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THE PSEUDO-OCTUPOLAR EFFECT IN THE AC AND AD RINGS

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The pseudo-octupolar effect in the AC and AD rings

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Introduction

Our aim is to establish to what degree a nonlinear effects associated with the end-field region of quadrupole magnets can account for the tune variation with the momentum deviation observed in the AC machine [1].

It is hoped that this analysis can be used to predict the importance of this effects for the AD ring and to suggest a cure if required.

We make use of two programs, AGILE [1] and MAD [2]. The results of the different codes are characterized by important discrepancies (typical of the subject treated) that, for the moment, we can only point out.

0.1 The experimental results

The fig. (0.1) shows the measured dependence [3] of the tunes Q_x, Q_y on the momentum deviation $\Delta p/p$ for the AC machine [2]. It is likely that the anomalous sextupole component has its origin in the wide bending magnets. As pointed out in [1], the parabolic shape of the horizontal tune can be due to the cubic component of the end-fields in quadrupoles.

We tried to reproduce these results numerically for the AC machine making use of AGILE and MAD. As a first step we used a lattice model which includes

[1] Alternating Gradient Interactive Lattice dEsign; see [3].
[2] Methodical Accelerator Design; see [4].
[3] Modeling neglecting octupoles.

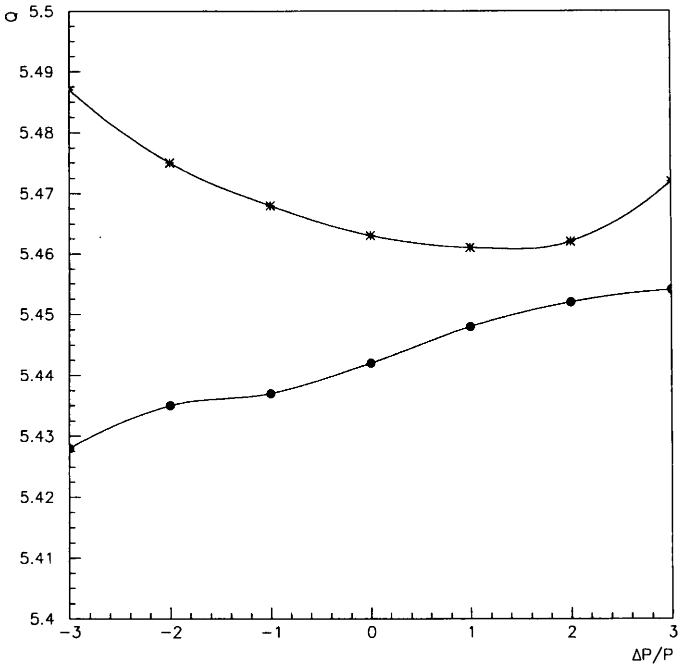

Figure 0.1: Measured tunes for the AC machine (see[2]): Q_x ($*$) and Q_y ($\bullet$) versus $\Delta p/p$ (%).

neither quadrupole stray fields nor other octupolar lenses. The resulting dependence of Q_x and Q_y on $\Delta p/p$ predicted by AGILE (fig. 0.2) and by MAD is displayed in figs. (0.2) and (0.3), respectively.

The difference between the two results is evident. We postpone the attempt to explain this inconsistency and we limit ourself to the observation that according to AGILE a nonzero second derivative of the tune with respect to the momentum deviation, seems to be present in the lattice even without the quadrupole end fields and octupole lenses. The shape of the curve given by AGILE has the same basic form as the measured one.

Making use of the Tracking module in MAD and analysing the data by a fast Fourier transform (FFT), we found the same behaviour of the tunes shown in fig. (0.3), that is a good agreement between the Twiss and the Tracking analysis in MAD.

In the vertical plane the result obtained with the Twiss command shows a good agreement with measured points even though the calculation does not include quadrupole end fields; the same is not true for the AGILE result (which agrees in essence with the curve shown in fig. (0.1) only for $\Delta p/p < 0$).

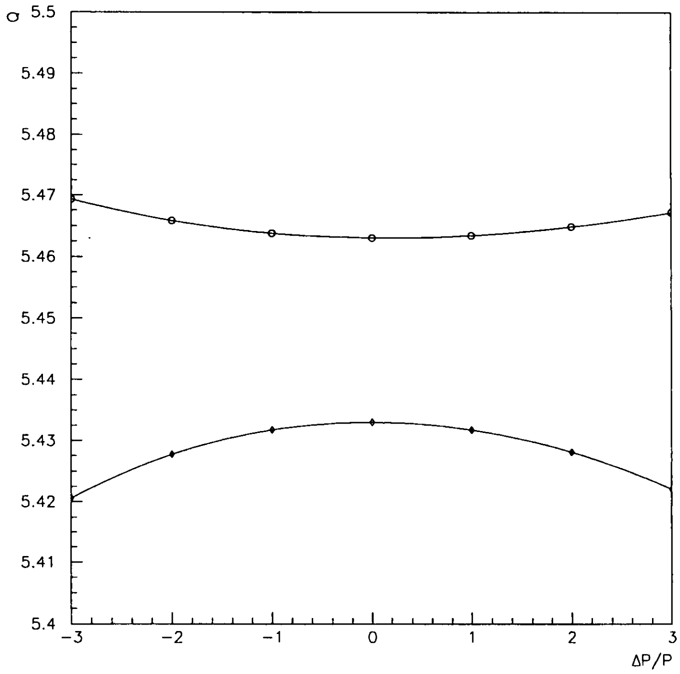

Figure 0.2: Tunes of AC machine given by AGILE: Q_x (∘) and Q_y (⋄) versus $\Delta p/p$ (%).

0.2 The model of the pseudo-octupoles

Making use of the Maxwell's equations $\nabla \wedge \mathbf{B} = \mathbf{0}$, $\nabla \cdot \mathbf{B} = \mathbf{0}$ and assuming, for a real quadrupole,

$$G = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = G(s) \tag{0.1}$$

one can demonstrate [5] that, up to the third order, the additional field components (respect to the hard edge model) due to the fringe field of a quadrupole lens can be expressed as

$$\begin{cases} B_x = \frac{1}{12} G'' y^3 + \frac{1}{4} G'' x^2 y \\ \\ B_y = \frac{1}{12} G'' x^3 + \frac{1}{4} G'' x y^2 \\ \\ B_s = G' xy \end{cases} \tag{0.2}$$

where $(G' \equiv \frac{\partial G}{\partial s})$. Due to the analogy with that generated by an octupole this field is called "pseudo-octupolar".

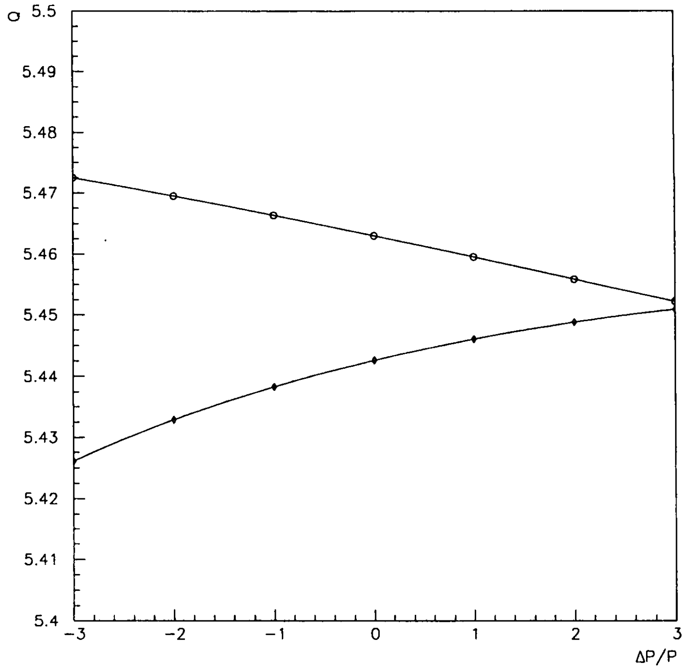

Figure 0.3: Tunes of AC machine given by MAD (Twiss): Q_x ($\circ$) and Q_y ($\diamond$) versus $\Delta p/p$ (%).

To take into account the pseudo-octupolar effect we follow the model proposed by P. Krejcik in [1]. Nevertheless the results we reached are different from those found by this author.

The simplest shape that can be attribuited to the end field (0.2) (characterized by a nonzero second derivative), is a quadratic dependence of the gradient G on s. We assume further that the gradient falls to zero within one aperture $2a$; then $G'' = \pm\frac{G_0}{a^2}$ where G'' has the same sign as G_0 (the gradient of the quadrupole in the hard-edge approximation) on the inner part of the quadrupole and opposite sign in the outer part.

Later on we neglect the effect of the longitudinal component of the field (0.2), that is $B(s)$. In fact, as pointed out by P. Belochitskii, considering the Lorentz force

$$\mathbf{F} = \mathbf{q}\mathbf{v} \wedge \mathbf{B} = \mathbf{q}(\mathbf{v_y B_s} - \mathbf{v_s B_y})\mathbf{i} + (\mathbf{v_s B_x} - \mathbf{v_x B_s})\mathbf{j} + (\mathbf{v_x B_y} - \mathbf{v_y B_x})\mathbf{s} \quad (0.3)$$

and evaluating the ratio $v_x B_s / v_s B_x$ (the same holds for $v_y B_s - v_s B_y$) we find ($v_s \simeq c \gg v_x, a \simeq x$)

$$\frac{v_x B_s}{v_s B_x} \simeq \frac{v_x}{v_s}\frac{G'/a}{G/a^2 x} = \frac{a}{x}\frac{v_x}{v_s} \ll 1. \quad (0.4)$$

It is worth noting that even if the contribution of B_s to the Lorentz force can in first approximation be neglected, the longitudinal field is of the same order as the two transverse components:

$$\frac{B_s}{B_{x,y}} \simeq \frac{a}{x} \simeq 1. \tag{0.5}$$

0.2.1 Bending and focusing of a pseudo-octupole

Assuming a zero dispersion in the vertical plane and a trajectory in the (x, s) plane of the form:

$$x = x_0 + x_0' s \tag{0.6}$$

(where x_0 and x_0' are, respectively, the position and the angle of the particle at the beginning of the region in which the pseudo-octupolar field is present), we can calculate the bending power δ_B and the focusing power δ_F due to the pseudo-octupolar field at each end of a quadrupole ($K_0 = \frac{1}{B\rho} G_0$):

$$\begin{aligned} \delta_B &\equiv \frac{1}{B\rho}\int B_y \mathrm{d}s = \pm\frac{1}{B\rho}\frac{G_0}{12a^2}\left[\int_{-a}^{0}(x_0+sx_0')^3\mathrm{d}s - \int_0^a (x_0+sx_0')^3\mathrm{d}s\right] = \\ &= \ldots = \mp\frac{1}{4}\frac{1}{B\rho}G_0[x_0^2x_0' + \frac{1}{6}x_0'^3a^2] = \mp\frac{1}{4}K_0[x_0^2x_0' + \frac{1}{6}x_0'^3a^2] \end{aligned} \tag{0.7}$$

$$\begin{aligned} (\delta_F)_x &\equiv \frac{1}{B\rho}\int \frac{\partial B_y}{\partial x}\mathrm{d}s = \pm\frac{1}{B\rho}\frac{G_0}{4a^2}\left[\int_{-a}^{0}(x_0+sx_0')^2\mathrm{d}s - \int_0^a (x_0+sx_0')^2\mathrm{d}s\right] = \\ &= \ldots = \mp\frac{K_0}{2}(x_0x_0') \end{aligned} \tag{0.8}$$

and $(\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y})$

$$(\delta_F)_y \equiv \frac{1}{B\rho}\int \frac{\partial B_x}{\partial y}\mathrm{d}s = \pm\frac{K_0}{2}(x_0x_0') \tag{0.9}$$

where the upper sign refers to the beginning (respect to the arbitrary positive direction of the s-axis) and the lower to the end of the quadrupole [4].

To insert the focusing effect in our lattice we have to subtract from (δ_F) the contribution due to each quadrupole in the hard edge approximation. The details of this calculation (still to be improved) are reported in the Appendix 2.

The result is a correction of $(\delta_F)_x$

$$[(\delta_F)_x]_{corr} = 2a(x_0')^2 \tag{0.10}$$

[4]In the paper of P. Krejcik the focusing power of the two ends of the quadrupole are found with the same sign. This seems to be a mistake.

while $(\delta_F)_y$ remains the same.

The expression of $(\delta_F)_x + [(\delta_F)_x]_{corr}$ is in agreement with the result found in [1] apart for a factor two in the correction term.

Comparing the two terms of the x-focusing power for the AC lattice

$$\frac{x_0 x_0'}{a(x_0')^2} \simeq \frac{D_x D_x'}{a(D_x')^2} = \frac{D_x}{aD_x'} \simeq 10^2 \qquad (0.11)$$

we conclude that our model does not depend significantly by the parameter a.

0.3 Numerical results

From the previous analysis we argue that the effect of the pseudo-octupolar field can be represented by placing at the two ends of each quadrupole a thin quadrupolar lens whose matrix is

$$(M)_{tl} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mp\frac{K_0}{2}[x_0 x_0' + a(x_0')^2] & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \pm\frac{K_0}{2}(x_0 x_0') & 1 \end{pmatrix} \qquad (0.12)$$

(the sign "-" holds for the entrance and the sign "+" holds for the exit).

The bending kick δ_B can be neglected since we are interested only in the tune correction up to the second order in $\Delta p/p$. In fact

$$\delta_F \propto (\frac{\Delta p}{p})^2 \qquad (0.13)$$

$$\delta_B \propto (\frac{\Delta p}{p})^3. \qquad (0.14)$$

0.3.1 AGILE results

Thanks to a modification introduced "ad hoc" by P. Bryant it is possible to insert in AGILE the pseudo-octupolar effect in the analytical way just specified.

Results for the horizontal plane are shown in fig. (0.4). The agreement with the experimental data is relatively good.

In the vertical plane, see figs. (0.5) and (0.6),

the effect [5] due to the finite extent of the fringing fields in dipoles is important. In the model developed in [6] this effect is specified by the quantities FINT and HGAP:

$$\text{FINT} = \int_{-\infty}^{+\infty} \frac{B_y(s)[B_0 - b_y(s)]}{gB_0^2} \mathrm{d}s \qquad (g = 2\text{HGAP}) \qquad (0.15)$$

[5] This effect does not influence the results of the horizontal plane.

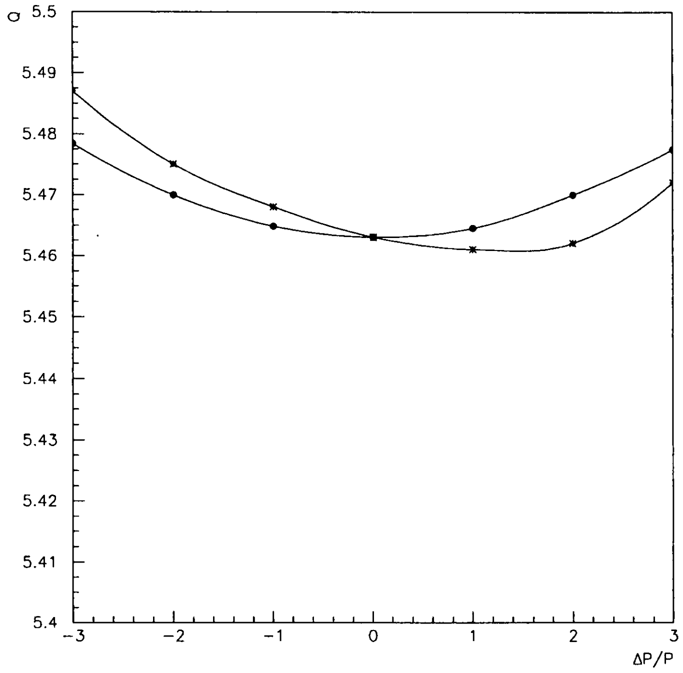

Figure 0.4: Horizontal plane: comparison between the measured (∗) curve and the theoretical one (•) given by AGILE adding pseudo-octupoles.

and HGAP=half of the dipole gap. The value FINT=0 corresponds to the hard edge approximation while, for example, the value FINT=1/6 corresponds to a linear drop-off of the field and the value FINT=0.45 to the square edged model.

We found that the AGILE's results in the vertical plane depend strongly on the choice of FINT.

In figs. (0.5) and (0.6) the curves are plotted for the hard-edge model and for the square-edged model. In both cases the theoretical results differ substantially from the experimental ones.

0.3.2 MAD results

For the moment is not possible to insert the pseudo-octupolar effect in MAD following the analytical way. Nevertheless, we managed to introduce it numerically using the model developped in the paragraph (0.2).

In the horizontal plane the results obtained using the Twiss command [6] are shown in fig. (0.7).

The agreement with the AGILE result is very good for $\Delta p < 0$. We find again

[6] The program so far does not allow a tracking analysis with the pseudo-octupoles include.

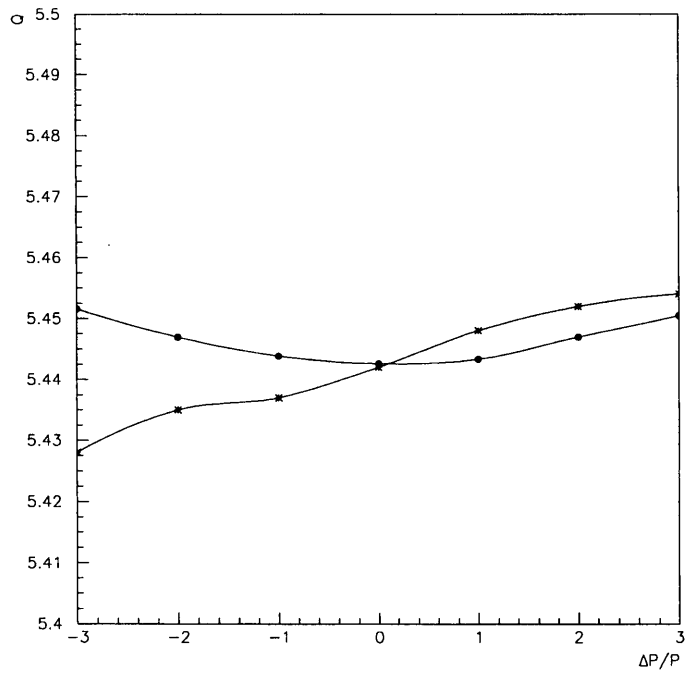

Figure 0.5: Vertical plane: comparison between the measured (∗) curve and the theoretical one (•) given by AGILE adding pseudo-octupoles. FINT=0.

that the effect goes in the right direction but it seems to be too weak to fit the measured curve.

In the vertical plane, as explained in the previous paragraph, the theoretical results are influenced by the choice of the parameter FINT. In this case this influence is less strong than in the case of AGILE but involve a shift of the theoretical curve. We show the plots for FINT=0 and FINT=0.45.

The agreement is better for the hard edge approximation.

0.4 Compensation

To compensate the pseudo-octupolar effect we introduced in the AC lattice one real octupole in a position with maximum dispersion (after the first QF8).

As shown in figs. (0.10) and (0.11) we obtained a good compensation using the integrated strengths $|K_3L| = 2\ \mathrm{m}^{-3}$ for AGILE and $|K_3L| = 1\ \mathrm{m}^{-3}$ for MAD. These values are much smaller then the typical strength that can be obtained with octupoles in AD. In fact starting from the expression of the octupolar strength

$$K_3 = \frac{1}{B\rho}\frac{\partial^3 B_y}{\partial x^3} \simeq \frac{6}{B\rho}\frac{B}{d^3} \qquad (d = \text{half aperture of the octupole}) \qquad (0.16)$$

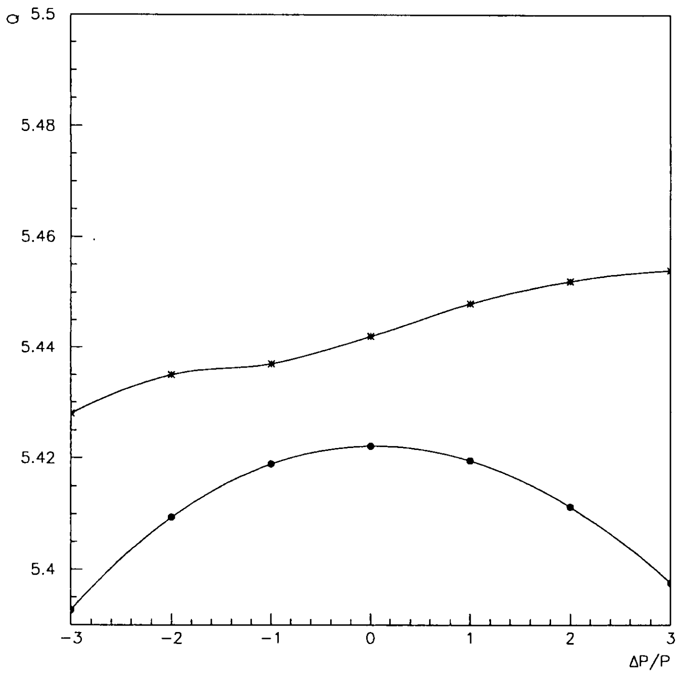

Figure 0.6: Vertical plane: comparison between the measured ($*$) curve and the theoretical one ($\bullet$) given by AGILE adding pseudo-octupoles. FINT=0.45.

and using the formula

$$\frac{1}{\rho} = 0.2998 \frac{B[\mathrm{T}]}{p[\frac{\mathrm{GeV}}{\mathrm{c}}]} [\mathrm{m}]^{-1} \tag{0.17}$$

we find, for p=3.57 Gev/c (the momentum in AD at injection),

$$(B\rho)^{-1} \simeq \frac{1}{12} (\mathrm{Tm})^{-1}. \tag{0.18}$$

Assuming B=0.5 T, d=0.15 m, L=0.2 m, we get the value of the integrated strength

$$K_3 L \simeq 15\mathrm{m}^{-3}. \tag{0.19}$$

0.5 Conclusion

The second order calculation of chromaticity in small machines is an open problem. The fact that it has not been possible to exactly riproduce the measured tunes of the AC is a serious problem for the confidence in the AD design.

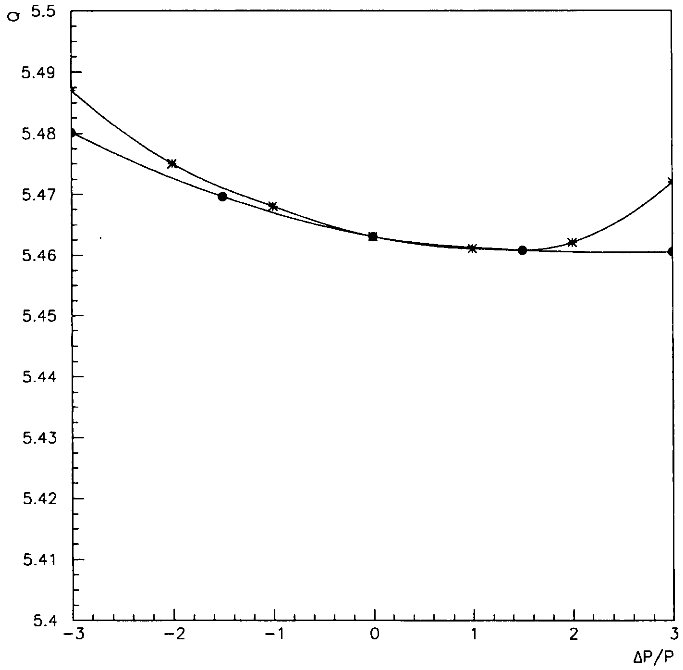

Figure 0.7: Horizontal plane: comparison between the measured (∗) curve and the theoretical (•) one given by MAD adding pseudo-octupoles.

For the benefit of the AD project it will be necessary to have an improvement of the understanding of the different approximations used by the two programs that follow a rather different philosophies for the off-axis orbit calculations.

In spite of these difficulties, we are able to draw some conclusion.

Following the model proposed in [1], we succeded in introducing in the AC and AD lattices the effect of the fringing field of quadrupoles. In this way we improved significantly the agreement between numerical and experimental data for the AC in the horizontal plane.

For the vertical plane the problem is more complex because in this case the choice of the model of the fringe field of dipoles became also important.

The cure of the pseudo-octupolar effect in the AD can be done using a real octupole with the characteristics shown in paragraph (0.4).

Acknowledgments

This work is leaded by the experience and disposability of P. Belochitskii, P. Bryant, S. Maury and D. Mohl.

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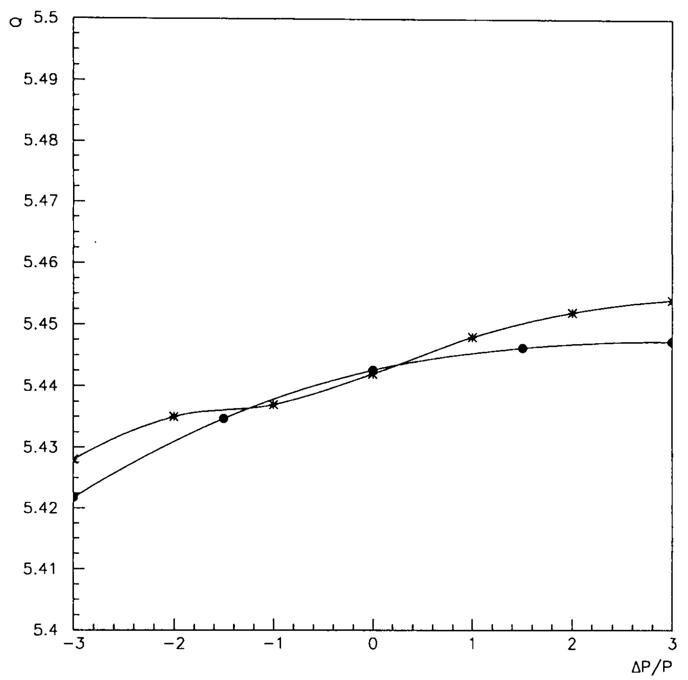

Figure 0.8: Vertical plane: comparison between the measured ($*$) curve and the theoretical one ($\bullet$) given by MAD adding pseudo-octupoles. FINT=0.

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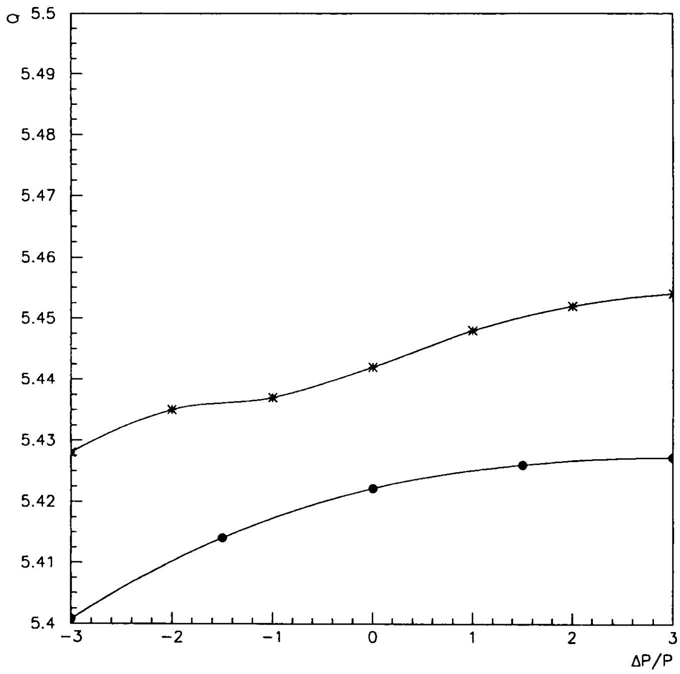

Figure 0.9: Vertical plane: comparison between the measured ($*$) curve and the theoretical one ($\bullet$) given by MAD adding pseudo-octupoles. FINT=0.45.

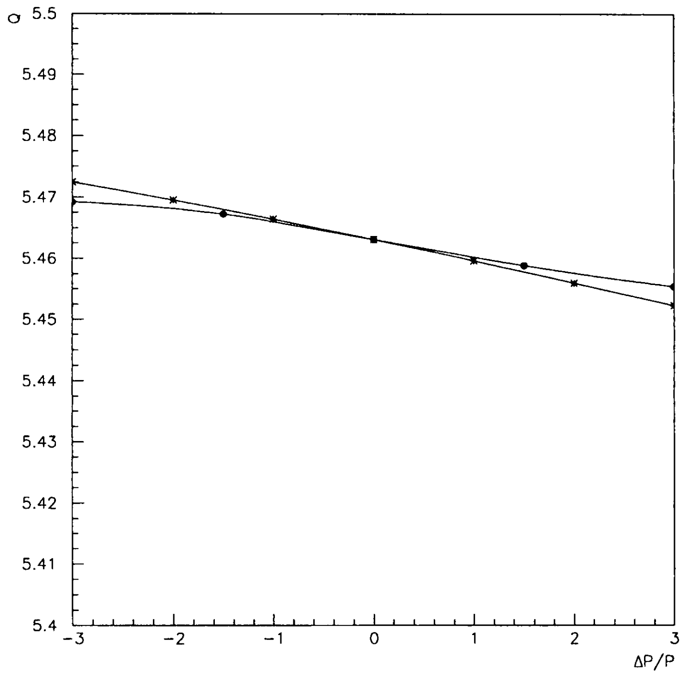

Figure 0.10: MAD program: comparison between the curve without pseudo-octupoles ($*$) and the compensated one ($\bullet$) for the horizontal plane.

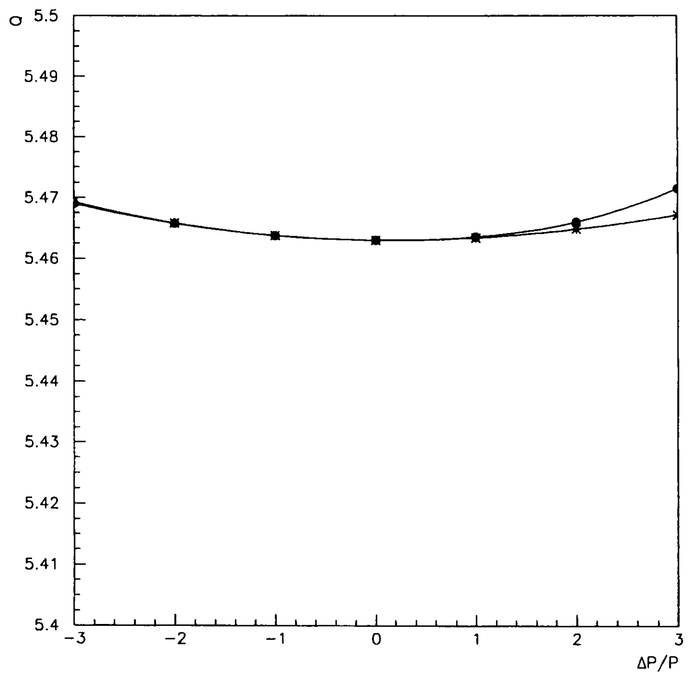

Figure 0.11: AGILE program: comparison between the curve without pseudo-octupoles ($*$) and the compensated one ($\bullet$) for the horizontal plane.

Appendix 1

Comparison between the "square-edged" model for the fringing field and a measurement for a LEAR quadrupole

The aim is to evaluate the ratio

$$\frac{\alpha}{\beta} = \frac{\int_{-a}^{0} K''\mathrm{d}s}{\int_{-a}^{0} \frac{K_0}{a^2}\mathrm{d}s} = \frac{\frac{2}{B\rho}\int_{-a}^{0} \frac{\partial^3 B_x}{\partial x^3}\mathrm{d}s}{\frac{1}{B\rho}\int_{-a}^{0} \frac{G_0}{a^2}\mathrm{d}s} \tag{0.20}$$

where α is the integral of the strength of a pseudo-octupole in the region in which the field raises from zero to the half of the internal value for the experimental mesure (see [3.]), and β is the same quantity calculated following the model shown in (0.2).

From the plot in fig. 1 of [3.] we got (after numerical integration)

$$\int_{-a}^{0} \frac{\partial^3 B_x}{\partial x^3}\mathrm{d}s \simeq 10.45 \ \ \mathrm{T/m^2} \tag{0.21}$$

and G_0=3 T/m.

Assuming $a \simeq 0.073\ m$ we find

$$\frac{\alpha}{\beta} = \frac{2\int_{-a}^{0} \frac{\partial^3 B_x}{\partial x^3}\mathrm{d}s}{G_0\frac{a}{a^2}} \simeq 0.5 \tag{0.22}$$

On the basis of this rough existimation the model seems to imply an overexistimation (about 50%) of the pseudo-octupolar effect.

Appendix 2

Focusing power of a pseudo-octupole

As reported in (0.2) the pseudo-octupole is associated to a focusing power

$$|\delta_F| = \frac{K_0}{2}(x_0 x_0') \tag{0.23}$$

To insert the effect of $|\delta_F|$ in our lattice we have to take into account the fact that by default the lattice programs deal with quadrupoles using the hard edge approximation. This fact implies that have to subtract to $|\delta_F|$ the contribution to the focusing power of the quadrupole in the hard edge approximation in the "raising" (or "lowering") region $[-a, a]$.

In the plane $y = 0$ (we assume a zero vertical dispersion) the correction for the horizontal focusing is

$$\begin{aligned} [(\delta_F)]_{corr} &= \frac{1}{B\rho}\int_0^a \frac{\partial B_y}{\partial x}\frac{\mathrm{d}s}{\cos x_0'} = \\ &= \frac{G_0}{B\rho}a\frac{1}{\cos x_0'} \simeq \frac{Q_0}{B\rho}a\frac{1}{1-\frac{(x_0')^2}{2}} \simeq \frac{Q_0}{B\rho}a[1+(x_0')^2] = \\ &= K_0 a[1+(x_0')^2]. \end{aligned} \tag{0.24}$$

Comparing the total focusing power $(\delta_F)_x - [(\delta_F)_x - [(\delta_F)_x]_{corr}$ with the one found in [1] by Kreycik

$$[(\delta_F)_x]_{Kr.} = -\frac{K_0}{4}[2x_0 x_0' + (x_0')^2] \tag{0.25}$$

we see that our result differs from the Kreycik's one for an extra-factor $4a$ and for a factor 2 in the correction term. In particular the extra-factor would be dominant leading to a too strong effect (as we tested using AGILE). For this reason we neglected "a fortiori" this term in our simulations. This calculation has to be evidently improved.

The factor $1/\cos x_0'$ takes into account the fact that the trajectory followed by the particle is not parallel to s.

In the vertical plane this factor is compensated by a term $\cos x'_0$ that comes out because the field seen by the particle is $B_x \cos x'_0$. Hence in the vertical plane the correction term is zero (neglecting also in this case a factor $4a$).

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