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IONIC OSCILLATIONS IN THE SPS

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The growth rate for both the dipole and the quadrupole mode can be described by a few parameters :

$$\mu_o = \sqrt{\frac{2\nu}{A_i}}\,\frac{c}{a}$$

μ_o = ion e-folding rate

ν = $r_p\bar{\lambda}$ = mean Budker parameter

A_i = atomic mass of ions

a = average beam radius

$$z = (Q - n)\frac{\omega_o}{\mu_o}$$

ω_o = angular revolution frequency

n = mode number

$Q = \begin{cases} Q_o & \text{for dipole oscillations} \\ 2Q_o & \text{for envelope oscillations} \end{cases}$

The growth rate is

$$\left(\frac{1}{t}\right)_g = \mu_o \frac{\eta A_i}{2\gamma Q_o \omega_o} \frac{z}{4+z^2}$$

γ = relative energy

η = ion production rate

η is of the order 10^3/s/μ Torr but slightly energy dependent as can be seen from table I.

GeV_{kin}	.03	.1	.3	1	3	10	30	100
$\frac{\eta}{10^3/s/\mu Torr}$	2	1.25	.9	.8	.83	.93	1.15	1.6

Numerical example for the SPS :

$P = 10^{-7}$ Torr; 10 GeV/c; $\eta = 93/s$; $N = 10^{13}$ ppp

$\nu = 2.2 \cdot 10^{-9}$; $A_i = 14$; $a_o = 15$ mm

$\mu_o = 0.35/\mu s$; $\omega_o = 0.27/\mu s$; $Q_o \simeq 28$

$\gamma = 10.7$; $z \simeq 2$ (n = 25 for dipole mode; n $\approx$ 53

for the envelope mode); $\frac{z}{4+z^2} \simeq \frac{1}{4}$ $\left(\frac{1}{t}\right)_g \simeq 0.7/s$.

Since the theory was made for a coasting beam with uniform transverse density, a smaller radius should be taken, so that the growth rate is a little larger than 1/s. It does not change much during the cycle but other modes become the most dangerous ones. For instance at 200 GeV/c we may have $a_o = 4$ mm; $\mu_o = 1.3/\mu s$ and the most dangerous mode numbers are n = 17 for the dipole mode and n = 46 for the envelope mode yielding a growth rate of $\left(\frac{1}{t}\right)_g \simeq 0.28/s$, whereas the previous mode numbers (n = 25 and n = 53) would yield a growth rate of about $\left(\frac{1}{t}\right)_g \simeq 0.15/s$.

This effect does not seem important even at a pressure of $3 \cdot 10^{-7}$ Torr.

References

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- H.G. Hereward, P.L. Morton, K.H. Schindl, The effect of ions on the symmetrical throbbing beam mode, IEEE Trans. Part. Acc. Conf. Washington, p. 180 (1969).

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