only cross terms between g_1 and g_2 and between g_3 and g_4 appear.

The cross terms are generally believed to vanish, but this assumption is not very well established experimentally. When cross terms exist, the so-called Kurie plot, where $K = [P/pEF]^{1/2}$ is plotted as a function of E is curved. As an example, consider a β-spectrum with $E^{max} \sim 3$. If a straight line is drawn through points for K_1 ($E = 1,2$) and K_2($E = 2.7$), the maximum deviation from this line is $\sim 4\%$ for $1.2 \leq E \leq 2.7$ for maximum cross terms (i.e. $b_F = b_{GT} = 1$). This is a rather small effect.

In a careful search for the influence of cross terms, one would have to work with transitions where the ratio $|\int 1|^2 / |\int \vec{\sigma}|^2$ can be estimated so that b_F and b_{GT} can be estimated independently. Work of this kind remains to be done and results obtained so far only indicate $b < .5$. Nevertheless, for the following discussion, we shall assume $b_F = b_{GT} = 0$.

Determination of $g_F^2\ g_{GT}^2$. [2)]

The total disintegration probability λ is given by

$$\lambda = \text{const.}\ f(Z, E^{max}) [(1-x)|\int 1|^2 + x|\int \vec{\sigma}|^2]$$

with

$$(1-x) = g_F^2/(g_F^2 + g_{GT}^2) \quad \text{and} \quad x = g_{GT}^2/(g_F^2 + g_{GT}^2)$$

and, thus, each β-decay permits the determination of a straight line

$$B(x) = ft[(1-x)|\int 1|^2 + x|\int \vec{\sigma}|^2]$$

if ft is measured and $|\int 1|^2$ and $|\int \sigma|^2$ can be determined theoretically.

It is generally believed that nuclear wave functions can most unambiguously be constructed for those mirror nuclei which have closed shells of (0, 2, 8, 20) protons and neutrons $\pm$ one nucleon. If, in these cases, the change in the radial part of the wave function is neglected, $|\int 1|^2$ and $|\int \vec{\sigma}|^2$ can be calculated from the spherical part of the wave functions alone and one obtains the $B(x)$ -lines in Fig.1. These lines are inside the experimental errors consistent with a common intersection point of $(B_0, x_0) = (2600 \pm 85, .50 \pm .05)$, where the errors are mean square deviations found from internal consistency of the data. However, these errors should not be taken too literally and several critical remarks in this connection will be given later.

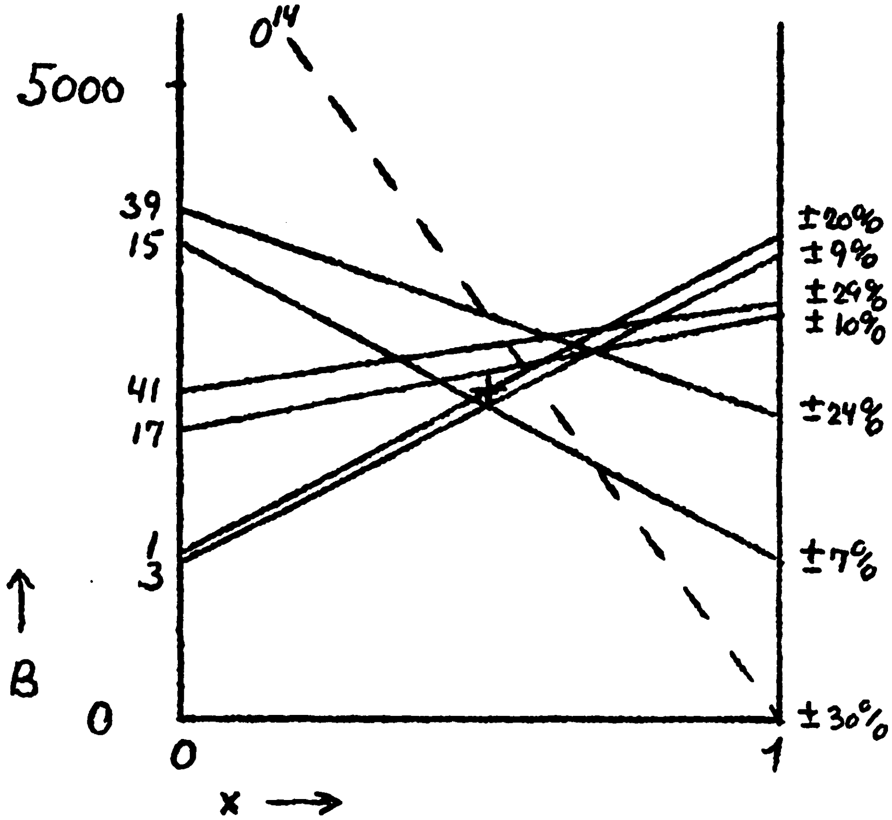

Fig 1.

Relation between $|\int\vec{\sigma}|^2$ and magnetic moments.[3]

If the extreme single particle picture is applied for the remaining mirror nuclei, the results in Fig.2 are obtained. The

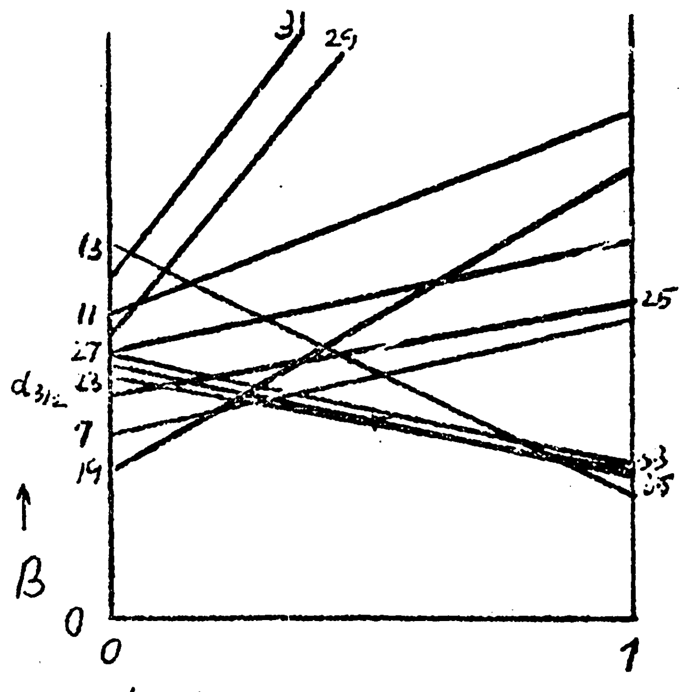

Fig. 2.

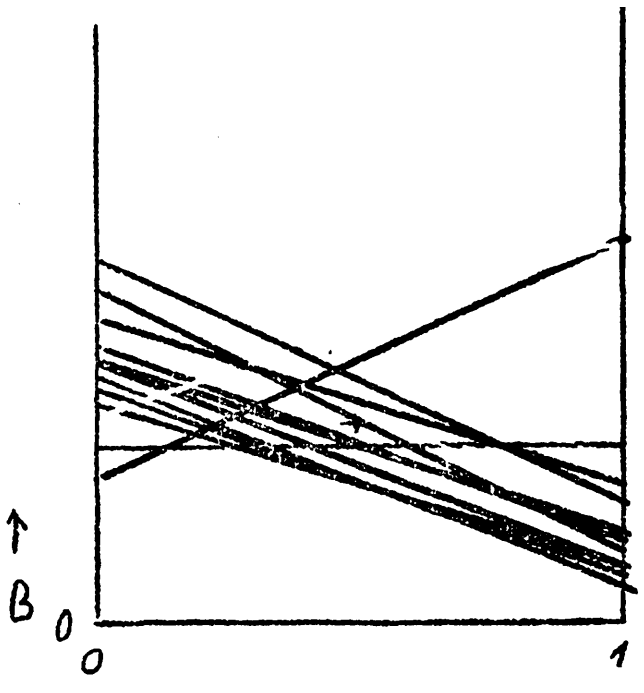

Fig. 3.

disagreement with a common intersection point is obvious. The application of charge symmetrized wave functions does not give better agreement. Also the magnetic moments for these nuclei disagree with the Schmidt lines, and it has been noted that better agreement for β-decay can be obtained if the wave functions are adjusted so as to give the correct magnetic moments.[3] This can be based on quite general arguments.[4] If the single particle shares the nuclear angular momentum with some other nucleons (e.g. the core as considered by A.Bohr and B.Mottelson[5]) we can write

$$\mu_{M_z=I} = g_s\langle s_z\rangle + g_l\langle l_z\rangle + g_R\langle R_z\rangle$$

where R and g_R describe the core or the group of particles participating in the angular motion.

Since

$$l_z \doteq I - R_z - s_z$$

we get

$$\mu_{M_z=I} = (g_s - g_l)\langle s_z\rangle + g_l I + (g_R - g_l)\langle R_z\rangle$$

as a first approximation we can neglect $(g_R - g_l)\langle R_z\rangle$ and, since for mirror nuclei we have

$$|\textstyle\int\vec{\sigma}|^2 = 4(I+1)/I\cdot\langle s_z\rangle^2$$

we get

$$|\textstyle\int\vec{\sigma}|^2 \cong 4\,\frac{I+1}{I}\left(\frac{\mu - g_l I}{g_s - g_l}\right)^2 \qquad (1)$$

We can thus from the experimental μ-values determine a half-empirical value for $|\int\vec{\sigma}|^2$. This leads to the improvement shown in Fig.3. The lines in Fig.1 are only changed a little.

In Fig.3, most of the lines deviate somewhat from crossing (B_o, x_o) but not more than can be explained by $(g_R - g_L)\langle R_z \rangle$ if one, e.g., takes $R_Z^{max} = \frac{I}{I+1}$ and $g_R = \frac{Z}{A}$ as obtained in the model used by A.Bohr and B.Mottelson[5] and taking into account the experimental errors in ft.

Although this relationship between μ and $|\int\vec{\sigma}|^2$ gives rise to some confidence inthe whole picture, it also may give rise to certain critical remarks as regards the determination of (B_o,x_o).

Possible errors in (B_o, x_o).

Since (1) for an odd neutron mirror nucleus gives

$$|\int\vec{\sigma}|^2 \simeq 4\frac{I+1}{I}\left(\frac{\mu}{g_s}\right)^2$$

it follows that the relative deviation from the single particle picture will be given by

$$\Delta \log |\int\vec{\sigma}|^2 \simeq 2\Delta\log|\mu|$$

For He^3, the observed μ deviates $\sim$ 10 % from the Schmidt line. Therefore, $|\int\vec{\sigma}|^2$ may be expected to deviate as much as 20 % from the single particle picture. This is much more than the experimental error indicated in Fig.1.

Also an admixture of D state to the He^3 wave function should be considered and the 4 % admixture estimated by Blatt[6] gives a change of 5 % in $|\int\vec{\sigma}|^2$ Blatt therefore concludes that a better estimate of (B_o x_o) would be obtained if more precise values for ft for the neutron and the $O^{14} \rightarrow N^{14*}$ decay were obtained, since here the matrix elements can be derived more unambiguously. $O^{14} \rightarrow N^{14*}$ gives the dotted line in Fig.1.

Both nuclear reaction data[7] and newly performed β-ray work[8] have shown that older measurements on He^6 and Ne^{23} are to be doubted. The same authors are responsible for the data used in the ft determinations for O^{15} and F^{17}. For F^{17} the result agrees with nuclear reaction data, for O^{15} no nuclear reaction data exist so far. Since, however, O^{15} is very important in the (B_0, x_0) determination, new measurements of the mass difference $O^{15} - N^{15}$ seem very desirable. A 25 % increase in ft for O^{15} would mean that all the lines in Fig.1 including O^{14} with $|\int 1|^2 = 2$ and He^6 with $|\int \vec{\sigma}|^2 = 5.4$ as mentioned by Wu et al.[8], would give a very close intersection in $(B_0, x_0) = (2800, .60)$. This result agrees closely with that obtained by Blatt,[6] and that obtained by Bouchez and Nataf[9].

Finally, it should be mentioned that cross terms also would influence (B_0, x_0) considerably and at the same time that large cross terms can be introduced without spoiling the internal consistency in Fig.1. If cross terms enter $B(x)$ would still be a straight line going from $f(b_F)t|\int 1|^2$ for $x = 0$ to $f(b_{GT})t|\int \vec{\sigma}|^2$ for $x = 1$ and an analysis of the six mirror nuclei considered in Fig.1 shows that even values of $(b_F, b_{GT}) \simeq (5, 5)$ would not spoil the consistency with a common intersection point.

Recoil experiments.

Further information on the coupling in β-decay can be obtained from recoil experiments. Results published hitherto have clearly demonstrated that momentum is not conserved for β-particle and recoil alone, and thus support the neutrino hypothesis, but very little can actually be said about the angular correlation, i.e. about the coupling constants.[10)]

Unpublished work on He^6 by Allen is quoted[5)] to give agreement with tensor interaction.

Work is in progress in Chalk River and Oak Ridge on the angular correlation in the neutron decay.

It has been mentioned[6)] that recoil experiments on O^{14} are very desirable. Unfortunately, O^{14} will probably form part of a molecule and therefore great difficulties will enter.

Forbidden β-spectra.

As reviewed by Wu[11)], the shape of forbidden β-spectra seems to support the hypothesis of mainly tensor interaction for the coupling.

Furthermore, if one believes that RaE → RaF is a 0 → 0 yes transition, it will be very difficult to explain the shape of this spectrum without a strong admixture of pseudoscalar coupling.

Since, however, the forbidden spectra involve many arbitrary constants, namely in addition to the g^s also ratios between different nuclear matrix elements, it is worth while to carry out a more detailed discussion of the subject. This program will be dealt with in a later report.

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