Delay time experienced by a laser pulse on transit through a substrate.

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Abstract

A general expression giving the delay time experienced by a laser pulse on encountering a substrate placed in the beam path is derived in terms of the phase refractive indices of the ambient and substrate material, the angle of incidence and substrate thickness. It is shown from calculation that in the ultraviolet region of the spectrum that the difference between the group and phase refractive indices of synthetic fused silica may not be neglected without introducing timing errors of the order of some picoseconds in the case of a 9.24mm thick substrate. The equation is verified experimentally for the case of a 209.42nm laser pulse of width 15±2ps (FWHM) travelling through air and incident normally on a stack of synthetic fused silica substrates.

1 Introduction

Development work into optical methods of setting the alignment and timing of 45° pulse train generator (PTG) systems [1],[2] required for the CLIC (CERN linear collider) Test Facility (CTF) demonstrated clearly that in the ultraviolet the group and phase refractive indices of synthetic fused silica are considerably different from each other. The purpose of this paper is to draw attention to this fact and to demonstrate the kind of timing errors which may be introduced if the indices are assumed equal when calculating the time taken by a laser pulse to travel through a substrate. The general formulae required to calculate the group and phase refractive indices of air and of different substrates materials are collected here. A general expression for the delay time generated by placing a substrate at a given angle of incidence in the path of a laser pulse is derived and verified experimentally for the case of a 209.42nm laser pulse of width 15±2ps (FWHM) travelling through air and incident normally on a stack of synthetic fused silica substrates.

2 General Formulae

Consider a laser pulse which arrives at some given point at a time τ as shown

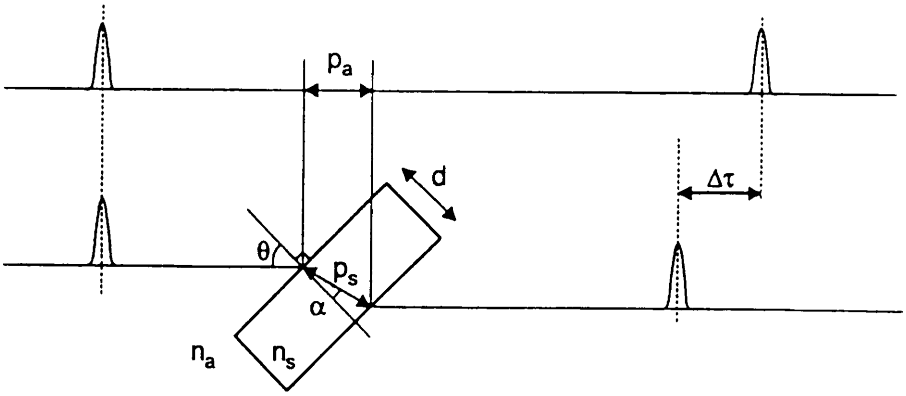

Figure 1: Illustration of the time delay $\Delta\tau$ experienced by a pulse when a substrate is inserted into the beam path. n_a, n_s, θ and α are respectively the phase refractive indices of the ambient and substrate mediums and the angles of incidence. d is the substrate thickness, p_s the path length the pulse takes in the medium and p_a the corresponding path length the pulse would have travelled if the substrate had not been introduced.

in figure (1). If a substrate were to be introduced in to the beam path the time of arrival of the pulse would be delayed by an amount $\Delta\tau$ given by

$$\begin{aligned} \Delta\tau &= |T_s - T_a| \\ &= \frac{p_s}{u_s} - \frac{p_a}{u_a} \end{aligned} \tag{1}$$

where T_s is the transit time of the pulse, which travels a path of length p_s through the substrate medium with a group velocity u_s and where T_a would have been the transit time of pulse travelling with a velocity u_a along a path length p_a though the ambient medium (eg air, N_2) had the substrate not been introduced. As will be shown below in order to calculate $\Delta\tau$ use is made of phase refractive indices to determine path lengths and group refractive indices to calculate the transit time of the pulse energy [3].

Group velocity (u) and phase velocity (v) are related by the equation

$$u = v - \lambda\frac{dn}{d\lambda} \tag{2}$$

where λ is the vacuum wavelength of the laser light and n its phase refractive index. By writing $v = c/n$ and $u = c/n'$ where c is the velocity of light in vacuum and n' the group refractive index of the wave packet the expression for group refractive index,

$$n' = \frac{n}{1 + \frac{\lambda}{n}\frac{dn}{d\lambda}}, \tag{3}$$

is obtained.

From figure (1) it can be seen that the path length p_s is given by

$$p_s = \frac{d}{cos\alpha} \tag{4}$$

where d is the thickness of the substrate material and α the angle of refraction of the pulse in the substrate as obtained using Snell's law

$$n_a.sin\theta = n_s sin\alpha \tag{5}$$

θ being the angle of incidence which the pulses make with the introduced substrate. From simple trigonometry eqn.(4) becomes

$$p_s = \frac{d.n_s}{\sqrt{n_s^2 - n_a^2 sin^2\theta}}, \tag{6}$$

thus

$$T_s = \frac{d.n_s.n_s'}{c\sqrt{n_s^2 - n_a^2 sin^2\theta}}. \tag{7}$$

where n_s' is the group refractive index of the substrate. For the case where the angle of incidence is altered by rotating the substrate inserted in a fixed beam path, cf reference[2] for converse case, p_a is written

$$\begin{aligned} p_a &= p_s.cos(\theta - \alpha) \qquad (8) \\ &= d\left\{cos\theta + \frac{n_a sin^2\theta}{\sqrt{n_s^2 - n_a^2 sin^2\theta}}\right\} \end{aligned}$$

yielding

$$T_a = \frac{d.n_a'}{c}\left\{cos\theta + \frac{n_a sin^2\theta}{\sqrt{n_s^2 - n_a^2 sin^2\theta}}\right\} \tag{9}$$

where n_a' is the group refractive index of air. Substituting eqns. (7) and (9) back into eqn.(1) yields,

$$\Delta\tau = \frac{d}{c\sqrt{n_s^2 - n_a^2 sin^2\theta}}\left\{n_s n_s' - \left(cos\theta\sqrt{n_s^2 - n_a^2 sin^2\theta} + n_a sin^2\theta\right) n_a'\right\}. \tag{10}$$

3 Synthetic fused silica substrate in air

The phase refractive index n and its derivative $dn/d\lambda$ may be evaluated, at a given wavelength and for a given substrate material, using Sellmeier's equation

$$n^2 = 1 + \sum_i \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2} \tag{11}$$

which yields

$$\frac{dn}{d\lambda} = -\frac{\lambda}{n}\sum_i \frac{A_i\lambda_i^2}{(\lambda^2 - \lambda_i^2)^2} \tag{12}$$

and where A_i and λ_i are constants[4] and where λ is in microns.

For synthetic fused silica Malitson[5] obtained the values of 0.6961663, 0.4079426, 0.8974794, 0.0684043, 0.1162414, and 9.896161 for the parameters $A_0, A_1, A_2, \lambda_0, \lambda_1$ and λ_2 respectively, used in eqns (11) and (12). Having evaluated n, $dn/d\lambda$ and hence the group velocity n', the plot of figure (2) was obtained for synthetic fused silica. As can be seen from the plot the phase and group refractive indices of this material are not equal. In fact the difference between them increases sharply with decreasing wavelength from 0.0129 at 10642Å to 0.2722 at 2094.2Å.

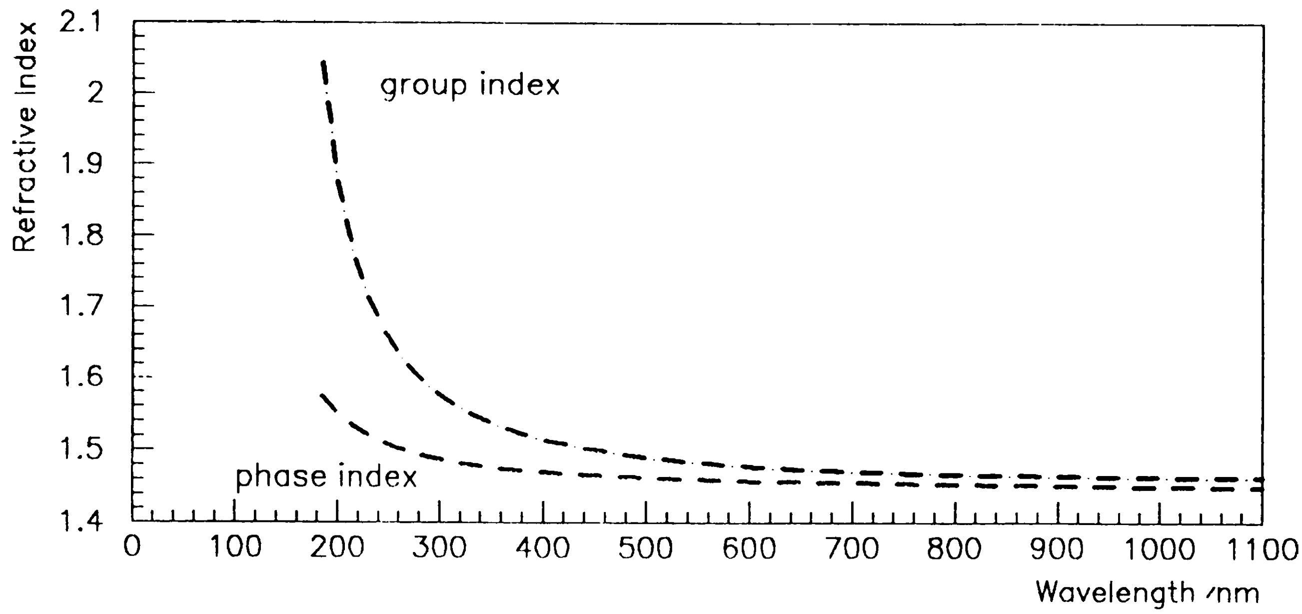

Figure 2: Plot of group and phase refractive indices as a function of wavelength for synthetic fused silica

The expression, where $\sigma = 1/\lambda$ is the vacuum wavenumber given in μm^{-1},

$$(n-1)_D \times 10^8 = 8342.13 + \frac{2406030}{(130-\sigma^2)} + \frac{15997}{(38.9-\sigma^2)} \tag{13}$$

for the phase refractive indice of dry air, denoted by subscript D, at 15°C, 760mmHg and 0.03% CO_2, was given by Edlén [5] in 1966, and is accurate to 1×10^{-8} or better. The corresponding group refractive index is given by

$$(n'-1)_D \times 10^8 = 8342.13 + \frac{2406030}{(130-\sigma^2)^2}(130+\sigma^2) + \frac{15997}{(38.9-\sigma^2)^2}(38.9+\sigma^2) \tag{14}$$

Some calculated values are given in references [7] and [8], from which it can be seen that the phase and group refractive indices of air, quoted here to six decimal places only, vary only slightly with wavelength , ranging from 1.000274 and 1.000276 respectively at 11290.5$\AA$ to 1.000338, 1.000572 respectively at 1854.73$\AA$. The formula

$$\begin{aligned}(n-1) \quad \times \quad & 10^8 = (n-1)_D \times 10^8 \qquad (15)\\ \times \quad & [\frac{p}{720.775}\frac{1+p(0.817-0.0133t)\times 10^{-6}}{1+0.0036610t}] - f[5.772 - 0.0457\sigma^2]\end{aligned}$$

takes into account the effects of water vapour of partial pressure f (torr) in an atmosphere of total pressure p (torr) and temperature t(°C)[8]. These formulae are good for pressures and ranges of temperature and humidities normally encountered in the laboratory. Improved formulae are only required for conditions of high temperature and humidity[8],[9].

In table (1), refractive indices, calculated from the above, have been listed for some of the observed fundamental wavelengths of some neodymium lasers [10],[11] and their harmonics. Listed also are the time delays that would be generated by inserting a 9.24±0.01mm substrate at normal incidence into the beam and the error given by

$$\Delta\tau_{error} = \Delta\tau - \frac{d}{c}(n_s - n_a) \qquad (16)$$

that would be induced if the group velocity were to be assumed equal to the phase velocity. In the table, the fundamental wavelengths are given to the nearest angstrõm and the indices correct to four decimal places. The accuracy to which the phase indices of synthetic fused silica may be calculated using Malitson's values for A_i and λ_i given above is estimated to be of the order of 3×10^{-5}. Matilson obtained these values by measuring the refractive indices of synthetic fused silica samples from three different companies each of which provided samples from four different production runs. One can expect [12] the refractive index of any synthetic fused silica substrate obtained commerically to agree to the fourth decimal place with the calculated values. Group refractive indices are similiarly accurate to the fourth decimal place for the wavelengths as tabulated. Correspondingly the indices for air are given only to the fourth decimal place.

As can be seen from table (1), the effect of failing to take into account the effect of group velocity will induce timing errors of less than a picosecond in the infrared and visible regions of the spectrum. However in the ultraviolet the errors introduced would be significant, being of the order of 8 ps for a 9.24mm substrate at normal incidence inserted into the path of a 209.42nm laser pulse.

Nd:YAG

λ(Å)	Syn. fused silica		Dry Air		$\Delta\tau$ (ps)	$\Delta\tau_{error}$ (ps)
	n	n'	n	n'	d=(9.24±.01)mm	d=(9.24±.01)mm
10642	1.4496	1.4625	1.0003	1.0003	14.25±2	0.50±4
5321	1.4607	1.4858	1.0003	1.0003	14.96±2	0.77±5
3547.3	1.4761	1.5353	1.0003	1.0003	16.49±2	1.82±4
2660.5	1.4997	1.6241	1.0003	1.0004	19.23±3	3.84±5
2128.4	1.5353	1.7914	1.0003	1.0004	24.38±3	7.89±5

Nd:YLF

λ(Å)	Syn. fused silica		Dry Air		$\Delta\tau$ (ps)	$\Delta\tau_{error}$ (ps)
	n	n'	n	n'	d=(9.24±.01)mm	d=(9.24±.01)mm
10530	1.4498	1.4626	1.0003	1.0003	14.25±2	0.50±4
5265	1.4610	1.4865	1.0003	1.0003	14.99±2	0.79±4
3510	1.4767	1.5375	1.0003	1.0003	16.56±2	1.88±4
2632.5	1.5009	1.6292	1.0003	1.0004	19.38±2	3.95±4
2106	1.5377	1.8042	1.0003	1.0004	24.78±3	8.22±5

Nd:YLF

λ(Å)	Syn. fused silica		Dry Air		$\Delta\tau$ (ps)	$\Delta\tau_{error}$ (ps)
	n	n'	n	n'	d=(9.24±.01)mm	d=(9.24±.01)mm
10471	1.4498	1.4627	1.0003	1.0003	14.25±2	0.49±4
5235.5	1.4611	1.4893	1.0003	1.0003	15.00±2	0.80±4
3490.3	1.4771	1.5386	1.0003	1.0003	16.59±2	1.90±4
2617.8	1.5016	1.6320	1.0003	1.0004	19.47±2	4.02±4
2094.2	1.5390	1.8112	1.0003	1.0004	25.00±3	8.40±4

Table 1: Calculated phase n and group n' indices for synthetic fused silica and air at wavelengths available from some neodymium lasers. The time delay $\Delta\tau$ introduced by inserting a 9.24mm thick substrate into a beam at normal incidence is calculated and the error $\Delta\tau_{error}$ that would be incurred by assuming $n' = n$ given. Errors bars refer to last decimal place given.

4 Experimental method

The experimental arrangement used to verify the effects of group refractive index is illustrated in figure (3). Pulses of wavelength 209.42nm, generated using a ND:YLF laser, and pulse width 15±2ps were input into a single stage of the 45° pulse train generator (PTG) system constructed for the CTF. The stage, which consists of two beam splitting optics and two mirrors, was aligned so as to provide at its output two pulses, separated by a time delay τ, travelling colinearly along a beam path and directed into a streak camera.

The experimental procedure consisted of first measuring the delay τ be-

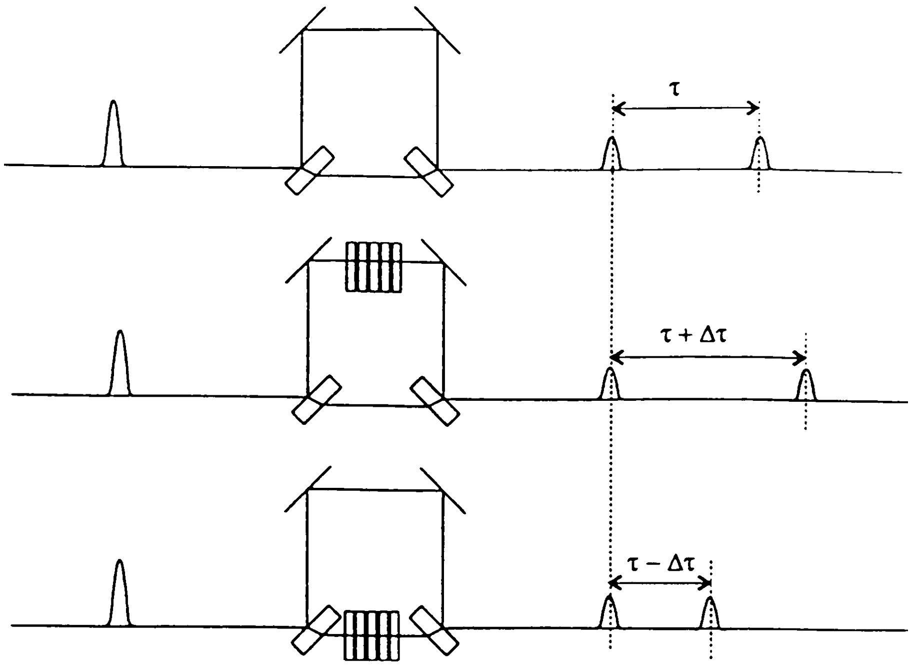

Figure 3: Experimental setup used to verify the effects of group velocity

tween the two pulses produced at the output of the stage. Five substrates, each of thickness 6.48mm, were then introduced and aligned normally with respect to the incident pulses, first into the detoured path of the PTG stage, thus increasing the delay between the pulses to $\tau + \Delta\tau$, cf figure 3, and then into the straight through path, thus reducing the the delay to $\tau - \Delta\tau$. In each case the delay between the pulses was measured five times to provide an estimate of accuracy.

The delays produced were measured by saving and analysing the integrated pulses profiles recorded using the streak camera. The profiles, cf example presented in figure 4, were saved as text files using the streak camera software, and the analysis was made off-line using Excel. These profiles show two peaks, each one corresponding to a pulse. To determine the delay between the pulses two windows of width 51 pixels were defined in the streak camera profile of total width 576 pixels. As illustrated in figure (4), the first window, identified by the black squares, starting at pixel n_1, was positioned to cover the first pulse. The second, starting at pixel n_2 and identified by the uncoloured squares, was likewise positioned to cover the second pulse. The window separation m is therefore given by

$$m = n_2 - n_1 \tag{17}$$

where m is in pixels. If m is chosen such that the contents of the second window is fitted pixel by pixel to that of the first then m will give the delay in pixels between the two pulses. An example of a case in which this has been done is given in figure (5).

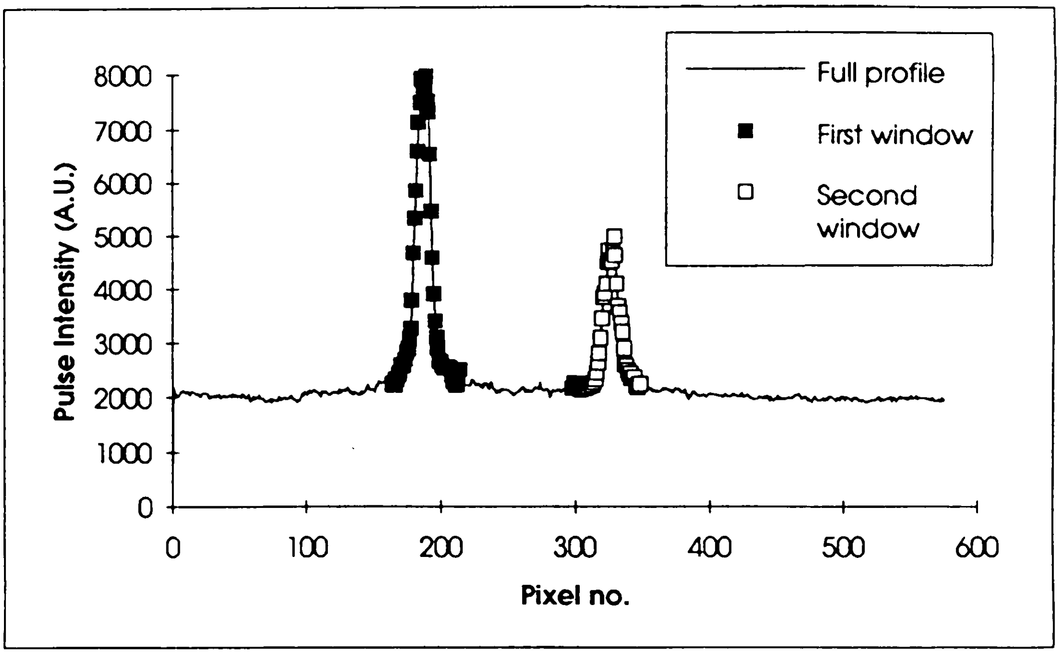

Figure 4: Example of a streak camera profile showing the position of the two 51 pixel windows positioned over each of the two pulses recorded.

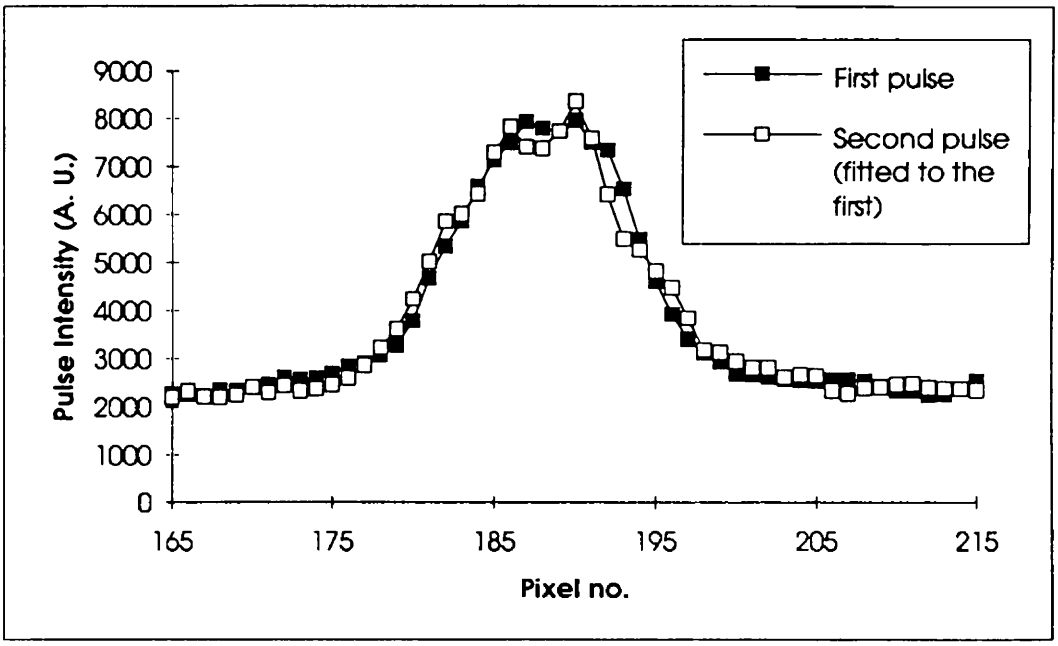

Figure 5: Example of fitting between the contents of windows 1 and 2 shown in figure 4

In figure (5) contents of the first window are presented unaltered along with the processed contents of the second window which gave the best fit. Fitting is achieved, for a given value of m, by calculating a scaling factor a, and a background level b, yielding for the fitted second pulse

$$I_i' = a(I_{i+m} - b) + b, \quad \text{for i} = n_1 \text{ to } n_{1+50} \tag{18}$$

where I is the intensity recorded in pixel i. The values of a and b are calculated to minimize the quadratic error between the intensity of first pulse, I_i, and that of the fitted second pulse, I_i' for different values of delay m. The value of m giving the best fit is taken as the delay between the two pulses.

5 Experimental results

The measurements of pulse separation, obtained using the configurations illustrated in figure (3), are presented in table (2). For each case the measurement was taken five times and is expressed in pixels.

Measurement Number	Pulse separation (pixels)		
	τ	$\tau + \Delta\tau$	$\tau - \Delta\tau$
1	140±.5	213±.5	64±1
2	140±.5	213±.5	63±1
3	140±.5	213±.5	63±1
4	140±.5	213±.5	64±1
5	139±.5	213±.5	65±1
Average	139.8±.5	213±.5	64.8±2
$\Delta\tau$		73.2±1	75.0±1.5

Table 2: Experimental results giving $\Delta\tau$ the increase or decrease in pulse separation due to the insertion of substrates into an arm of the PTG stage.

The accuracy to which τ and $\tau+\Delta\tau$ may be measured was estimated to be of the order of ±0.5 pixels. For the case of τ-$\Delta\tau$ the inaccuracy was found to be larger, in the order of ±1 pixel. These differences in accuracy arise due to the fact that inserting the substrates into an arm of the stage appears to change the state of polarization of the pulses traversing them and so, since the beam splitting ratio obtained at the beam splitters in the stage used depends on the state of polarization of the incident light, pulses of equal magnitude are not obtained at the stage output. When the difference in magnitude between the pulses is great, as it was in the case of the τ-$\Delta\tau$ measurements, the magnitude of the second pulse being about one sixth that of the first, the accuracy to which fitting may be achieved is reduced. Also, in the case of the τ-$\Delta\tau$ measurements the trailing edge of the first peak overlapped with the leading edge of the second.

Using a streak camera calibration of 1.168±16 ps/pixels obtained using two electron pulses known to be synchronised to the 33Ghz rf of the CTF to an accuracy of ±3ps, the $\Delta\tau$ introduced by the substrates was calculated to be 85.5ps±2.4ps. This is the value obtained by considering the measurements of $\tau + \Delta\tau$ and agrees with the expected value of 87.6ps obtained on evaluation of eqn.(10) for normal incidence and using the values of 1.8112 and 1.0003 for group refractive indices of synthetic fused silica and air respectively as tabulated in table (1).

6 Conclusion and Discussion

An general expression was derived for the delay time introduced by inserting a substrate into the path of a laser pulse and experimentally verified for the case of a 15±2ps pulse of wavelenght 209.42nm travelling through air and incident normally on a stack of five 6.48mm thick substrates of synthetic fused silica. It was also shown, by calculation, that for the infrared and visible regions of the spectrum that the group and phase refractive indices of synthetic fused silica may be taken to be equal without introducing timing errors greater than one picosecond when the laser pulse encounters at normal incidence a substrate of thickness 9.24mm in air. However, for the ultraviolet, timing errors of the order of some picoseconds can be incurred if the group and phase indices are assumed equal.

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