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CONCENTRIC STORAGE RINGS FOR THE CERN-PS.

by

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1. Introduction.

In a previous report (de Raad 1962 a) we have made a preliminary study of a pair of concentric intersecting storage rings (ISR) for the CPS. In the present report we shall make a more systematic study of the ISR. In general the present results confirm the figures given in our first report. The main difference is that we shall now base ourselves on an ISR design with strong focusing magnets. This is a more economical solution and for reasons set out below we believe that its flexibility for experiments is comparable to that of ISR with split focusing and bending. Since it is preferable to make this report self-contained, we shall repeat some of the arguments given in our first report. Fig. 1 shows the layout of the CPS with the ISR. Their design (total) energy is 28 GeV, the mean radius 150 m.

2. General Considerations about Concentric Storage Rings.

In order to exploit fully the advantages of the concentric arrangement there should be at least 6 interaction regions. Since the circumference increases with increasing number of interaction regions N we shall only consider N = 6 and N = 8. The number of superperiods is then 3 or 4. The Q-values that should be avoided in a machine with superperiods and equal Q-values for the horizontal and vertical motion are given by the relation

$$n\,Q \;=\; p\,S \qquad (1)$$

where n and p are integer numbers.

We shall restrict ourselves to Q-values between 6 and 9. Taking n = 2 we find that the stopbands due to the superperiods occur at Q = 6 ; 7.5 and 9 for N = 6 and at Q = 6; 8 and 10 for N = 8. To minimize the wiggles on the closed orbit and betatron oscillations the Q-value should be chosen about halfway in between these stopbands but, of course, always avoiding the integral and half-integral Q-values. Additional Q-values, forbidden by n = 3 and n = 4 (quadratic and cubic resonances) are Q = 6.75 and 8.25 for N = 6 and Q = 6.67 for N = 8. The quadratic resonances for N = 6 leave only the possibilities Q = 6.25 ; 7.25 ; 7.75 and 8.75 , all of which are rather close to the stopbands of the superperiod. For N = 8 we can choose between

between $Q = 7.25$ and $Q = 8.75$ that are both nearly halfway in between the stopbands. Therefore we shall restrict ourselves to ISR with 8 intersections and for reasons discussed later in this report we consider $Q = 8.75$ as the most suitable value.

The magnet structure of the ISR is different from that of the CPS because the ISR must have much longer field free sections for experimental purposes and have a variable mean radius. The latter is a consequence of their concentric arrangement. Another difference with the CPS is that in an accelerator the energy spread of the protons is small and the aperture requirements are mainly determined by the injected beam size and closed orbit deviations due to magnet imperfections, whereas in a storage ring the beam is injected with a high energy and small diameter. Most of the horizontal aperture, which is a few times larger than the vertical aperture, is required for the energy spread in the stacked beam and for the injector itself.

Changes in the magnet periodicity along the machine circumference (e.g. long straight sections) lead to an azimuthal variation of the betatron oscillation amplitude and the momentum compaction function, in addition to the regular strong focusing wiggle, which is always present. We define the momentum compaction function by

$$\Delta r\ (s) = \alpha_p\ (s)\ \frac{\Delta p}{p} \qquad (2)$$

where s is the distance along the orbit and $\alpha_p(s)$ is in m *). In general one finds that by a suitable choice of parameters the azimuthal variation of either the betatron oscillations or the momentum compaction function can be suppressed but it is never possible to eliminate both. Due to the energy spread in the stacked beam we shall be specially interested in matching the momentum compaction function.

In the middle of an F and a D sector the closed orbits for all momenta are parallel to the central orbit. We can therefore at these positions interrupt the regular machine lattice and insert straight sections of arbitrary length without changing the momentum compaction function. This makes it possible to obtain the long straight sections for colliding beam experiments and a

* All equations are written in MKS units.

variable mean radius by slimply choosing different spacings in between the magnet units.

Let us now consider the influence of this procedure on the betatron oscillations. The focusing properties of an alternating gradient structure are completely determined by specifying how the amplitude function β , defined by Courant and Snyder (1958) varies around the circumference. The betatron oscillations are given by the equation

$$x = a\,\beta^{1/2} \sin\left[\int \frac{ds}{\beta} + \delta \right] \qquad (3)$$

where s is the distance along the orbit and δ a phase angle. The beam width is therefore everywhere proportional to $\beta^{1/2}$. The maximum and minimum values of β occur in the middle of a focusing and defocusing sector respectively. The Brookhaven AGS, which has $Q = 8.75$ would have $\beta_F = 27$ m and $\beta_D = 11.6$ m if it were scaled up to a mean radius of 150 m.

If at N places, regularly spaced along the circumference, straight sections of length L are inserted in a regular structure, the change in Q-value is approximately

$$\Delta Q = \frac{N\,L\,(1 + \alpha^2)}{4\,\pi\,\beta} \qquad (4)$$

where β is the value of the amplitude function at the place of the insertion and where

$$\alpha = -\tfrac{1}{2}\,\frac{d\beta}{ds} \qquad (5)$$

at that position. The amplitude function β will now fluctuate along the circumference and its maximum increase is approximately

$$\left(\frac{\Delta\beta}{\beta}\right)_{max} = \frac{L\,(1 + \alpha^2)}{2\beta \left| \sin \frac{2\pi Q}{N} \right|} \qquad (6)$$

In a typical machine one finds $\alpha \approx 2$ halfway in between the F and D sectors and, of course, $\alpha = 0$ in the middle of an F or D sector. Therefore the position of the long straight sections chosen on the basis of matching the momentum compaction function also gives the smallest blowup of the betatron oscillations. The increase in β depends only on the length, but not on the number of the long straight sections, provided that Q/N is suitably chosen. To get some idea of what we can expect for the ISR we consider again the scaled up Brookhaven AGS and take $L = 11$ m, $\sin 2\pi Q/N \approx 1$. We assume that the long straight sections are inserted in the middle of a number of F sectors. Using eq. 6 we then find $\beta_{max}(H) = 32$ m and $\beta_{max}(V) = 40$.

3. Basic Parameters.

The layout of 1/8 of the ISR circumference is shown in Fig. 2. All magnet units are identical and consist of a half F and a half D sector. Only the yokes are periodically reversed to improve the accessibility for experiments injection and ejection. We have chosen this so-called FOFDOD structure for the following three reasons

a) The equality of all magnet units is desirable from an engineering point of view.

b) Correcting elements can conveniently be inserted in the mid F and mid D straight sections.

c) It is rather insensitive to parallel displacements of the magnet units, which might e.g. be caused by movements of the foundations.

The FOFDOD structure has the disadvantage of requiring rather high magnetic field gradients. We shall come back to this point later.

Each storage ring has 4 outer arcs, where the magnet units are placed as close as possible, leaving only some space for correcting elements and 4 inner arcs, with about 13 m long mid F straight sections and about 5 m long mid D straight sections. The number of colliding beam events is inversely proportional to the beam height and therefore the interaction regions have been chosen to be in mid F straight sections. In half the interaction regions the beams go towards the outside of the ISR, and in the other half they go towards the inside. The

The latter interaction regions are most suitable for small angle experiments on account of the longer field free sections.

Concerning the general layout the main parameters to choose are the angle of intersection α and the number of magnet periods M. The former should be reasonably large to avoid mutual interference of the magnets of the two ISR, but this is limited by the fact that the ISR mean radius R increases with increasing α. This increases the cost and reduces the interaction rate, which is given by

$$N_{IR} = \frac{c\,\sigma}{h\ \mathrm{tg}\ \alpha/2} \left(\frac{N_s}{2\pi R}\right)^2 \qquad (7)$$

because of the factors $\mathrm{tg}\ \alpha/2$ and R^2 in the denominator. M should be divisible by 4, since there are 4 superperiods and preferably some other numbers to allow a reasonable subdivision of the number of magnet units between inner and outer arc and a convenient distribution of correcting elements. To reduce the gradient of the magnetic field and to make the straight sections long, M should be as small as possible. The most convenient choice appears to be $\alpha = 15^{\circ}$ and $M = 48$. Each outer arc then has 16 magnet units and each inner arc has 8 magnet units. The distance between the middle of the inner and outer arc is 9.1 m. The circumference of the central orbit is $2\pi \times 150$ m, but the 8 interaction regions lie on a circle with radius 148.73 m.

The maximum value of the magnetic field is subject to serious limitations. Whereas in an accelerator like the CPS a few cm of good field at top energy are sufficient, in the ISR the field shape must remain correct over the whole aperture, so that a much better correction is required. The fact that the magnet operates at d.c. is of no help. On the contrary, heating of the poleface windings limits the possibilities of correction. We have therefore chosen a maximum magnetic field on the central orbit of 1.2 Wb/m^2.

The air gap between the F and D half sectors is 0.20 m *), This value was chosen to allow a convenient assembly of the ultra high vacuum chamber and

*) All lengths quoted in this report refer to the actual magnet steel and straight section lengths. To calculate the proton orbits we have included the end effects which are known from measurements on the CPS magnet.

to have room for clearing electrodes to suppress neutralisation of the circulating beam by the residual gas.

Since the mid F straight sections are much longer than the mid D straight sections, the focusing strength of the F sectors must be slightly larger than that of the D sectors. We follow the same method as was used for the CPS, namely to choose equal n-values but different lengths.

Regarding the lengths and locations of the long straight sections there is a large variety of possible choices, but one feels intuitively that the structure should be as periodic and symmetrical as possible. The straight section lengths of the magnet layout shown in Fig. 2 are given in the following Table.

Table I.

$$a_0 = 1.663 \text{ m}$$
$$a_1 = 3.353 \text{ m}$$
$$a_2 = a_6 = 13.243 \text{ m}$$
$$a_3 = a_5 = 5.043 \text{ m}$$
$$a_4 = 12.493 \text{ m}$$
$$b_1 = 5.183 \text{ m}$$
$$b_2 = 8.060 \text{ m}$$

The variation of the amplitude function β along one half of a superperiod is shown in Fig. 3. These curves were calculated with the help of a computer programme written by S. van der Meer. The maximum values of β are 34.5 m and 38.5 m in the horizontal and vertical planes, respectively. These values are reasonably close to those derived from eq. 6, which means that the distribution of the straight sections is satisfactory. The following Table summarizes the main parameters of the proposed ISR design.

Table II.

Maximum energy (total)	E_{max}	28 GeV
Peak field on the central orbit	$B_{c\ max}$	1.2 Wb/m^2
Magnetic radius	ρ	79.2 m
Average radius	R	150 m
Profile parameter	n/ρ	4.133 m^{-1}
Field index	n	327.4
Number of periods	M	48
Number of magnet units		96
Number of superperiods		4
Number of intersections		8
Q-value		8.75
Average β horizontally	$\beta_{av}(H)$	17.9 m
Maximum β "	$\beta_{max}(H)$	34.5 m
Average β vertically	$\beta_{av}(V)$	22.2 m
Maximum β "	$\beta_{max}(V)$	38.5 m
Mid F momentum compaction function	$\alpha_p(F)$	2.30 m
Mid D " " "	$\alpha_p(D)$	1.62 m
Total energy spread in a 6 cm wide stack		2.5 o/o
Length of a half F sector	L_F	2.465 m
Length of a half D sector	L_D	2.435 m
Gap in between F and D sectors	d	0.20 m
Horizontal aperture		150 mm
Vertical aperture		50 mm
Gap height at centre line		100 mm

4. Discussion of the Choice of Parameters.

In this section we shall provide some additional justification for the choice of our parameters by discussing various alternatives.

a) Variation of the straight sections.

Inspection of Fig. 3 shows, that the three maximum values of $\beta(V)$ are approximately equal. In fact, the slightly different lengths of the long mid F straight sections were chosen on the basis of this criterium. A reduction of the height of one peak will in general increase the height of another peak so that vertically there is little room for improvement. In the horizontal plane, however, the β-curve has one dominant peak and it is worthwhile to see how this could be improved. One obvious trial is to make all long mid F straight sections equal, namely

Table III

$$a_o = 1.663 \text{ m}$$
$$a_1 = 3.353 \text{ m}$$
$$a_2 = a_4 = a_6 = 12.943 \text{ m}$$
$$a_3 = a_5 = 5.043 \text{ m}$$

In that case we find $\beta_{max}(H) = 34.7$ m and $\beta_{max}(V) = 39.9$ m , which is somewhat worse than the values corresponding to the straight sections of Table I.

A very interesting possibility is the following

Table IV.

$$a_o = 1.403 \text{ m}$$
$$a_1 = a_3 = a_5 = 5.043 \text{ m}$$
$$a_2 = a_4 = a_6 = 12.943 \text{ m}$$

The β-curves for this combination of straight sections are shown in Fig. 4. Horizontally there are 5 and vertically 4 peaks of about the same height with $\beta_{max}(H) = 29.6$ m and $\beta_{max}(V) = 38.1$ m. The difficulty is, that now the free space between the coils of adjacent magnet units in the outer arc has been reduced to about 0.65 m, since we must subtract from a_o about 0.75 m for the coil ends. This looks too small and we shall therefore in this report stick to the value $a_o = 1.663$ m.

b) Variation of the relative lengths of mid F and mid D straight sections.

In each superperiod the total length of long mid F and long mid D straight section is $2\ a_2 + 2\ a_4 + a_6$ and $2\ a_1 + 2\ a_3 + 2\ a_5$ respectively. The total length of straight section is determined by the general layout we have chosen, but one may ask how the ratio between the total mid F and the total mid D straight section length influences the β-curves. For simplicity we have taken $a_2 = a_4 = a_6$ and $2\ a_1 = a_3 = a_5$. This means e.g. also that $a_2 + a_3 =$ const. For these conditions we have calculated $\beta_{max}(V)$ and $\beta_{max}(H)$ for different ratios a_2/a_3. The result is shown in Fig. 5 and turns out to be rather insensitive to the ratio a_2/a_3, so that in this respect we have a large freedom of choice.

c) Variation of the Q-value.

Since the superperiodicity of our magnet structure leads to a stopband at Q = 8, we must investigate its influence on the betatron oscillations. We have therefore, with the straight sections of Table I, changed the n-value of the magnet and calculated the resultant Q-value and maximum values of β. The results are shown in Fig. 6 and we see that the variation of β in the working diamond from Q = 8.5 to Q = 9.0 is very small. Since the mid F straight sections are the longest the stopband should be more pronounced in the vertical plane, but nevertheless the difference between horizontal and vertical plane is larger than one might have expected.

d) Q = 7.25.

Since we have mentioned in sec. 2 the possibility of choosing Q = 7.25 we give the corresponding β-curves in Fig. 7. The maximum values are $\beta_{max}(H) = 38.3$ m and $\beta_{max}(V) = 56.8$ m. The shape of the β-curves is rather different from those for Q = 8.75 and especially in the vertical plane there is a pronounced peak. The straight section values of Table III only make the situation worse and give $\beta_{max}(H) = 43.6$ m and $\beta_{max}(V) = 61.1$ m. The mid F value of the momentum compaction function $\alpha_p(F) = 3.20$ m. The only advantage of Q = 7.25 is that it has a lower gradient, namely $n/\rho = 3.45\ m^{-1}$. Calculating the aperture requirements on the same basis in the two cases (see sec. 11) we find 182 x 56 mm^2 for Q = 7.25 and 150 x 50 mm^2 for Q = 8.75. The variation of the magnetic field across the aperture is about the same in the two cases and in addition $Q = 7.25$ needs a somewhat larger vertical aperture. For these reasons we feel that $Q = 8.75$ is a better choice.

e) FODO structure.

In a FODO structure the outer arcs would be made of 5.1 m long F units and 5.1 m long D units. To form the inner arcs the F and D units are split up in their middle and similar straight sections as given in Table I are inserted at these places. The inner arc then consists of 2.5 m long separate magnet units The orbit properties are essentially the same as for the FOFDOD structure, namely

Table V.

$\beta_{max}(H)$	=	35.3	m
$\beta_{max}(V)$	=	40.4	m
$\alpha_p(F)$	=	2.36	m
$\alpha_p(D)$	=	1.57	m

The attractive feature of the FODO structure is its low gradient, namely $n/\rho = 3.0\ m^{-1}$, which facilitates the magnet design.

The disadvantage is a somewhat larger sensitivity to misalignments and the fact that in the outer arc there are no free mid F and mid D straight sections. The latter are necessary if one wants to make Q-changes of the same sign for both the horizontal and vertical betatron oscillations by means of correcting lenses. Resegotti has suggested to place short homogeneous field magnets in between the F and D units. If these are energized the ratio between total bending and focusing strength of the ISR is changed and this results in approximately equal horizontal and vertical Q-changes. If one places a bending magnet in between each F and D unit the closed orbit distortion for $\Delta Q = 0.25$ is only about $\pm$ 2 mm. The bending magnets take a considerable amount of space and do not solve the problem of non-linear (sextupole) corrections that might be required. We feel therefore, that a thorough study of the correction problems and magnet design is necessary before a final choice between a FOFDOD and FODO structure can be made.

f) Split focusing and bending.

This structure was discussed in a previous report (de Raad 1962 a). We have now discarded it because a more detailed study has shown, that the cost of the magnet with its power supplies and the power consumption would be respectively 1.4 and 1.7 times larger than for the machine with strong focusing magnets. The orbit properties of a split focusing and bending structure with the same period lengths as in the design of sec. 3 would be

Table VI.

$$\beta_{max}(H) = 39.7 \text{ m}$$

$$\beta_{max}(V) = 43.5 \text{ m}$$

$$\alpha_p(F) = 2.40 \text{ m}$$

$$\alpha_p(D) = 1.53 \text{ m}$$

5. Special Magnet Sections for Experiments.

For small angle scattering experiments it would be desirable to extend the ISR magnet units downstream of the interaction region in the radial direction so that their magnetic field can also be used for momentum analysis of the scattered particles (de Raad 1962 b). This procedure is somewhat difficult with strong focusing magnets. A split focusing and bending (sfb) structure has the attractive feature that the radial extension of the homogeneous field of the bending magnets can be made arbitrarily large. We shall therefore investigate what happens, if we simply replace in each storage ring the first half period downstream of some of the interaction regions by a bending magnet with two quadrupoles, as shown in Fig. 8. We first assume that Q_M and Q'_M are absent.

Comparison of Tables II and VI shows, that in the sfb half period $\alpha_p(F)$ is larger and $\alpha_p(D)$ is smaller than in the strong focusing structure. This mismatch produces a beating of α_p around the ISR and we shall first derive a formula for the reduction in beam intensity due to this effect. Let us assume, that the horizontal aperture available for the stacked beam is A and the distance from the injection orbit to the bottom of the stack is B (see Fig. 17). The latter is fixed and depends on the construction of the fast kicker magnet of the injection system. The maximum intensity of the stacked beam in normal conditions is, in arbitrary units.

$$I_1 = \frac{A}{\alpha_p(F)} \qquad (8)$$

We define $\alpha_p(\max)$ and $\alpha_p(\text{inj})$ as the maximum value of α_p at any place around the circumference and the value of α_p in the kicker magnet, respectively. The maximum stacked beam intensity in the presence of a beating of α_p is

$$I_2 = \frac{A - B\left[\dfrac{\alpha_p(\max) - \alpha_p(\text{inj})}{\alpha_p(\text{inj})}\right]}{\alpha_p(\max)} \qquad (9)$$

so that the reduction in intensity is

$$r = \frac{\alpha_p(F)}{\alpha_p(\max)} \left[1 - \frac{B}{A}\left\{\frac{\alpha_p(\max) - \alpha_p(\text{inj})}{\alpha_p(\text{inj})}\right\}\right] \qquad (10)$$

In our design $B/A = 0.6$. Eq. 10 shows, that the reduction in beam current depends both on the amplitude and the phase of the perturbation of α_p.

We have studied the following two cases

a) One sfb half period downstream of the 4 interaction regions where the beams go towards the inside.

b) One sfb half period downstream of all 8 interaction regions.

The following Table summarizes the results.

Table VII.

	$\beta_{max}(H)$	$\beta_{max}(V)$	$\alpha_p(\max)$	r
4 sfb sections	36.5 m	41.8 m	2.49 m	0.86
8 sfb sections	33.3 m	44.2 m	2.63 m	0.73

The increases in β_{max} are rather modest, but the values of r should be improved The fast kicker magnet was assumed to be in the position marked "inflector" in

Fig. 13.

Let us now consider, how the momentum compaction function could be matched. The fact that $\alpha_p(F)/\alpha_p(D)$ is larger for the sfb half period than for the strong focusing structure cannot be changed. If the length of the sfb half period is multiplied by 1.62/1.53 its $\alpha_p(D)$ becomes equal to $\alpha_p(D)$ of the strong focusing period. This has the additional advantage that the bending magnet B1 can be made 10 o/o longer so that its magnetic field becomes 1.45 Wb/m^2 instead of 1.6 Wb/m^2. The latter value is required for a sfb structure with the same period length as the strong focusing structure. Of course the mismatch of $\alpha_p(F)$ has now been increased, but if we place at the upstream end of the interaction straight section, as shown in Fig. 9, a horizontally defocusing matching quadrupole Q_M with a strength $C = -0.0093\ m^{-1}$ and increase the strength of Q1 by approximately the same amount, a perfect matching of α_p is obtained. The maximum value of α_p occurs in Q1 and is 2.53 m. Therefore the aperture of Q1 must be somewhat larger than the rest of the ISR aperture. The following Table summarizes the β-values for this matched arrangement.

Table VIII

	$\beta_{max}(H)$	$\beta_{max}(V)$
4 matched sfb sections	37.2 m	41.9 m
8 matched sfb sections	33.6 m	44.6 m

Comparison with Table VII shows that the values of β_{max} are practically not affected by this matching procedure. We conclude from this discussion that sfb sections can be inserted in a strong focusing structure with little loss in aperture.

6. Superposition of Closed Orbits for Different Momenta.

For some experiments it is necessary that the beams have a small width in the interaction region. This can be achieved by local superposition of the closed orbits for different momenta. We shall give an approximate theory of such superposition schemes by neglecting the strong focusing wiggles and assuming that the amplitude function has everywhere the value

$$\beta = R/Q \tag{11}$$

One possibility, suggested by Hereward (1960) and shown in Fig. 10a is to place two horizontal F quadrupoles with strength

$$C = 1/\beta \tag{12}$$

half a wavelength (= $\pi\,\beta$) apart. The closed orbits for all momenta are then tangent to the central orbit ($\alpha_p = 0$) halfway in between the quadrupoles. To keep the Q-value constant the ISR must be retuned with other correcting quadrupoles. The two quadrupoles with $C = 1/\beta$ are a serious gradient perturbation and this leads to large wiggles on the amplitude function. It can readily be shown, that

$$\beta_{max} = (1 + \sqrt{2})\ \beta \tag{13}$$

so that the maximum amplitude of the betatron oscillations is increased by a factor 1.55.

If we want to have $\alpha_p = 0$ in several interaction regions, we arrive at a situation where there are a number of F quadrupoles near the interaction regions and a number of D quadrupoles halfway in between. It is then rather natural to distribute the quadrupoles in a regular way around the machine, as shown in Fig. 10 b. This is equivalent to the proposal made by Terwilliger (1959) who showed that one can make $\alpha_p = 0$ in a number of places by applying a harmonic gradient perturbation with a periodicity close to Q. In the present design each ISR has 8 F and 8 D quadrupoles. The phase shift between adjacent F and D quadrupoles (see Fig. 10 b) is

$$\mu = \frac{\pi Q}{8} \tag{14}$$

If all quadrupoles have the same strength

$$C = \frac{2 \cot g\ 1/2\ \mu}{\beta} \tag{15}$$

we obtain $\alpha_p = 0$ in all F quadrupoles, while the Q-value remains unchanged in a first approximation. For our choice of parameters $\cot g\ 1/2\ \mu = 0.15$. The maximum increase in β is approximately

$$\left(\frac{\Delta\beta}{\beta}\right)_{max} = \cot g^2\ 1/2\ \mu + \left| \frac{\cot g\ 1/2\ \mu}{\cos \mu} \right| \qquad (16)$$

Substitution gives $(\Delta \beta/\beta)_{max} = 0.16$ which is acceptable. The disadvantage of this scheme is that the momentum compaction function has been doubled in the D quadrupoles.

The Terwilliger scheme can be used in two ways. One possibility is, to inject in normal conditions and to excite the Terwilliger quadrupoles only after the stacking process is finished. By changing at the same time the radial position of the stack (e.g. by increasing the magnetic field) the horizontal aperture that was first required by the injector can now be used to accommodate part of the increased stack width. In that case the reduction in intensity is

$$r = 1/2\ \frac{A + B}{A} \approx 0.8 \qquad (17)$$

where A and B have been defined in the previous section. For certain experiments it is necessary to have the smallest possible vacuum chamber in the interaction region and in that case the Terwilliger quadrupoles must be excited before injection starts. The beam intensity is then reduced a factor 0.5 and obviously the fast kicker magnet must be placed in the vicinity of a D quadrupole.

Fig. 11 shows the variation of α_p along the ISR circumference for 2 different strengths of the Terwilliger quadrupoles. The T_D quadrupoles are placed in the centre of the inner and outer arcs, and the T_F quadrupoles are near the interaction regions. For $C = 0.0123\ m^{-1}$ we get $\alpha_p = 0$, but this occurs not in the interaction region. The reason is that the outer arc contains more magnets than the inner arc and therefore its betatron phase shift per unit length is larger. One possibility to obtain $\alpha_p = 0$ in all interaction regions is to increase the quadrupole strength to $C = 0.016\ m^{-1}$. This causes an even larger reduction of beam intensity while the protons at the edge of the stack pass through the interaction region with angles up to ± 1 mrad with respect to the central orbit.

This leads to a loss in angular definition which may be unacceptable in some experiments.

A better solution is to displace the Terwilliger quadrupoles azimuthally. As shown in Fig. 12 it is possible to obtain perfect superposition in 4 interaction regions. In the other 4 interaction regions there is only a moderate reduction in stack width and an increased angular spread, but in these places one can do experiments which do not need a small beam width. To obtain the optimum phase of the α_p curve it was necessary to divide the D quadrupoles into two halves, each with a strength $C = 0.0066\ m^{-1}$, located in different straight sections. We propose to choose the azimuthal location of the Terwilliger quadrupoles in such a way that the good superposition of closed orbits is obtained in those interaction regions where the beams go to the inside. As we shall see in sec. 9 this is also the most convenient for injection. With the kicker magnet for injection in the position marked "inflector" in Fig. 13 the intensity reduction is $r = 0.46$.

The maximum values of β for the three curves shown in Figs. 11 and 12 are as follows

Table IX.

	$\beta_{max}(H)$	$\beta_{max}(V)$
Symmetrical quadrupoles, $C = 0.0123\ m^{-1}$	38.7 m	39.2 m
" " , $C = 0.0160\ m^{-1}$	39.5 m	39.5 m
Displaced " , $C = 0.0132\ m^{-1}$	36.2 m	42.9 m

so that in all these cases the increase of the betatron oscillation amplitude is small. This result is in agreement with eq. 15. The average value of $\beta(H)$ at the position of the D quadrupoles is about 24 m. The values of C calculated with the help of eq. 15 are in excellent agreement with those obtained with the computer for the more irregular ISR structure.

7. Corrections and Correcting Devices.

a) Poleface Windings.

In an accelerator corrections to the magnetic field distribution are mainly necessary at high fields when the beam diameter is about one tenth of the aperture. In that case a reasonable correction in the vicinity of the central orbit can be obtained with a moderate number of quadrupoles and sextupoles. In a storage ring the full aperture is required, even at maximum field. In addition the reduced periodicity of the magnet does not lend itself very well for corrections with lenses. We consider it essential, therefore, to correct the deviations in the magnetic field distribution due to saturation, by means of poleface windings (pfw). This increases the magnet cost, but apart from a better correction it also saves a considerable amount of straight section length and therefore reduces the circumference. Since the ISR operate at dc one can adjust the currents in the individual conductors of the pfw separately. In this way it is possible to correct for the saturation of the steel over the whole aperture and at all field levels. The magnitude of the correction that can be made with pfw is severely limited by heating and therefore other corrections, that do not depend on saturation should be made in a different way.

The use of pfw can lead to complications if one wants to stack electrons. In this case the stacked beam has a continuous energy loss of several MW in the form of X-radiation. The pfw are nearest to the vacuum chamber and will be subject to serious radiation damage. Although the radiation is concentrated in the median plane of the ISR, some of it will be vertically scattered in the walls of the vacuum chamber. The magnitude of this effect and also the radiation damage to other ISR components must be calculated before one can decide about the feasibility of stacking electrons.

b) Sextupole Correction of the Pole Profile.

If the momentum of a proton deviates from that which corresponds to the central orbit, the effective focusing strength of the ISR is also different. The corresponding variations of the Q-value are

$$\Delta Q_H = -8.1 \frac{\Delta p}{p} \qquad (18)$$

$$\Delta Q_V = -10.7 \frac{\Delta p}{p}$$