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RESONATOR METHOD FOR IMPEDANCE DETERMINATION

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Abstract

A method is presented for the measurement of small imaginary impedances. For a transverse electromagnetic mode (TEM) resonator, shifts of resonance frequency are used to determine the impedance. The measurement set-up and procedure are described. A theoretical derivation is given and comparison of measurement results with calculations of impedance with Bethe hole coupling is done.

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1 INTRODUCTION

Measurements of a low coupling impedance are useful for studying the properties of the LHC liner, a structure shielding the synchrotron radiation. The approach presented here is the resonator method. Shifts of the resonance frequencies in a transverse electromagnetic mode (TEM) resonator are used to determine a change of the electrical length. This change can be attributed to a purely inductive impedance caused by the liner perforations which are inductive up to several gigahertz. Thus from the measured change of electrical length one can calculate the imaginary impedance. The straightforward, simple, theoretical model explained below provides a practical tool for studying low-impedance devices.

2 EXPERIMENTAL SET-UP

In Fig. 1 the experimental set-up is shown. The TEM resonator consists of the liner tube, with or without the perforations, and an inner conductor. Measurement of the tube without holes is necessary to give a reference. For centring the inner conductor some dielectric supports are required (here PVC foam). The two electrodes are needed to excite the resonator and measure the S_{21} parameter. Spring contacts are necessary for both good mechanical sliding and electrical shielding against outside influence. Ferrites are positioned behind the electrodes to damp higher modes. The 50 Ω cables are equipped with subminiature adaptor (SMA) plugs. The inner conductor (Fig. 1) has a length of 1.5 m and a diameter of 1 cm and is placed inside a tube of 3.2 cm diameter. This leads to a resonance frequency of about 100 MHz for the fundamental TEM mode and for the harmonics of 100 MHz. In fact the resonance frequencies are a little lower than multiples of 100 MHz as the capacities at both ends of the inner conductor increase the electrical length of the resonator.

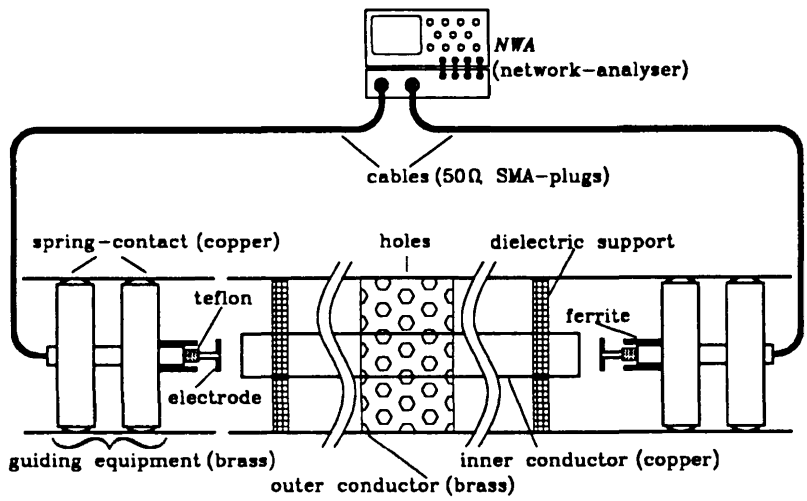

Figure 1: Experimental set-up for the resonator impedance measurement method

3 CALCULATIONS

The analytical treatment of the longitudinal liner impedance discussed here is based on the Bethe hole coupling theory[1]. For the longitudinal beam coupling $Z(\omega)$ impedance of a single hole in an infinitely thin wall, Kurennoy[2] gives the following expression:

$$Z(\omega) = \mathrm{j}Z_0 \frac{\omega}{c_0} \frac{(\alpha_m + \alpha_e)}{4\pi^2 b^2} \,. \tag{1}$$

Here $Z_0 = 377\ \Omega$ is the impedance of free space, b is the tube radius, α_m and α_e are the magnetic and electric polarizability, c_0 is the velocity of light.

The polarizabilities are given by Kurennoy as:

$$\alpha_m = \frac{4}{3}a^3 \quad , \qquad \alpha_e = -\frac{2}{3}a^3 \,, \tag{2}$$

with a the radius of the hole.

Two corrections have to be applied to these polarizabilities. The first is due to the finite wall thickness. Here Gluckstern's[3] numbers are used. Then for a single hole in the liner tube the following formula is obtained:

$$Z(\omega) = \mathrm{j}Z_0 \frac{\omega}{c_0} \frac{(\tau_m \alpha_m + \tau_e \alpha_e)}{4\pi^2 b^2} \tag{3}$$

with τ_m as the magnetic and τ_e as the electric correction coefficients[3] for finite wall thickness.

Using the liner tube as a resonator a further correction has to be carried out. There is a sine- or cosine-shaped field distribution in the area of the holes due to the standing waves. This has to be taken into account with a standing wave form factor f_e and f_m for the electric and magnetic field, respectively. It is done by integrating the squares of the fields over the length of the tube area. Squares are used because the polarizabilities are connected with stored energy. For the electric field one obtains:

$$\begin{aligned} E_\varrho &\sim -\cos(\beta_z z) \\ &\Rightarrow \\ f_e &= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \cos^2(\beta_z z) \mathrm{d}z \\ &= \frac{1}{l_2 - l_1} \Big[\frac{1}{2}z + \frac{1}{4\beta_z} \sin(2\beta_z z)\Big]_{l_1}^{l_2} \,. \end{aligned} \tag{4}$$

In the same way for the magnetic field:

$$\begin{aligned} H_\varphi &\sim \sin(\beta_z z) \\ &\Rightarrow \\ f_m &= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \sin^2(\beta_z z) \mathrm{d}z \\ &= \frac{1}{l_2 - l_1} \Big[\frac{1}{2}z - \frac{1}{4\beta_z} \sin(2\beta_z z)\Big]_{l_1}^{l_2} \,. \end{aligned} \tag{5}$$

Here β_z is the propagation constant of the coaxial line, l_1 the start position and l_2 the end position of the area covered by perforations. Using the form factors one finally obtains for the impedance in the TEM line resonator:

$$Z(\omega) = \mathrm{j} Z_0 \frac{\omega}{c_0} \frac{(f_m \tau_m \alpha_m + f_e \tau_e \alpha_e)}{4\pi^2 b^2} \,. \qquad (6)$$

Some of the form factors f_e and f_m for the measurement set-up used are shown in Table 1.

Figure 2 shows the relative frequency shift versus the resonance frequency as a result of calculation. The impedance formula applied here is valid only for a single hole. As an approximation neglecting interactions between the holes, it is used for a number of perforations by straightforward multiplication.

In comparison with the standard coaxial wire method[4], where essentially the change of S_{21} for a single pass of a travelling wave is considered, the configuration described above takes advantage of multiple passes in a standing wave structure.

TABLE 1: Standing wave form factors for some resonance frequencies

ν[MHz]	f_e	f_m	ν[MHz]	f_e	f_m
100	0.0145	0.9855	900	0.5782	0.4218
200	0.9435	0.0565	1000	0.3965	0.6035
300	0.1217	0.8783	1100	0.6079	0.3921
400	0.7965	0.2035	1500	0.4997	0.5003
500	0.2936	0.7064	2000	0.5514	0.4486
600	0.6166	0.3834	2500	0.5414	0.4586
700	0.4649	0.5351	3000	0.5003	0.4997
800	0.4687	0.5313			

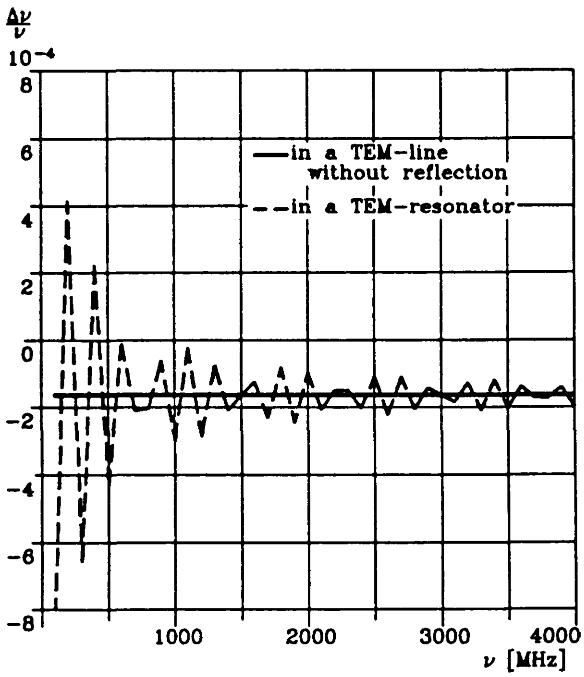

Figure 2: Calculated relative frequency shift versus resonance frequency for 200 holes (each 4 mm diameter)

4 RESULTS

Table 2 shows the frequency shift of the resonance frequency due to 200 holes with 4 mm diameter.

In Fig. 3(a) results from calculation for the TEM structure without reflections from the ends (solid line, coaxial wire method) and for the TEM resonator (dotted line) are presented. These discrete numbers (Table 2 and Fig. 3) are obtained by measurement.

The same is done for 200 holes of 3 mm diameter [Fig. 3(b)]. Note that the correspondence between measurement and theory here is better than for the 4 mm holes.

TABLE 2: Frequency shifts for several resonance frequencies

ν[MHz]	$\Delta\nu$[kHz] measured	$\Delta\nu$[kHz] calculated	ν[MHz]	$\Delta\nu$[kHz] measured	$\Delta\nu$[kHz] calculated
98.8	-87.8 ± 6	-79.63	890.6	-77.8 ± 8	-55.38
197.6	$+69.8 \pm 7$	$+83.05$	988.5	-284.1 ± 10	-298.41
296.8	-186.4 ± 5	-196.99	1088.6	-67.7 ± 7	-25.09
395.2	$+92.2 \pm 8$	$+89.37$	1484.6	-272.1 ± 10	-245.93
494.7	-215.2 ± 5	-216.33	1978.6	-206.4 ± 10	-192.84
593.0	-1.4 ± 3	-6.86	2475.3	-278.8 ± 10	-273.65
692.6	-137.0 ± 8	-146.49	2970.4	-492.3 ± 10	-489.13
790.7	-165.0 ± 5	-163.50			

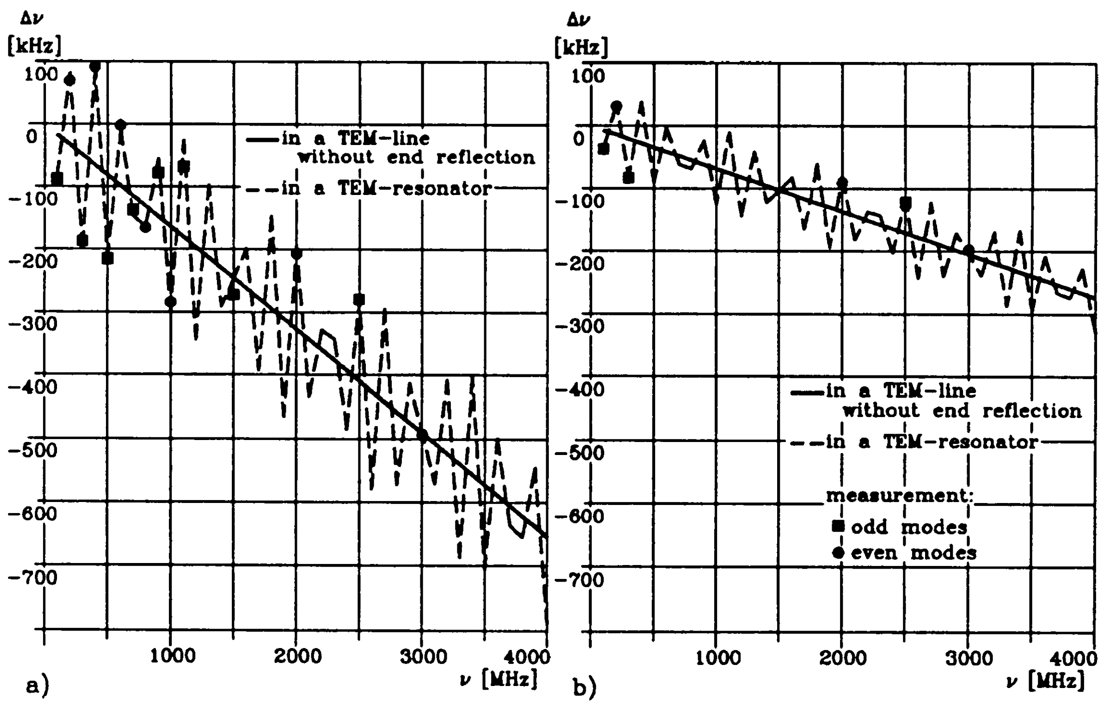

Figure 3: Frequency shift versus resonance frequency for 200 holes with (a) 4 mm diameter and (b) 3 mm diameter, calculation (dotted line) and measurement (points)

Possible reasons are:

- Bethe hole coupling is strictly valid only for a planar surface. If the curvature of the wall becomes comparable with hole diameter, deviations occur.
- Mutual coupling of holes are not taken into account in this calculation. These interactions grow up with larger diameter.

5 QUALITY FACTOR MEASUREMENTS

The quality factor of the resonance frequencies is measured with and without the holes. The differences of these two values

$$\Delta Q = Q_{\text{reference}} - Q_{\text{perforated tube}} \tag{7}$$

for some frequencies are displayed in Fig. 4 with a 200-hole pattern each with 4 mm diameter. This difference of the quality factor is related to the real part of the coupling impedance.

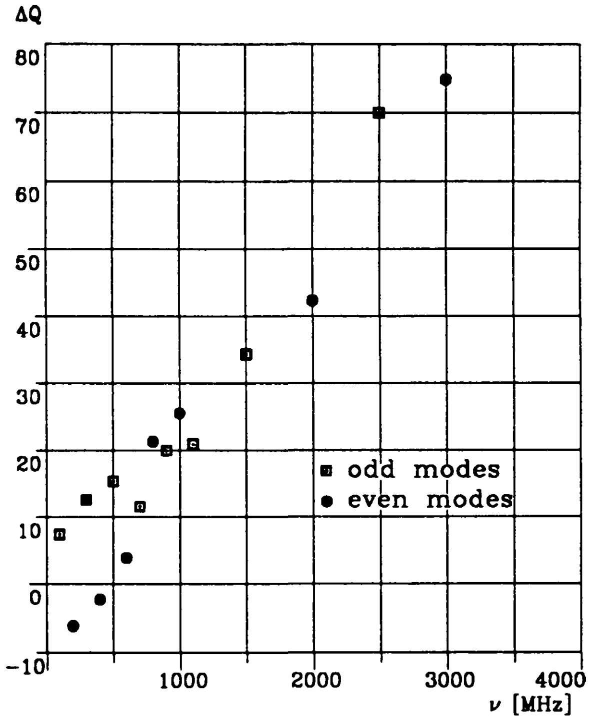

Figure 4: Quality factor for 200 4 mm holes versus frequency

The Q-values are about 800 for 100 MHz and up to about 3800 for 3000 MHz. Thus the differences amount to about 2% of these values and this is near to the limit of measurement accuracy. This may explain the negative Q-differences at 200 and 400 MHz which would otherwise be difficult to explain. But the tendency of the measurement is as expected. The higher the frequency the greater the real part of the impedance due to radiation through the holes. Another effect can also be visualized. At positions of perforations where the locally stored energy in the magnetic field is greater than in the electric field, the losses are larger. This can be seen for the lower frequencies in Fig. 4.

6 COMPARISON WITH S_{21} TRANSMISSION MEASUREMENT

Measuring the imaginary part of an impedance by means of the coaxial wire[4] method is done by looking at phase differences between the device under test (DUT) (for example the liner tube) and a reference tube. The tubes are TEM lines with the ends matched to the cables (50 Ω) as well as possible. The inner conductor simulates the beam ($v = c_0$). When measuring the reference tube, the electrical delay setting of the analyser is chosen such that the phase becomes zero. The DUT is measured while tuning the phase to zero again. The difference of delay given to the analyser is the phase difference due to the imaginary impedance. The electric delay for 200 holes with 4 mm diameter is about 3.3 psec ($\triangleq$ 1 mm), this implies a phase shift of about 4° at 3 GHz. But the uncertainty of the S_{21} measurement is of the same order. This makes the measurement of such small impedances rather delicate even with advanced hardware and sophisticated calibration procedures.

7 CONCLUSIONS

To get good results the following points are important:

- The coupling factor has to be reproducible and equal for all measurements. For that each electrode is adjusted in the beginning to $\Delta S_{11} = \Delta S_{22} = 0.1$ dB. Fine adjustment is done by transmission measurement adjusting the peak of each resonance to $S_{21} = -40.5$ dB.
- One has to be very careful when placing the dielectric supports. They change the resonant frequency due to their interaction with the electric field. The best way to avoid a noticeable influence is to put them at minima of the electric field. Some fixing to prevent sliding on the inner conductor is necessary for a good repeatability.
- There has to be a good electrical contact (achieved by the springs) between the outer conductor and the guiding equipment.

The resonator method has the following advantages:

- There is high sensitivity due to measuring just changes of resonance frequencies instead of amplitudes.
- Results do not depend on the length of the outer conductor (liner tube). Only the length of the inner conductor is significant. This means that the liner tube with perforations, and the reference tube necessary for comparison, do not need to have known lengths with an accuracy $\sim$ 0.1 mm. The same inner conductor has to be centred in both tubes.
- The reproducibility of the method is of about 10–20 ppm or better with regard to the absolute frequency.
- Reproducibility should be at least an order of magnitude better as compared with the coaxial wire method.

There are still some open questions:

- How exact is the correction for wall thickness in the calculations? What are the limits of validity?
- How relevant are interactions between the holes?

It maybe worth while to mention that the imaginary part of the beam coupling impedance of holes increases approximately linearly with the frequency up to beam-pipe cut-off. Thus measurements at several frequencies allow for a further increase in accuracy by interpolation.

References

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