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NEGATIVE MASS EFFECT AT TRANSITION IN THE SPS

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An estimate, which has still to be written up, allows to check whether the negative mass effect, which is unavoidable after crossing transition, leads to a dangerous blow-up of the bunch height. The main feature is to integrate the growth rate for the most dangerous mode through the unstable region and to make an estimate of the initial condition of the perturbation to see whether the total number of e-folding times is too large. There is little danger if the following expression F remains smaller than unity.

$$F = \frac{\omega_1 k \; n^2 \; A^{\frac{1}{2}} \; (4-\pi) \; \Theta_n(0)}{32 \; \pi \left(\frac{\pi}{2} \gamma^2 \left|\cot \phi_s\right|\right)^{1/3} (\dot{\gamma}_s - \dot{\gamma}_t) \; \ln \sqrt{N_{\text{bunch}}}} \left(\frac{\dot{\gamma}_s}{\nu_{RF}}\right)^{5/6}$$

The symbols have the following meaning :

$\omega_1 = \frac{c}{R} = \frac{\omega}{\beta}$ = revolution frequency for $\beta = 1$

k = most dangerous mode number $\quad k \simeq \frac{\gamma R}{2b}\left(1.6 + 0.5\,\frac{b}{a}\right)$

b = pipe radius ; a = beam radius

A = bunch area in units $\Delta(\phi_{RF}) \times \Delta(\beta\gamma)$

ϕ_s = synchronous phase angle

$\dot{\gamma}_s$ = rate of change of synchronous γ

$\dot{\gamma}_t$ = rate of change of γ_t. In the following it is assumed $\dot{\gamma}_t = 0$ since it is only necessary to make $\dot{\gamma}_t < 0$ in case of high space charge.

ν_{RF} = frequency of the RF system for $\beta = 1$

$$n = 0.7723 \times \eta_o(0) = \frac{3\pi^2 r_p N g}{R\, A^{3/2}} \left(\frac{\nu_{RF}}{\dot{\gamma}_s}\right)^{\frac{1}{2}}$$

$$\Theta_n(0) = \begin{cases} \dfrac{3^{1/6}}{\pi^{\frac{1}{2}}}\,\Gamma\left(\dfrac{2}{3}\right) \simeq 0.92 & \text{for } n = 0 \\ \simeq n^{1/3} + \dfrac{0.21}{n} & \text{for large } n \quad (n > 2) \end{cases}$$

N_{bunch} = particle number per bunch.

Parameters independent of bunch area and beam size are :

$b \simeq 30$ mm; $\omega_1 = 2.73 \cdot 10^5/s$; $\gamma = \gamma_t = 24$; $\dot{\gamma}_s = 175/s$;

$N_{bunch} = 2.5 \cdot 10^9$; $\cot\phi_s \simeq 1$; $\nu_{RF} \simeq 2 \cdot 10^8/s$.

For the two most relevant cases we have :

$A = A_{ct} = 0.08$ (continuous transfer)

$A = A_{bt} = 0.18$ (bunch by bunch transfer).

We take $a_{ct} = 8$ mm ;	$a_{bt} = 11$ mm
$k_{ct} = 1.5 \cdot 10^6$;	$k_{bt} = 1.3 \cdot 10^6$
$g_{ct} = 3.2$;	$g_{bt} = 2.5$
$n_{ct} = 7 \cdot 10^{-2}$;	$n_{bt} = 1.5 \cdot 10^{-2}$
$F_{ct} = 3.5 \cdot 10^{-3}$;	$F_{bt} = 1.5 \cdot 10^{-4}$.

These small numbers for F show that there is no danger of negative mass blow-up at transition.

If, however, a mode is used as described by D. Möhl (MPS/DL/Note 72-9), using the SPS as injector for storage rings, we can expect the following: the beam is contained in 60 buckets only instead of 4620 (ideally). n_{bt} increases by a factor 78 which brings it up to, say $n_{bt} = 1.2$. $\Theta_n(0) \simeq 1.22$ $F_{bt} \simeq 1.2$.

A bunch blow-up of the order of 1.5 might occur which can then be avoided by a γ_t-jump. $|\dot{\gamma}_t|$ has not to be large, only of the order $\dot{\gamma}_s$. (The magnitude of the $\Delta\gamma_t$ has to be such that at the end of the jump we have

$$\Delta\gamma = \gamma_s - \gamma_t = \Delta\gamma_{th} = \frac{n\,\Theta_n(0)\,\gamma^2}{4\left(\frac{\pi}{2}\gamma^2 |\cot\phi_s|\right)^{1/3}} \left(\frac{\dot{\gamma}_s}{\nu_{RF}}\right)^{1/3} \simeq 0.22.)$$

REFERENCE

A. Sørenssen, A review of transition problems in the SPS, CERN/MPS/DL 72-4 (1972)

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