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BEAM LIFETIME IN THE ANTIPROTON DECELERATOR (AD)

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1 Introduction

In an earlier study [1], the deterioration of an antiproton beam in the AD due to multiple scattering on the rest gas was discussed, and led to the conclusion that a vacuum improvement of approx. a factor of 20 compared to the AC was essential for the success of AD experiments. The beam loss at low energies is dominated by emittance blow up due to multiple scattering which leeds to beam loss on the ring aperture, if blow up is prevented, for instance by cooling, the lifetime is dominated by nuclear scattering and single Coulomb scattering (as opposed to multiple Coulomb scattering). At high energies the loss due to multiple scattering decreases, and we would expect the lifetime to begin to be limited by nuclear and single Coulomb scattering.

In this note we consider the contributions from nuclear scattering (where we, due to the large forces involved, can consider all participating particles lost), and single Coulomb scattering to the beam lifetime. Combining these contributions with the earlier studied multiple scattering we get a full picture of the loss mechanisms in the AD for the whole energy range. The calculations are compared to AC lifetime measurements, and a study, of how critical the gas composition of the vacuum is, is conducted. As in [1] most of our calculations follow the recipe of [2] and [3].

2 Nuclear Scattering

A proton involved in a nuclear scattering event (nuclear reaction or diffraction scattering) can be assumed to be scattered into so large angles or loose so much energy that it is lost from the beam. This will result in a simple exponential decay

$$I_n(t) = I_0 \exp\left(-\frac{t}{\tau_n}\right) \tag{1}$$

where

$$\tau_n = \frac{1}{\beta c n_a \sigma_a} \tag{2}$$

where βc is the beam velocity, n_a is the number of atoms per unit volume, and σ_a is the nuclear scattering cross section.

The nuclear scattering cross section for the AD momentum range can be approximated by using the results of [4] and [5].

$$\sigma_a = A^{2/3}\left(\frac{53}{p[GeV/c]} + 66\right) \text{mbarn} \tag{3}$$

then, for convenience, we define a nuclear scattering equivalent density n_{ns}(as we in [1] defined a multiple scattering equivalent density) which is given as follows

$$n_{ns} = \sum n_i A_i^{2/3} \tag{4}$$

resulting in the following expression for the nuclear scattering life time

$$\tau_{ns} = \frac{1}{\beta c n_{ns} \sigma_{ns}} \tag{5}$$

where σ_{ns} is the Hydrogen cross section (A=1).

Using the vacuum description from [1], which is repeated in appendix A, we obtain a total nuclear scattering equivalent density of $1.28{\cdot}10^{15}\mathrm{m}^{-3}$, and a nuclear scattering lifetime in the AD, with AC pressure as plotted in figure 1.

$$\tau_{ns} = \frac{7.23 \cdot 10^3 \text{ hours}}{\beta\,(53/(p[GeV/c]) + 66)} \tag{6}$$

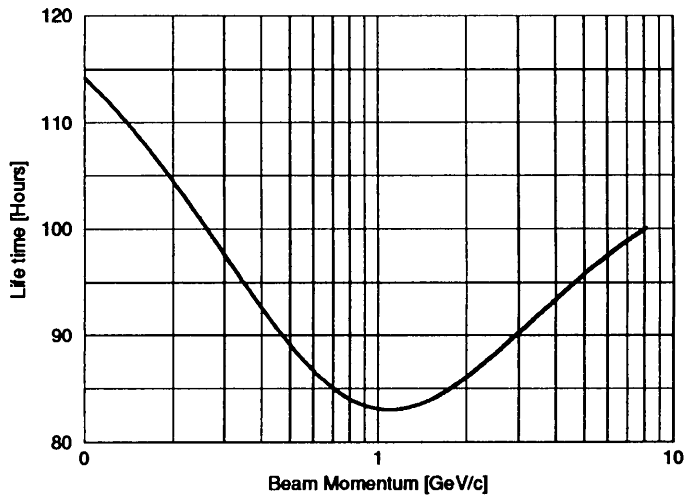

Figure 1: Lifetime in the AD (with standard vacuum conditions (appendix A)) due to nuclear scattering.

3 Single Coulomb Scattering

Beam loss due to Coulomb scattering occurs when a beam particle is deflected in a collision with the residual gas by an angle which brings it outside the ring acceptance. Integrating the Rutherford cross section we obtain an expression for the cross section for scattering by an angle larger than θ [6]

$$\sigma_{sc} = 4\pi \left(\frac{Ze^2}{4\pi\epsilon_0 \gamma m_p c^2 \beta^2} \right)^2 \frac{1}{\theta^2} \tag{7}$$

where Z is the charge of the rest gas nucleus, γ and β relativistic factors, and m_p the rest mass of the proton. The minimum angle causing loss of a particle is described by the machine acceptance, for the AD the horizontal and vertical acceptances are about the same, thus we have to use an 'average' angle as calculated in [7]

$$\frac{1}{\theta^2} = \frac{1}{2\theta_H^2} + \frac{1}{2\theta_V^2} \; ; \; \frac{1}{\theta_{H,V}^2} = \frac{\beta_{H,V}}{\epsilon_{accep,H,V}} \tag{8}$$

where $\beta_{H,V}$ are the average beta functions ($\beta_H \sim 7.5$ m and $\beta_V \sim 7.0$ m for the AD lattice). Inserting this into equation (7) and using equation (2) we obtain the beam life time due to single Coulomb scattering

$$\tau_{sc} = \frac{\gamma^2 \beta^3 \epsilon_{accep}}{2\pi r_p^2 c \beta_z n_{sc}} \tag{9}$$

where we for convenience have introduced the classical proton radius r_p=1.535$\cdot10^{-18}$ m and the single Coulomb scattering density n_{sc} given by :

$$n_{sc} = \sum Z_i^2 n_i \tag{10}$$

which for the standard vacuum conditions in the AC (appendix A) equals $n_{sc} = 6.35\cdot10^{15}$ m^{-3}.

Inserting the single Coulomb scattering density using a ring acceptance of $\epsilon_{accep} = 150$ π mm mrad from [1], and the average beta function for the AD we obtain the following expression for the single Coulomb scattering lifetime

$$\tau_{sc} = 102.0 \text{ hours} \cdot \beta^3 \gamma^2 \tag{11}$$

which has been plotted on figure 2.

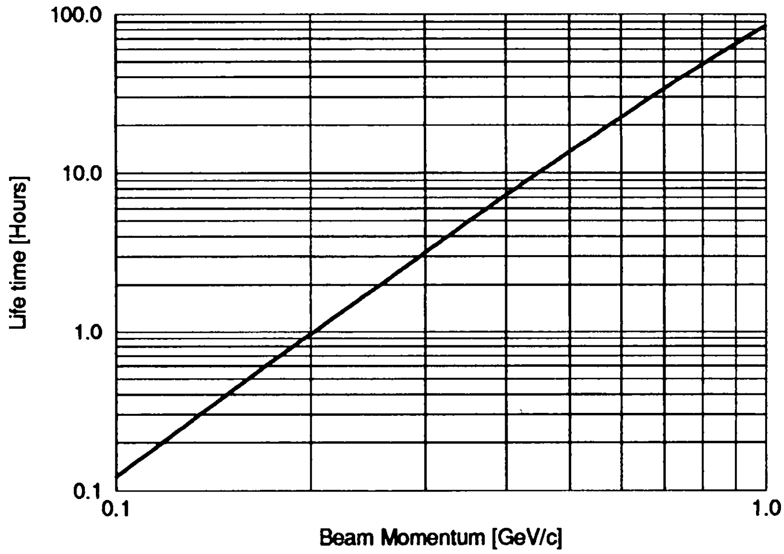

Figure 2: Single Coulomb Scattering lifetime in the AD with standard vacuum conditions (appendix A) as a function of beam momentum. An acceptance of 150 π mm mrad has been assumed.

4 Total Lifetime

We now have two new contributions to beam loss, and assuming that cooling keeps the emittance down, these are the dominant contributions. As they are both exponential decays, we can calculate the total beam lifetime as

$$\tau_{n+sc} = \left(\tau_n^{-1} + \tau_{sc}^{-1}\right)^{-1} \tag{12}$$

a plot of which is shown in figure 3.

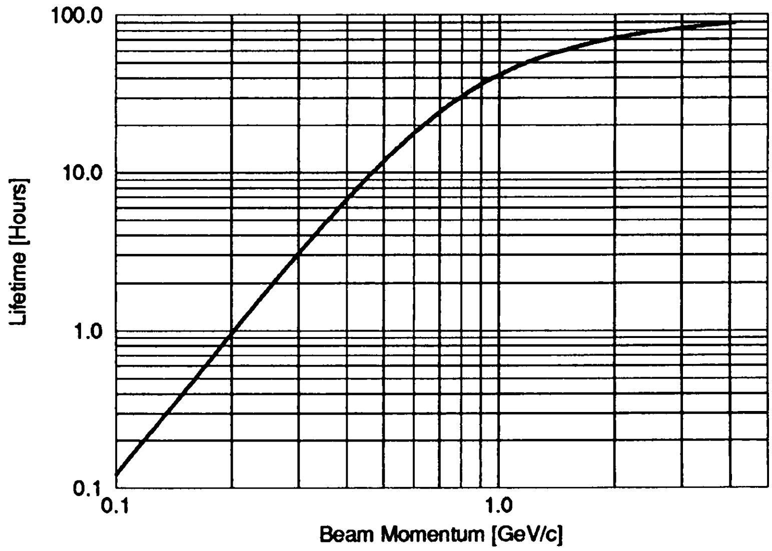

Figure 3: Total lifetime without beam blowup caused by multiple scattering for the AD with standard vacuum conditions (appendix A).

Comparing these results with the time it takes for the uncooled beam to start being scraped by the aperture of the vacuum chamber (figure 4 in [1]) it is clear that for the uncooled beam multiple scattering will dominate the losses for low energies, only at energies above approximately 1 GeV the influence from nuclear and single Coulomb scattering start to be of same order of magnitude as the losses induced by multiple scattering.

But in order to compare the multiple scattering results directly with the nuclear and single Coulomb scattering, we need to extract a lifetime from the emittance growth.

We use the basic assumption that particles beyond the acceptance are considered lost. The 2σ emittance growth was given by [1]:

$$\epsilon_{2,z}(t) = 12.3\text{mm mrad} + \frac{1.270 \cdot 10^{-2}\text{ mm mrad}}{\beta \cdot (p[Gev/c])^2} \cdot t[s] \tag{13}$$

assuming a gaussian distribution of particles in each transverse phase space, and chosing a coordinate system, where the particle motion in phase space is a circle, we can estimate the fraction η of particles within the acceptance as follows :

$$\begin{aligned} \eta(t) &= \frac{1}{(\sigma_z(t))^2}\int_0^{r_{accep}} \exp(-\frac{-r^2}{2(\sigma_z(t))^2})rdr \\ &= 1-\exp(-\frac{r_{accep}^2}{2(\sigma_z(t))^2}) \\ &= 1-\exp(-\frac{\epsilon_{accep}}{2(\epsilon_{2,z}(t)/4)}) \end{aligned} \tag{14}$$

which using the above given numbers results in the behavior illustrated on figure 4.

The behavior is obviously not an exponential decay, but as the exponential decay is faster than the decay due to the aperture, we can make a lower estimate on the lifetime

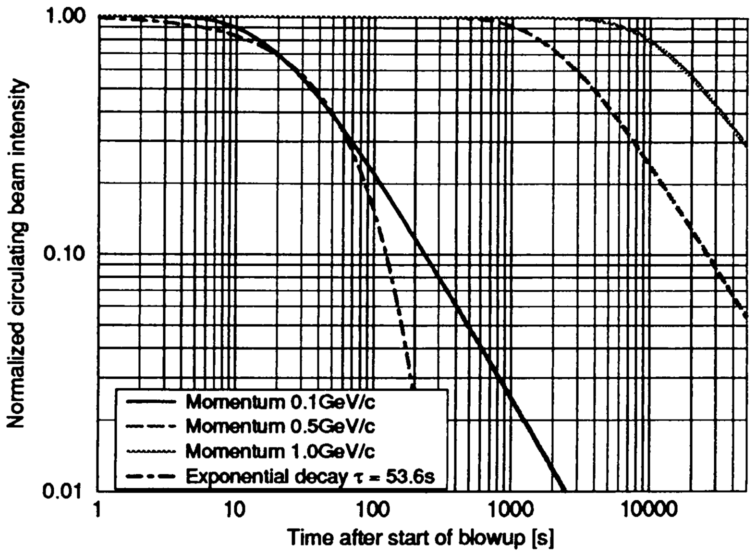

Figure 4: Calculation of the normalized beam intensity left in the AD, assuming AC vacuum conditions, as a function of time, when only losses induced by multiple scattering is taken into account. The exponential decay shown is the approximation used for the lifetime calculations.

of the beam as a function of energy by approximating the behavior with an exponential decay. We do this by calculating the time at which the intensity has fallen to a fraction e^{-1}, i.e.

$$\begin{aligned}\tau_{ms} &= \frac{-2\epsilon_{accep}/\ln(1-e^{-1}) - \epsilon_{init}}{1.270 \cdot 10^{-2}\text{mm mrad}} \cdot \beta(p[Gev/c])^2 \\ &= 14.0 \text{ hours} \cdot \beta(p[Gev/c])^2 \end{aligned} \tag{15}$$

which has been plotted as the dashed curve on figure 5. Combining this estimate with the lifetime calculations for nuclear and single Coulomb scàttering we obtain the lifetime for an uncooled beam in the AD as a function of the beam energy. A plot of this is shown in figure 5.

Keeping in mind that the plot in figure 5 is log-log we observe that at approximately 2.5GeV the contribution from multiple scattering and the contributions from nuclear and single Coulomb scattering are equal. Thus as expected multiple scattering dominates for low energies, and it's at the high energies that the direct losses become important. It should however also be noted that in the range where the different contributions are the same order of magnitude, the lifetime is rather long compared with the expected cycle times of the AD [8].

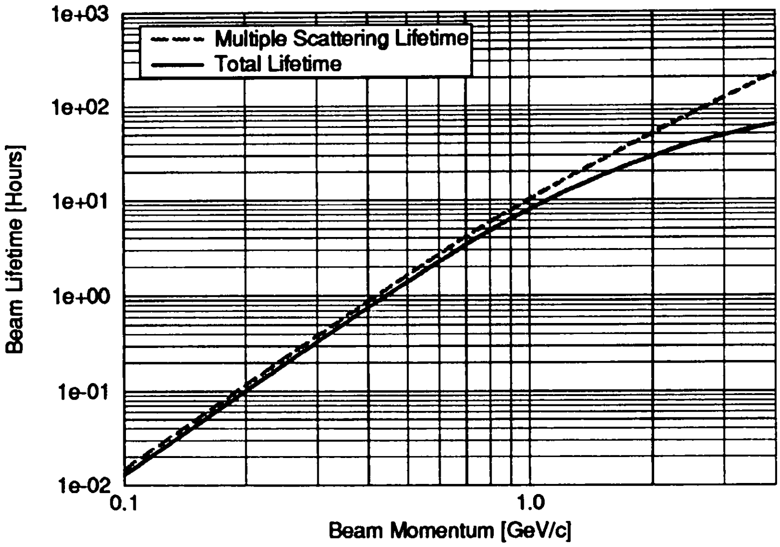

Figure 5: Lifetime of an uncooled circulating beam in the AD with AC vacuum conditions, including all contributions from multiple scattering (beam blow up), nuclear scattering and single Coulomb scattering.

5 Dependence on vacuum composition

An interesting question is of course how much the changes in vacuum composition alters the results. A simple estimate of this can be done by using the different measured compositions already at hand [9]. The different measurements are listed (converted into densities) in table 4 in appendix A. In table 1 are given the corresponding effective densities for the different loss mechanisms.

Density type.	Dens No.1 $[m^{-3}]$	Dens No.2 $[m^{-3}]$	Dens No.3 $[m^{-3}]$	Dens No.4 $[m^{-3}]$
n_{ms}	$5.81 \cdot 10^{16}$	$7.17 \cdot 10^{16}$	$5.98 \cdot 10^{16}$	$7.41 \cdot 10^{16}$
n_{ns}	$1.29 \cdot 10^{15}$	$1.41 \cdot 10^{15}$	$1.28 \cdot 10^{15}$	$1.38 \cdot 10^{15}$
n_{sc}	$6.35 \cdot 10^{15}$	$7.87 \cdot 10^{15}$	$6.57 \cdot 10^{15}$	$8.16 \cdot 10^{15}$
variation	-	9% - 24%	1% - 3.5%	7% - 31%

Table 1: Effective densities for the compositions listed in table 4

The last row in table 1 shows the range of deviations from the vacuum description the calculations through out this text was based on. The largest variations seems to be on the multiple scattering and the single Coulomb scattering densities, this is because these both scale with Z^2 whereas the nuclear scattering scales only with $A^{2/3}$. As the multiple scattering dominates for small energies and the nuclear for high energies, the variations in the single Coulomb scattering probability will not influence the losses much. Thus in the worst case we can expect the lifetime in the low energy range to be $\sim$ 30% less than the estimates otherwise given here, and in the high energy range $\sim$ 15% less.

6 Comparison with AC measurements

Before the shut down of the AC a few measurements of the beam lifetime as a function of energy were done [10], these should give us a clue as to how good the approximations done in the previous sections are. Figure 6 shows the results plottet together with estimates calculated from the equations in these notes.

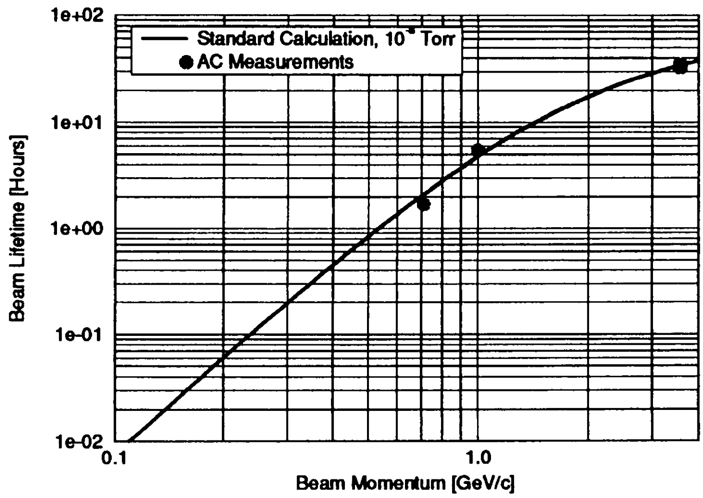

Figure 6: Comparison between the AC lifetime measurements and the results from these notes. It should be noted that the AC has a slightly different betafunction average than the AD, this has not been taken into account. The densities were scaled to a N_2 equivalent pressure of $1.0{\cdot}10^{-8}$ Torr $= 1.3{\cdot}10^{-6}$ Pa. to coincide with the actual AC pressure during the lifetime measurements [10]. Standard vacuum composition (appendix A) was used.

The small deviations between the measured points, which themselves hold an unknown uncertainty, and the theoretical estimates, indicates that the validity of the calculations are rather good. The theoretical estimate has been calculated using the standard vacuum composition (appendix A).

7 Conclusions

As mentioned in [1] it is clear that for low energies, where the lifetime is short, the lifetime in the AD is dominated by multiple scattering in the uncooled case. As the lifetimes all scale linearly with the pressure, a factor 20 reduction of the pressure will increase the lifetimes with a factor 20, which should be enough for most operation.

The observed rather strong variation in the effective densities due to variations in the vacuum composition, which did not alter the gauge readings, also supports the decision of improving the vacuum as much as possible, as these variations have been up to an order of 30% from the mean value.

The beam lifetime measurements in the AC turned out to be in rather good agreement with the theoretical values calculated using the observed pressure and the standard vacuum composition (Number 1, appendix A), which shows promise for the use of these calculations for predicting the conditions in the AD.

A Vacuum Conditions in the AC

As the cross section of the interaction between antiprotons and rest gas depends on the nature of the rest gas, the composition of the rest gas needs to be taken into account. Relevant numbers are given in table 2.

Species	Ion Gauge Cal.	RGA[1] Cal.	Composition [arb. units]	Pressure N_2 eq. [Pa]	Pressure Abs [Pa]	Density [m^{-3}]
H_2	2.50	1.54	178	$4.48 \cdot 10^{-7}$	$1.12 \cdot 10^{-6}$	$2.70 \cdot 10^{14}$
CH_4	0.78	0.71	10	$5.45 \cdot 10^{-8}$	$4.25 \cdot 10^{-8}$	$1.03 \cdot 10^{13}$
H_2O	1.00	1.00	40	$1.55 \cdot 10^{-7}$	$1.54 \cdot 10^{-7}$	$3.74 \cdot 10^{13}$
CO	0.83	1.00	35	$1.36 \cdot 10^{-7}$	$1.12 \cdot 10^{-7}$	$2.71 \cdot 10^{13}$
N_2	1.00	1.00	0	0	0	0
Ar	0.69	0.83	0	0	0	0
CO_2	0.72	1.54	3	$7.54 \cdot 10^{-9}$	$5.43 \cdot 10^{-9}$	$1.31 \cdot 10^{12}$

Table 2: Residual gas description for the AC, $P_{IG,N_2} = 8.00 \cdot 10^{-7}$Pa (=$6.0 \cdot 10^{-9}$ Torr)

The calibrations of the Ion Gauge and the rest gas analyzer (RGA) are given in terms sensitivity to N_2 divided by the sensitivity to the gas in question. The pressure returned by a standard Ion Gauge is the Nitrogen equivalent pressure (P_{IG,N_2}).

As the relevant densities for the calculations are the atom densities, these have been listed in table 3.

Species	Atomic No. (A)	Charge (Z)	Density [m^{-3}]	n_{ms} [m^{-3}]	n_H [m^{-3}]	n_{sc} [m^{-3}]
H	1	1	$6.561 \cdot 10^{14}$	$6.925 \cdot 10^{15}$	$6.561 \cdot 10^{14}$	$6.561 \cdot 10^{14}$
C	12	6	$3.874 \cdot 10^{13}$	$1.273 \cdot 10^{16}$	$2.030 \cdot 10^{14}$	$1.395 \cdot 10^{15}$
N	14	7	0	0	0	0
O	16	8	$6.717 \cdot 10^{13}$	$3.842 \cdot 10^{16}$	$4.265 \cdot 10^{14}$	$4.299 \cdot 10^{15}$
Ar	40	18	0	0	0	0

Table 3: Atomic densities for the vacuum description in table 2. The total multiple scattering densities, Hydrogen equivalent densities (nuclear scattering) and single Coulomb scattering are, $n_{ms} = 5.81 \cdot 10^{16}$ m^{-3}, $n_H = 1.29 \cdot 10^{15}$ m^{-3} and $n_{sc} = 6.35 \cdot 10^{15}$ m^{-3}.

Table 4 shows 4 different vacuum compositions, and the resulting effective densities (with which the lifetimes scale linearly). Measurement no.1 and 2. are measured in the AC. Measurement 3. and 4. are measured elsewhere, but in vacuum systems fairly equivalent to the conditions to be expected in the AC. Measurement no.4. is for a system where baking hasn't finished. All the measurements have been scaled to the same Nitrogen equivalent pressure.

Species	Dens No.1 [m^{-3}]	Dens No.2 [m^{-3}]	Dens No.3 [m^{-3}]	Dens No.4 [m^{-3}]
H_2	$2.70 \cdot 10^{14}$	$2.67 \cdot 10^{14}$	$2.82 \cdot 10^{14}$	$1.95 \cdot 10^{14}$
CH_4	$1.03 \cdot 10^{13}$	$2.78 \cdot 10^{12}$	$7.09 \cdot 10^{12}$	$6.60 \cdot 10^{12}$
H_2O	$3.74 \cdot 10^{13}$	$2.14 \cdot 10^{13}$	$3.27 \cdot 10^{13}$	$8.48 \cdot 10^{13}$
CO	$2.72 \cdot 10^{13}$	$4.64 \cdot 10^{13}$	$2.67 \cdot 10^{13}$	$1.46 \cdot 10^{13}$
N_2	0	0	$3.40 \cdot 10^{12}$	$2.61 \cdot 10^{12}$
Ar	0	0	$1.31 \cdot 10^{12}$	0
CO_2	$1.31 \cdot 10^{12}$	$7.13 \cdot 10^{12}$	$8.16 \cdot 10^{11}$	$1.16 \cdot 10^{12}$
n_{ms}	$5.81 \cdot 10^{16}$	$7.17 \cdot 10^{16}$	$5.98 \cdot 10^{16}$	$7.41 \cdot 10^{16}$
n_n	$1.29 \cdot 10^{15}$	$1.41 \cdot 10^{15}$	$1.28 \cdot 10^{15}$	$1.38 \cdot 10^{15}$
n_{sc}	$6.35 \cdot 10^{15}$	$7.87 \cdot 10^{15}$	$6.57 \cdot 10^{15}$	$8.16 \cdot 10^{15}$

Table 4: Results from different measurements of the vacuum composition. All cases have been normalized to a Nitrogen equivalent pressure of $8.0 \cdot 10^{-7}$ Pa. The No.1 densities are those used throughout the text.

B MATHCAD calculation file

A Mathcad file doing the calculations presented in this note has been compiled. The file takes the following inputs

- Vacuum composition
- Ion Gauge calibration
- Rest gas analyzer (RGA) calibration
- Emittance after last deceleration sequence (0.3 GeV/c $\rightarrow$ 0.1 GeV/c)
- Transverse acceptance of the storage ring.
- Average transverse beta function.

With these numbers in hand the program calculates the equivalent densities for multiple, nuclear and single Coulomb scattering, as well as the total beam lifetime in the machine.

The intermediate parts of the document includes the calculations from [1], thus if necessary one can extract more detailed information by investigating the details of the calculations performed.

The file can be found at the following location :

`G:\home\d\deskad\Mathcad\beamloss.mcd`

and is in Mathcad 6.0Plus format.

References

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[10] T. Eriksson, CERN PS/OP, Private Communications (1997)