PS/OP/Note (MD-LEAR) 89-3
PS/AR/Note (MD-LEAR) 89-10
Revised 24.11.1989

PS Division

J.Bosser,F.Lenardon,D.Manglunki

Diffusion measurements

November 22, 1989

Measurements of the diffusion constant on a coasting beam experiencing RF noise.

Abstract

Diffusion constants were measured in LEAR, by applying RF noise to a coasting beam and watching the time evolution of its momentum distribution. The calibration of the kicker voltage as experienced by the beam has been verified from these measurements.

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1. Introduction

Diffusion constants were measured in LEAR, by applying RF noise to a coasting beam and watching the time evolution of its momentum distribution. The calibration of the kicker voltage as experienced by the beam has been verified from these measurements.

2. Experimental set up

2.1 Noise generation

The noise is applied on the beam via the longitudinal kicker used for stochastic extraction (Figure 1 [1]).

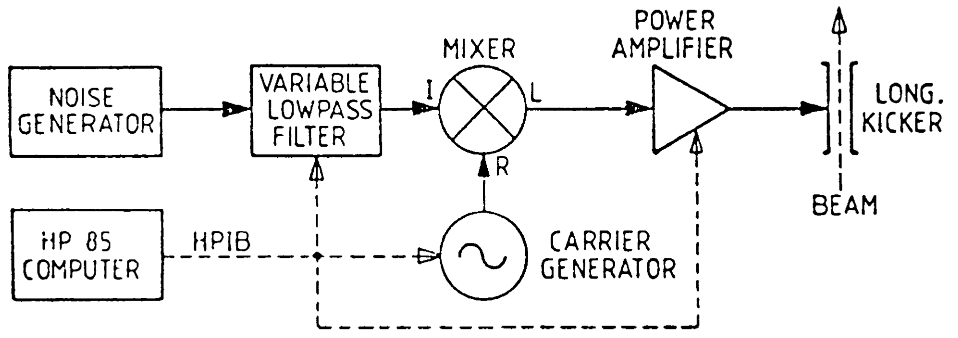


Figure 1: Noise generator, schematic (from [1])

2.2 Signal observation

The momentum distribution is measured by observing a stochastic cooling sum pick−up signal on a spectrum analyser (Figure 2 on page 2). The output of the spectrum analyser [2] is proportional to

$$\sqrt{\frac{dN}{df} + \frac{dN_{noise}}{df}}$$

Where $\frac{dN}{df}$ is the particle density and $\frac{dN_{noise}}{df}$ is the particle density equivalent of the noise level. The noise level is estimated by measuring the power on the edges of the display. Squaring the signal, then removing the noise contribution gives the true density $\frac{dN}{df}$. Assuming a gaussian shape for the distribution, one can deduce $\left(\frac{\Delta p}{p}\right)_{rms}$. For practical reasons, the program does not compute σ, but $6\sqrt{2}\sigma$.

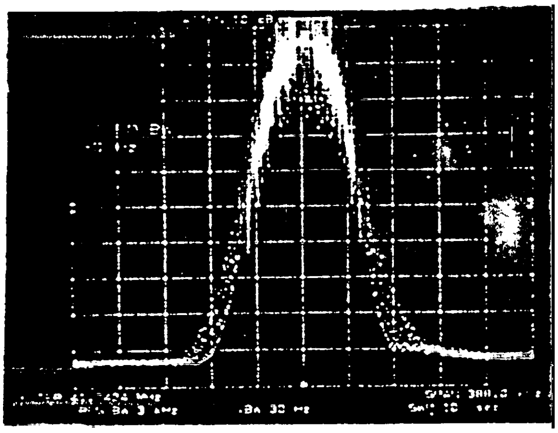


Figure 2: Spectrum analyser display. The different traces show the 309 MeV/c beam distribution spreading as it diffuses in the 0.1 μW/Hz noise. Pictures were taken every 10 seconds.

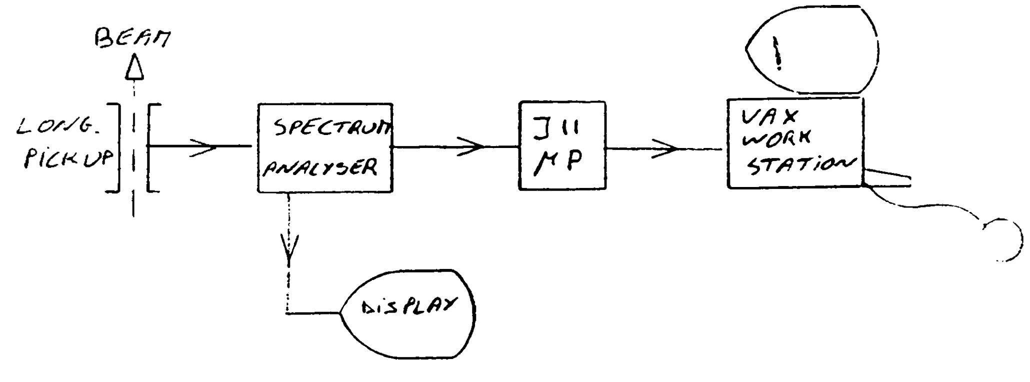


Figure 3: Signal acquisition, schematic

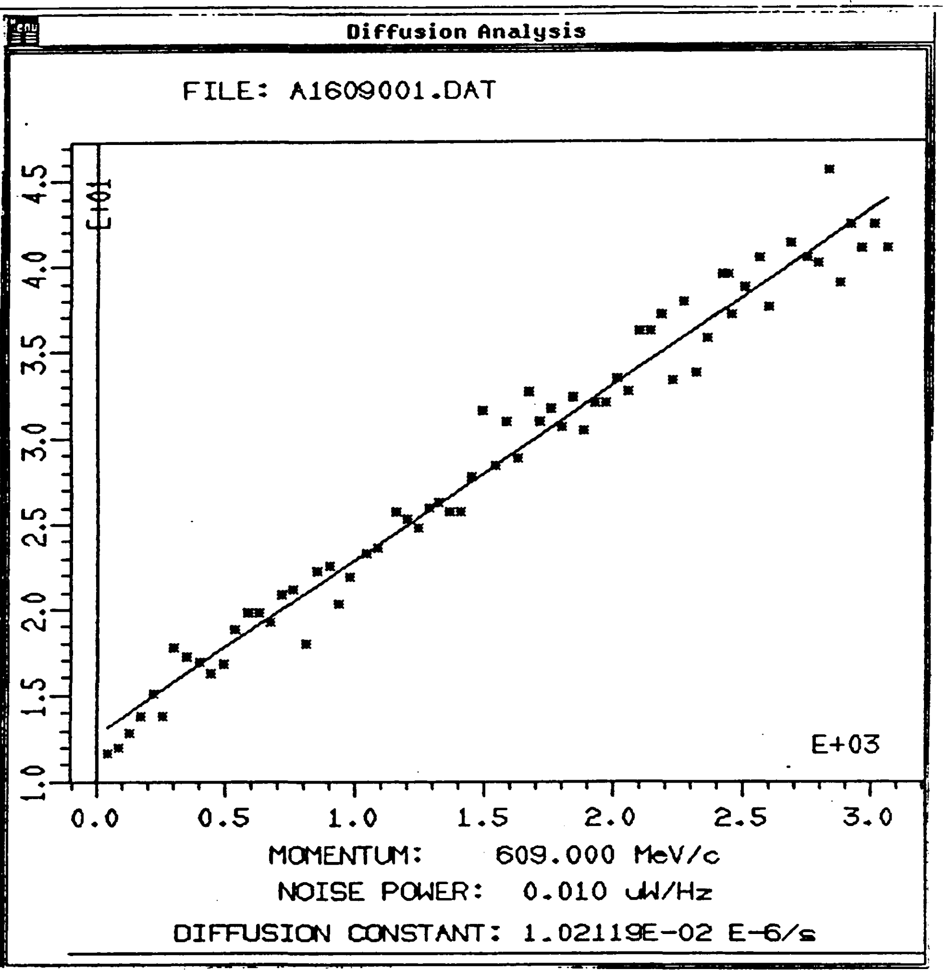
3. Measurements

The applied noise was powered to 0.01, 0.02, 0.05 and 0.1 μW/Hz. Those values in fact correspond to the calibration of the former kicker arrangement, measured by R.Cappi and G.Molinari in 1984 [3]. Since that time, the kicker has been modified in such a way that it is now equivalent to 4 gaps of 50Ω characteristic impedance each. [4] As we shall see, the original calibration is still valid, provided that the number of gaps is taken into account in the computations.

For each set of measurements, the beam was first cooled down to the limit of the stochastic cooling system, then the cooling was turned off and the acquisition program started. Noise was then applied on the beam while the program computed the rms momentum spread every minute. The time and momentum spread were then saved on a file for each measurement.

Each set of measurement was done at 309 and 609 MeV/c.

Another program was used to display the relation between the squared momentum spread and the time. As expected, this gives a straight line, the slope of which is the diffusion constant (Figure 4 on page 4).



```
FILE: A1609001.DAT
MOMENTUM:       609.000 MeV/c
NOISE POWER:    0.010 uW/Hz
DIFFUSION CONSTANT: 1.02119E-02 E-6/s
```

Figure 4: Output of the analysis program. The value of the diffusion constant computed by the program should be divided by 144 to get the standard definition of $D=\frac{1}{2}\frac{d\sigma^2}{dt}$. The program used $D=<\frac{d\left(\frac{\Delta p}{p}\right)^2}{dt}>$ with $\frac{\Delta p}{p}=6\sqrt{2}\sigma$

4. Results

Table 1 on page 6 and Figure 5 on page 5 show the results of the measurements. One can see on the plot that the agreement is quite good with the theoretical predictions, apart from a constant offset, thus showing that the former calibration of the kicker is still valid. The constant offset is interpreted as the contribution of other phenomena (interaction with the residual gas, intra−beam scattering and vacuum chamber impedance) to the spreading of the momentum distribution. In terms of equivalent−diffusion constant, this contribution amounts to $3.1*10^{-11}s^{-1}$ at 609 MeV/c, and to $6.1*10^{-11}s^{-1}$ at 309 MeV/c.

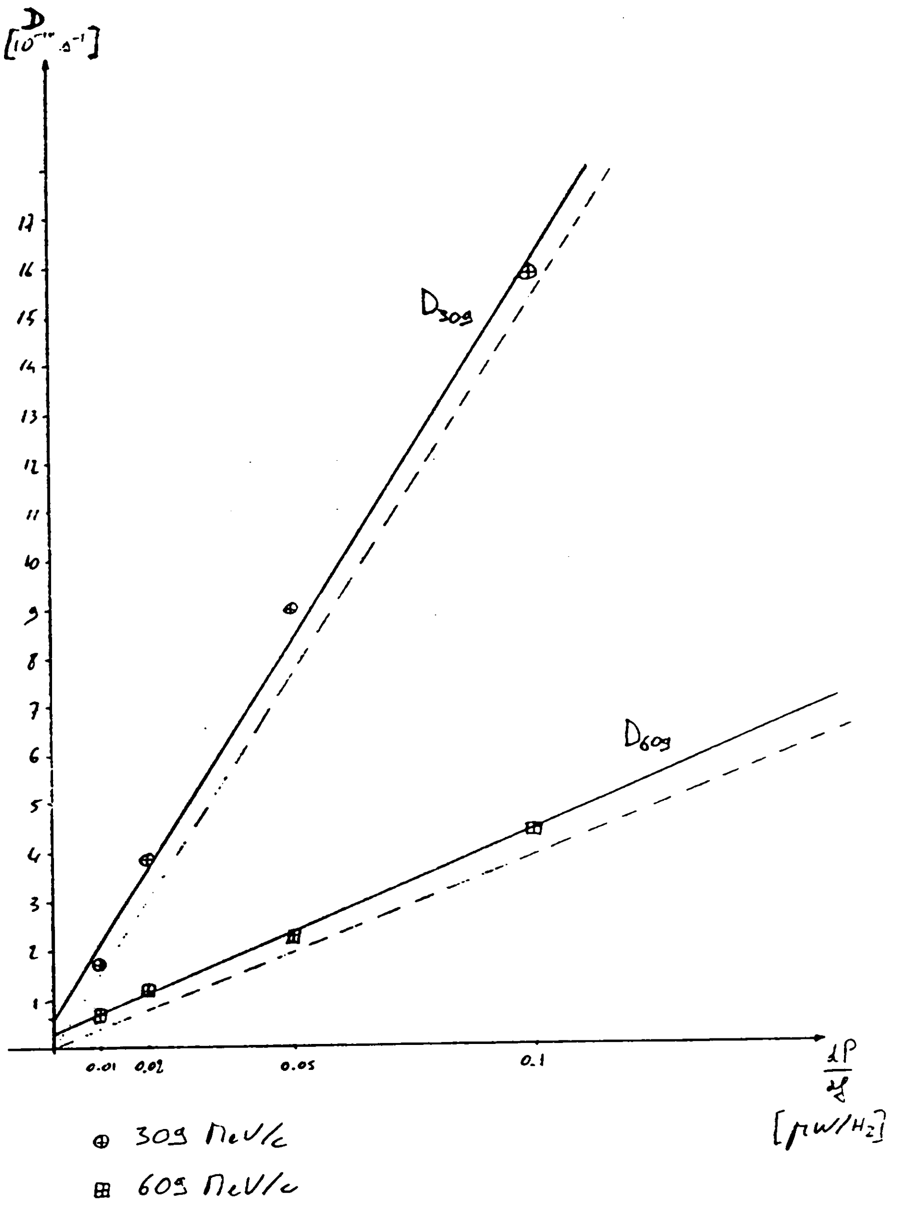


Figure 5: Plot of the measurements. The squares correspond to the measurement points at 609 MeV/c, and the circles to 309 MeV/c. The solid lines are obtained from linearly fitting the experimental points, and the dashed lines are the theoretical curves.

Table 1: Results from the measurements

p	dP/df	measured D	computed D
[GeV/c]	[μW/Hz]	[$10^{-12}s^{-1}$]	[$10^{-12}s^{-1}$]
0.309	0.01	169.	153.
	0.02	386.	305.
	0.05	894.	764.
	0.10	1580.	1530.
0.609	0.01	70.8	39.34
	0.02	116.7	78.7
	0.05	227.1	196.7
	0.10	439.6	393.4

Appendix A

Derivation of the diffusion constant.

The diffusion constant $D_{\frac{\Delta p}{p}}$ that describes the evolution of the momentum distribution

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial x} D_{\frac{\Delta p}{p}} \frac{\partial \psi}{\partial x}$$

where $x = \frac{\Delta p}{p}$ is the momentum deviation, t is the time and $\psi(x,t)$ is the particle density $\frac{dN}{dx}$, is given by:

$$D_{\frac{\Delta p}{p}} = \frac{1}{2} \frac{d<(\frac{\Delta p}{p})^2>}{dt}$$

Using the Fokker−Planck equation, F.Ruggiero shows that $D_p = \frac{1}{\beta^2 c^2} D_E$ ([5]), where $D_E = \frac{1}{2} \frac{dE^2}{dt}$ is the diffusion constant in *energy* space.

We have used $D_{\frac{\Delta p}{p}} = \frac{1}{p^2} D_p$

Now for a particle exposed to white noise, the rate of change of the average squared energy is simply: $\frac{dE^2}{dt} = f_{rev}(eV_n)^2$, where V_n is the rms noise voltage, e is the particle charge, and f_{rev} the revolution frequency. This has to be corrected by a factor $\frac{f_{rev}}{W}$ if the noise with a bandwidth W is only applied on one harmonic of the revolution frequency [6].

$$\Rightarrow D_{\frac{\Delta p}{p}} = \frac{1}{2} \frac{f_{rev}^2}{p_0^2 \beta^2 c^2} \frac{(eV_n)^2}{W}$$

Using $f_{rev} = \frac{\beta c}{2\pi R}$, where $2\pi R$ is the total machine length, $p_0 = eB\rho$, and $\frac{V_n^2}{W} = Z_0 n_{gaps} \frac{dP}{df}$ (characteristic impedance times the number of gaps times the power density), we finally get:

$$D_{\frac{\Delta p}{p}} = \frac{1}{2(2\pi RB\rho)^2} Z_0 n_{gaps} \frac{dP}{df}$$

Bibliography

1. R.Cappi, R.Giannini, W.Hardt *Ultraslow Extraction (status report)* CERN/PS/LEA 82−3
2. D.Boussard *Schottky noise and beam transfer function diagnostics* in CAS Advanced Accelerator Physics proceedings, CERN 87−03
3. R.Cappi and G.Molinari *Measurement of the gap voltage as seen from the beam* MD 15/3/84, private communication
4. G.Molinari *Notes on the extraction kicker arrangement* Private communication
5. F.Ruggiero *Diffusion in energy for non−relativistic particles* Private communication
6. S. van der Meer *Stochastic extraction, a low−ripple version of resonant extraction* CERN/PS/AA 78−6

Acknowledgements

We wish to thank Roberto Cappi, Dieter Möhl, Francesco Ruggiero and Simon van der Meer for helping us understand the diffusion process, and Gianni Molinari for his technical assistance and documentation about the hardware involved in the measurements.

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CERN PS/89-60 (AR)
CERN-CLIC-NOTE 101

November 21, 1989

Intense Charged-Particle Emission in a Diffuse Vacuum Discharge

H. Gundel and H. Riege

CERN, CH-1211 Geneva 23, Switzerland

Abstract

The well-known effects of plasma-assisted emission of electrons from a cathode and of ions from an anode are used in experiments in order to generate intense particle beams in a vacuum gap. The plasma is produced mainly by sputtering of the electrons and ions impinging on the electrode surfaces. When the initial electron beam, which here is generated by ferroelectric emission, reaches a threshold current density of about 1 A/cm^2, a homogeneous vacuum discharge takes place provided an electric field of the order of 10 kV/cm is applied between the anode and the cathode. The vacuum gap is short-circuited and the sum of the electron and ion beam currents is uniquely determined by the charging voltages and by the parameters of the external electric circuit. Beam currents of electrons and ions are extracted through holes or grids in the corresponding electrodes and measured with Faraday cups.[1]

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[1] Paper submitted to Applied Physics Letters