EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE

CERN - PS DIVISION

PS/ HP/ Note 98-15
AD Note043

THE LINEAR COUPLING CORRECTION IN THE AD MACHINE

G. De Ninno

A general overview of the two different theories dealing with linear coupling is given. These formalisms are both applied to the correction of the main coupling source in the AD machine, that is the presence at the electron cooling of a solenoid fed with a constant current during the deceleration from p=300 MeV/c to p=100 MeV/c. The simple configuration adopted for the LEAR (two corrector solenoids adjacent to the main one) is not yet possible and a number of different possibilities having at disposal an additional pair of correctors (skew quadruples) are investigated.

Geneva, Switzerland
30 November 1998

The linear coupling correction in the AD machine

by

G. De Ninno

1 Introduction

The linear coupling is driven:
• by solenoidal fields that can be produced by detectors installed at the interaction points in circular colliders or, as in the case of AD, by an electron cooling system necessary to produce the dense beam required by the experiments at low energy;
• by the skew quadrupole component arising from imperfections in the fabrication and alignement of the magnets;
• by vertical closed orbit excursion in sextupole fields.
The most important consequence of linear coupling is the fact that it drives the difference resonances $Q_x - Q_z = integer$ and the sum resonances $Q_x + Q_z = integer$.
The first ones do not lead to instability but cause amplitude beating and energy exchange between the two transverse directions.
This phenomenon produces a "round" beam that is not suitable for a wide number of accelerators and prevents operating these machines close to the main diagonal in the tune diagram (where the resonance-free tune space is larger), unless a compensation is provided.
In fact the interchange of horizontal and vertical oscillations is not to be necessarily avoided: the Mobius accelerator [1] is an example of the way the coupling can be exploited to modify (and in some case improve) the property of an existing circular accelerator.
The sum resonances lead to instability. Also if the tunes of accelerators are usually far away from it, its effect shows up in a distorsion of the generalized orbit functions and in the planes of the eigenmodes of the coupled transverse motion not being horizontal and vertical.
In computer tracking it has been observed [2] that linear coupling reduces the dynamic aperture and modifies the strengths of the nonlinear resonances close to the working point: it reduces the strength of resonances excited to the first order when it is absent and increases the strength of the other ones.

The AD is forseen to be commissioned with the working point $(5.39, 5.37)$; the closest cou-

pling resonances are $Q_x - Q_z = 0$ and $Q_x + Q_z = 11$.
The criteria that can be used for the correction of the coupling effects are essentially two:
- Following the matrix formalism ([3], [4], [5]) one can block- diagonalize the 4×4 matrix for the transverse motion between one or more points around the ring.
This requires four families of correctors (solenoids and/or skew quadrupoles) for each point where the block diagonalization is to be achieved. In the particular case of AD, for the compensation in one particular point of the off-axis blocks of the one turn matrix, due to symmetry and do considering field errors, two families are sufficient.
- Following the Hamiltonian formalism ([6], [7]) one can cancel the driving terms for the difference and sum resonances close to the working point. This compensation requires two families of correctors for each resonance to be removed. In the particular case of AD, due to symmetry and not taking into account errors, one can compensate the closest difference and sum resonances making use of two families.
As it will be pointed out these two approaches lead to the same results only in special cases.

2 The matrix formalism

The matrix approach to deal with coupling is the generalization of the Courant-Snyder parametrization to the two-dimensional linear coupled motion.
The single-turn transfer matrix

$$\mathbf{T} = \begin{pmatrix} M & n \\ m & N \end{pmatrix} \tag{1}$$

of a periodic system is symplectic and periodic; this allows to express it in terms of 10 periodic parameter. A way to proceed is to make use of a "symplectic rotation" [3]:

$$\mathbf{T} = \begin{pmatrix} I\cos\phi & D^{-1} \\ -D\sin\phi & I\cos\phi \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} I\cos\phi & -D^{-1}\sin\phi \\ D\sin\phi & I\cos\phi \end{pmatrix} \equiv \mathbf{RUR^{-1}} \tag{2}$$

where A, B and D are 2×2 unimodular (symplectic) matrices each of which requires 3 parameters; the tenth parameter is the angle ϕ. I is the unit 2×2 matrix. The matrix $\mathbf{R}$ defines a canonical transformation from the coupled coordinates $\overrightarrow{x}$ to a new set of coordinates $\overrightarrow{v}$:

$$\overrightarrow{v} = \mathbf{R^{-1}}\overrightarrow{x} = (\mathbf{u}, \mathbf{p_u}, \mathbf{v}, \mathbf{p_v}) \tag{3}$$

such that the motion described by (u, p_u, v, p_v) is decoupled;
$\mathbf{U}$ is the single-turn matrix for $\overrightarrow{v}$. It is natural to parametrise A and B in Courant-Snyder form:

$$A = I\cos\mu_1 + J_1\sin\mu_1 \tag{4}$$

$$B = I \cos \mu_2 + J_2 \sin \mu_2 \tag{5}$$

where

$$J_{1,2} = \begin{pmatrix} \alpha_{1,2} & \beta_{1,2} \\ -\gamma_{1,2} & -\alpha_{1,2} \end{pmatrix}. \tag{6}$$

The phase advances μ_1 and μ_2 relate in the usual way to the eigenvalues of A and B. Due to simplecticity, if λ is an eigenvalue of T the same holds for λ^{-1}; supposing $|\lambda| = 1$ (linear stability of the motion) one can show that [4]

$$\lambda_1 + \lambda_1^{-1} = e^{i\omega_1} + e^{-i\omega_1} = 2\cos\omega_1 \tag{7}$$

$$\lambda_2 + \lambda_2^{-1} = e^{i\omega_2} + e^{-i\omega_2} = 2\cos\omega_2. \tag{8}$$

The 10 periodic parameters specified by [3] can be expressed in terms of the elements of $\mathbf{T}$. In particular

$$(\cos\mu_1 - \cos\mu_2)^2 = (\mathrm{Tr}M - \mathrm{Tr}N)^2/4 + \det|m + \overline{n}| \tag{9}$$

($\overline{n}$ is the symplectic conjugate of n)

$$\cos 2\phi = \frac{1}{2}\frac{\mathrm{Tr}(M-N)}{(\cos\mu_1 - \cos\mu_2)}. \tag{10}$$

From eq. (9) it can be seen that the sign of $\det|m + \overline{n}|$ has a special importance: if it is negative and the traces of M and N are equal, then μ_1 and (or) μ_2 are complex and the motion become unstable. This can happen in correspondence of a difference resonance $Q_x - Q_z = p$ (p integer) or of a sum resonance $Q_x + Q_z = p$.
The difference resonance is shown to be inherently stable and the sum resonance is inherently unstable. This fact allows to run an accelerator near to the bisecting line in the (Q_x, Q_z) plane (close to which the "room" free of nonlinear resonances is large).
The eigenvectors of A and B correspond to the normal modes of the motion. Such a motion is not restricted to a single plane but rather the phase point moves on an ellipse in the (x, z) space.

3 The Hamiltonian formalism

The Hamiltonian of the linear betatron motion coupled by the presence of a skew quadrupole and solenoidal field components is

$$H = \frac{1}{2}[K_1 x^2 + K_2 z^2 + 2Kxz + (p_x - Sz)^2 + (p_z + Sx)^2] = H_0 + U \tag{11}$$

where

$$K_{1,2} = \frac{R^2}{B\rho} G_{1,2} \tag{12}$$

(with $G_{1,2}$ field gradients),

$$K = \frac{R^2}{B\rho} \frac{\partial B_x}{\partial x} \tag{13}$$

is the skew quadrupole strength and

$$S = \frac{R}{2B\rho} B_s \tag{14}$$

is the solenoidal field strength.
The perturbation due to coupling is given by

$$U = Kxz + Sxp_z - Sxp_x + \frac{1}{2} S^2 (x^2 + z^2). \tag{15}$$

According to Floquet's theorem the solution of the unperturbed motion can be written in the form

$$x = a_1 u e^{iQ_x \theta} + \bar{a}_1 \bar{u} e^{-iQ_x \theta} \tag{16}$$

$$z = a_2 v e^{iQ_z \theta} + \bar{a}_2 \bar{v} e^{-iQ_z \theta} \tag{17}$$

together with $p_x = \mathrm{d}x/\mathrm{d}\theta$, $p_z = \mathrm{d}z/\mathrm{d}\theta$, where $Q_{x,z}$ are the transverse tunes, θ the azimuthal angle and u, v the Floquet functions

$$u(\theta) = \sqrt{\beta_x(\theta)/2R} \cdot \exp[i \int_0^\theta (\frac{R}{\beta_x(\xi)} - Q_x) \mathrm{d}\theta] \tag{18}$$

$$v(\theta) = \sqrt{\beta_y(\theta)/2R} \cdot \exp[i \int_0^\theta (\frac{R}{\beta_y(\xi)} - Q_y) \mathrm{d}\theta]. \tag{19}$$

The general solution of the perturbed motion can be obtained studying the variation of the four costants a_1, a_2, $\bar{a}_1$, $\bar{a}_2$ due to the perturbation H_1:

$$\dot{a}_j = [a_j, U] \qquad \text{([...]is the Poisson bracket).} \tag{20}$$

The function U is periodic and can be developed in a Fourier series; the low frequency part of it (supposed to give the main contribution to the variation of the costants [1]) can be written

$$\begin{aligned} U &= h^{(2)}_{11000} a_1 \bar{a}_1 + h^{(2)}_{00110} a_2 \bar{a}_2 + \\ &+ h^{(2)}_{1010-p} a_1 a_2 \mathrm{e}^{i(Q_x+Q_z-p)\theta} + \text{c.c.} + \\ &+ h^{(2)}_{1001-p} a_1 \bar{a}_2 \mathrm{e}^{i(Q_x-Q_z-p)\theta} + \text{c.c.} + \\ &+ h^{(2)}_{2000-p} a_1^2 \mathrm{e}^{i(2Q_x-p)\theta} + \text{c.c.} + \\ &+ h^{(2)}_{0020-p} a_2^2 \mathrm{e}^{i(2Q_z-p)\theta} + \text{c.c.} \end{aligned} \tag{21}$$

where c.c. stands for complex conjugate. The terms containing

$$h^{(2)}_{11000,00110} = \frac{1}{8\pi R} \int_0^{2\pi} S^2 \beta_y \mathrm{d}\theta \tag{22}$$

($y = x$ for $h^{(2)}_{11000}$ and $y = z$ for $h^{(2)}_{00110}$) cause tune shifts with amplitude whereas the terms containing

$$\begin{aligned} h^{(2)}_{1001-p,1010-p} &= \frac{1}{4\pi R} \int_0^{2\pi} \sqrt{\beta_x \beta_z} \left[K + RS \left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_z}{\beta_z} \right) - iRS \left(\frac{1}{\beta_x} \pm \frac{1}{\beta_x} \right) \right] \times \\ &\times \ \exp \left\{ i \left[\mu_x \mp \mu_z \right) - (Q_x \mp Q_z - p)\theta \right] \right\} \mathrm{d}\theta \end{aligned} \tag{23}$$

(where the sign has to be chosen according to the indices of $h^{(2)}$: top sign for $h^{(2)}_{1001-p}$ and lower sign for $h^{(2)}_{1010-p}$) lead to sum and difference coupling resonances.
The terms with the coefficient $h^{(2)}_{2000-p}$ and $h^{(2)}_{0020-p}$ depend only on S and lead to one-dimensional quadrupole resonances. Introducing the expression of U for one resonance at time in the equation (20) one gets

$$\frac{\mathrm{d}a_1}{\mathrm{d}\theta} = i\lambda_x a_1 + i f_a \mathrm{e}^{-i\theta\Delta} \tag{24}$$

$$\frac{\mathrm{d}a_2}{\mathrm{d}\theta} = i\lambda_z a_2 + i g_a \mathrm{e}^{\pm i\theta\Delta} \tag{25}$$

where $\lambda_x = h^2_{11000}$, $\lambda_z = h^2_{00110}$ (tune shifts),

$$f_a = \frac{1}{2} \bar{C}^- a_2, \quad g_a = \frac{1}{2} C^- a_1 \tag{26}$$

[1] This is the first and more delicate approximation of this method.

(for the difference resonance),

$$f_a = \frac{1}{2}\bar{C}^+\bar{a}_2, \quad g_a = \frac{1}{2}\bar{C}^-\bar{a}_1 \tag{27}$$

(for the sum resonance), $C^+ = 2h_{1010-p}$, $C^- = 2h_{1001-p}$ (driving terms of the sum and difference resonances, respectively) and $\Delta \equiv \Delta^{\mp} = Q_x \mp Q_z - p$ for the difference and sum resonances.

Solving the equations (26) and (27) near a single difference resonance [2] the solution of the motion can be written

$$a_1 = \frac{1}{2}\bar{C}\left(\frac{A_1}{\omega_2}\mathrm{e}^{i\omega_{x2}\theta} + \frac{A_2}{\omega_1}\mathrm{e}^{i\omega_{x1}\theta}\right) \tag{28}$$

$$a_2 = A_1\mathrm{e}^{-i\omega_{z2}\theta} + A_2\mathrm{e}^{-i\omega_{z1}\theta} \tag{29}$$

where

$$\omega_{x1,2} = \frac{1}{2}\left[-(\Delta^- - \lambda_x - \lambda_z) \pm \sqrt{(\Delta^- + \lambda_x - \lambda_z)^2 + |C^-|^2}\right] \tag{30}$$

$$\omega_{z1,2} = \frac{1}{2}\left[-(\Delta^- + \lambda_x + \lambda_z) \mp \sqrt{(\Delta^- + \lambda_x - \lambda_z)^2 + |C^-|^2}\right]. \tag{31}$$

This equations indicates that in this case the motion is always stable with energy exchange between the two transverse direction.

The solution near a (single) sum resonance is analogous to the previous one but this time the motion is stable only if

$$|C^+| \leq |\Delta^+ + \lambda_x + \lambda_z|. \tag{32}$$

The equations (28)-(31) show that the perturbated motion is made up of two modes associated with two different frequencies ω_1, ω_2. It can be shown [7] that these modes have an elliptical shape and are inclined at an angle α respect to the (x, z) system. The area of the ellipses can be related to the intensity of the coupling generated by the solenoidal field (so that if S vanishes the area is zero) and the tilt of the eigenplanes can be related to the intensity of the coupling due to skew quadrupoles (if K vanishes α is zero).

In practice [8] a given working point (Q_x, Q_y) can be considered far enough from the most close sum and difference resonances when the following conditions are both satisfied

$$|Q_x - Q_y - p_1| \geq \frac{1}{2}\frac{\Delta e^-}{\delta} = \frac{2C^-}{\delta} \tag{33}$$

[2] This is the second approximation assumed in the Hamiltonian approach.

$$|Q_x + Q_y - p_2| \geq \frac{1}{2}\Delta e^+(1+\frac{1}{\delta}) = 2C^+(1+\frac{1}{\delta}), \quad (34)$$

where p_1 and p_2 are integer the most close to $Q_x - Q_y$ and $Q_x + Q_y$ (respectively),

$$\Delta e^- = 4C^- \quad (35)$$

is the bandwidth for the difference resonance,

$$\Delta e^+ = 4C^+ \quad (36)$$

is the bandwidth for the sum resonance and δ is the (small) tolerable beam blow up with respect to the umperturbed beam amplitude.
All the formulae (33), (34), (35) and (36) refer to the semplified case in which the horizontal and the vertical emittances are almost equal.

4 Comparison between the two formalism

It has been shown that both approaches describe the main characteristic of the betatron motion in presence of coupling; nevertheless they follow two completly different philosophies and this has to be taken into account for coupling compensation.
The matrix formalism is exact and permits to redifine the Twiss parameter when the coupling is present. The block-diagonalization of the one turn matrix involves the global compensation of all the sum and difference resonances for one point. This means that the effect of the nearest sum and difference resonances is not compensated individually but together with the contribution of all other resonances. If the latter is small one can expect a small residual value of driving terms of the closest resonances (that is agreement between the two criteria of compensation), but if the contribution of the "far" resonances is large, then the one turn matrix compensation leaves a residual value of the closest driving terms significantly different from zero (that means disagreement between the two methods).
The Hamiltonian approach contains approximations leading to a definition of tunes that is meaningful only if the working point is close to a resonance excited by coupling and remote from the others.
The tunes show a "fine structure" not shown by the matrix formalism.
This tune splitting is caused by the presence of solenoidal fields (see eqs. (30) and (31)) and so, even if the working point is very close to one particular resonance, the comparison between the formulae for tunes given by the two approaches can be made only if the coupling is due to a skew quadrupolar field.
The general link between the different definitions of tunes given by the two formalism is an open problem [9].
The Hamiltonian formalism gives an indication for the best position of the corrector magnets. Supposing in fact that one wants to cancel one particular resonance, the aim is to create in the

plane $(\mathrm{Re}(C), \mathrm{Im}(C))$ a vector exactly compensating the one generated by the coupling source. The combination of the two correctors to be used is most effective when the phase difference between the vectors they generate is close to $\pi/2$.

5 Compensation of the coupling due to the AD electron cooling solenoid

The main coupling source in AD is the presence at the electron cooling (see AD layout) of a solenoid that is fed with a constant current[3] I=400 A during the deceleration from p=300 MeV/c to p=100 MeV/c. This implies that not only the optics used at high energy must be modified at low energy (because of the coupling correction) but also that it is necessary to vary the low energy optics for the different values of momentum.
The strength of the electron cooling solenoid is most critical (and the coupling most dangerous) for p=100 MeV/c and in the following the compensation strategy that can be adopted for this case is illustrated.
The available correctors forseen for the start up of the machine are two skew quadrupoles (connected in series) and two solenoids (connected in series with the electron cooling solenoid).
In paragraph 5.1 the future possibility of having one independent power supplies for the two corrector solenoids togheter with that of varying the position of the two skew quadrupoles in section 14 and 44 (see AD layout) is considered. This would allow a perfect coupling compensation (in the sense of matrix approach) in one region of the lattice and a minimization of the driving terms of the nearest sum and difference resonances.
In paragraph 5.2 the possibility to replace the two solenoids with an other pair of skew quadrupoles is illustrated.
Finally in paragraph 6 we come back to the case of the two corrector solenoids connected in series with the main one with the two skew quadrupoles in a positon given by the configuration required for the AD start up.
Another source of coupling to be taken into account [10] is due to errors, namely, tilt of normal quadrupoles and vertical orbit distorsion in sextupoles.

To get the compensation of the coupling generated by the electron cooling solenoid (whose center coincides with the symmetry point of the machine) the ideal case should be to place two corrector solenoids adjacent to it. This configuration, used for the LEAR, would allow an almost perfect compensation according to the relation

$$k_c l_c + kl = 0 \tag{37}$$

[3]From the point of view of the machine operation it would be better to keep the solenoidal strength $k = eB_0/p$ constant. This requires ramping of the solenoidal field which complicates the operation. In addition a strong field is desired for the dynamics of the electron cooling at 100 MeV/c.

where k_c and l_c are the strength and the total length of the correctors and k and l are the ones of the main solenoid.
Unfortunatly, with the hardware recuperated from LEAR and AC this would cause a reduction of machine acceptance of about 30% as the quadrupoles placed on both sides of the cooling insertion would have to be further away from the electron cooler. As a consequence the corrector solenoids will be housed several meters upstreams and downstreams of the main solenoid and to compensate the off-axis elements of the one turn matrix or the two closest resonances an additional pair of correctors is needed (skew quadrupoles).
The exact position of the two solenoids is thus determined by the space required by the insertion quadrupoles while the skew quadrupoles can be placed between QDN13(45) and QFN14(44). For the positioning of both pairs of correctors the symmetry of the machine has been kept.
The one turn matrix in correspondence of the middle point of QDS01 and the driving terms of the nearest sum and difference resonances ($Q_x - Q_y = 0$ and $Q_x + Q_y = 11$) when the electron cooling solenoid is switched on (no coupling compensation) are

$$\mathbf{T}_{coup} = \begin{pmatrix} -0.81 & 1.56 & -0.17 & 1.09 \\ -0.18 & -0.81 & -0.03 & -0.21 \\ 0.21 & -1.09 & -0.70 & 7.65 \\ 0.03 & 0.17 & -0.06 & -0.70 \end{pmatrix} \tag{38}$$

$$C^+ = 0.0314, \qquad C^- = 0.0894. \tag{39}$$

These two values have to be compared with the conditions (33) and (34). Considering as tolerable a beam blow of about 20% for both the sum and the difference resonances the (33) and (34) give

$$|Q_x + Q_y - 11| \geq 10C^+ \longrightarrow C^+ < 0.024 \tag{40}$$

$$Q_x - Q_y \geq 10C^- \longrightarrow C^- < 0.002. \tag{41}$$

While C^+ given by eq. (39) is not far to satisfy the previous stability requirement the same does not hold for C^-. Turning on the main solenoid and the coupling correctors the β functions are perturbed (see eq. (23)). This causes a tune shift that can be compensated using normal quadrupoles.
The values of the tunes when the main solenoid is not compensated are $Q_1 = 5.44$ and $Q_2 = 5.35$[4].

[4]The tunes are here called Q_1 and Q_2 and not Q_x and Q_y because (see paragraph 2) in presence of coupling the motion is not restricted to the (x, p_x) and (y, p_y) planes.

5.1 One turn matrix compensation using two independent solenoids and two skew quadrupoles

To choose the best position of the skew quadrupoles in the straight sections (see AD layout) between QDN13(45) and QFN14(44) three different configurations have been considered: one with the skew quadrupoles as close as possible to QDN13(45), one with the skew quadrupoles in the middle and one with the skew quadrupoles as close as possible to QFN14(44).
The strategy is the following: using the code MAD [11] the off-axis elements of the 4×4 matrix (using the coupling correctors) and the consequent tune shifts (leaving two families of normal quadrupoles free to vary) are compensated at the same time. The resulting variation of the β functions reintroduces the coupling and a new set of values of the strengths of the correctors is needed. A new tune shift is introduced. Instead of using an (converging) iterative procedure one can ask the program to correct both the coupling and tune shift at the same time.
As it has been pointed out in paragraph 2 the matrix correction is local whereas the compensation of the resonances does not depend on the position of the observation point along the ring. This implies that the position where the correction of the off-axis elements of the 4×4 matrix is applied has to be carefully chosen. In our case the requirements taken into account are two: the attempt to have pure horizontal and vertical modes in the position where tunes will be measured and in the electron cooling region (where a non ambigous definition of the β functions is needed). According to this, a good reference point for the matrix correction is the middle of straight section 1 (that is the center of the quadrupole QDN01): after the compensation, the one turn motion referred to any position in the arc (QDN45, QDN13) is decoupled and so the mesurement of the tunes [5] is not affected by coupling; the motion in the arc (QDN13, QDN45) turns out to be coupled but, due to symmetry, the coupling is minimum in the middle of the electron cooling region.
Once the coupling (in the sense of the matrix formalism) and tune shift compensation are established the new strengths of the correctors are put into the code AGILE [12] to evaluate the residual values of the driving terms of the most close resonances ($Q_x - Q_z = 0$ and $Q_x + Q_z = 11$). For all the three configurations an accetable values of the driving terms C^+ and C^- have been found. Taking into account other important "quality factors" like flexibility (that is the number of different good solution varying the quadrupole families used for the tune shift correction) and smallness of the perturbation of the corrected optics with respect to the umperturbed one, the configuration with the skew quadrupoles as close as possible to QFN14(44) appears preferable. In particular the QFN14(44) configuration is characterized by a wide number of good solutions; two of them are shown in tab.1. The choice of the families KF6 and KDEC to compensate the tune shifts is the most natural because the high sensitivity of Q_x to the change of KF6 and of Q_z to the change of KDEC. A good alternative "knob" to KF6 is KFEC. The other two configurations are less flexible and in particular the QDN13(45) one allows for a good solution only using KF79 and KF8 as tune shift "knobs" (see tab.1). The QFN14(44) configuration is also characterized by a relatively small value of the strength $|KSKEW|$ of the skew quadrupoles

[5] With pick-ups and kickers located in this part of the machine

Conf	KSOLG (rad/m)	KSKEW (m^{-2})	$\|C^-\|$	$\|C^+\|$	$\Delta K_1(m^{-2})$	$\Delta K_2(m^{-2})$
1	$-3.283 \cdot 10^{-1}$	$\pm 8.69 \cdot 10^{-2}$	10^{-4}	$2.45 \cdot 10^{-2}$	$\Delta K_{KF6} = 4.6 \cdot 10^{-3}$	$\Delta K_{KDEC} = 7.6 \cdot 10^{-3}$
1	$-3.377 \cdot 10^{-1}$	$\pm 1.135 \cdot 10^{-1}$	$1.3 \cdot 10^{-3}$	$2.97 \cdot 10^{-2}$	$\Delta K_{KFEC} = 3.7 \cdot 10^{-2}$	$\Delta K_{KDEC} = 1.4 \cdot 10^{-2}$
2	$-2.518 \cdot 10^{-1}$	$\pm 1.936 \cdot 10^{-1}$	$12 \cdot 10^{-3}$	$67 \cdot 10^{-2}$	$\Delta K_{KF6} = 5.2 \cdot 10^{-3}$	$\Delta K_{KDEC} = 5.8 \cdot 10^{-3}$
2	$-2.46 \cdot 10^{-1}$	$\pm 2.433 \cdot 10^{-1}$	$28 \cdot 10^{-3}$	$78 \cdot 10^{-3}$	$\Delta K_{KFEC} = 4.2 \cdot 10^{-3}$	$\Delta K_{KDEC} = 1.2 \cdot 10^{-3}$
3	$-5.744 \cdot 10^{-1}$	$\pm 1.705 \cdot 10^{-1}$	$3 \cdot 10^{-4}$	$28 \cdot 10^{-3}$	$\Delta K_{KF8} = 9.5 \cdot 10^{-3}$	$\Delta K_{KF79} = 9.5 \cdot 10^{-3}$

Table 1: Parameters of the coupling compensation using a pair of independent solenoids and a pair of skew quadrupoles. The skew quadrupoles are placed as close as possible to QFN14(44) (conf. 1), in the middle point between QFN14(44) and QDN13(45) (conf. 2) and as close as possible to QDN13(45) (conf. 3.). The ΔK are the variations of the retuning strengths respect to the umperturbed (no coupling) case.

(weaker perturbation respect to the uncoupled optics) and by the fact that the two conditions (40) and (41) can be more easly satisfied.
Due to the symmetry the solutions found require the same absolute value for the strengths of the two skew quadrupoles. This allows to connect them in series using only one power supply providing a current less then 9 A (see Appendix 1). However if one takes into account the coupling generated by random errors (a non symmetric effect) the strengths needed are different and then two power supplies are desiderable.

5.2 One turn matrix compensation using four skew quadrupoles

Following the same strategy illustred in section 5.1 in this paragraph the possibility to compensate the electron cooling solenoid using an additional pair of skew quadrupoles (identical with the one already present in the machine) instead of the two solenoids will be considered. The position of the two skew quadrupoles at 3 and 9 o'clock is the one corresponding to the configuration number 1 in tab. 1. The results (making use of KF6 and KDEC or KFEC and KDEC as retuning knobs) are summarized in tab. 2. In both cases one gets a perfect compensation of the

KSKEWn (m^{-2})	KSKEW (m^{-2})	$\|C^-\|$	$\|C^+\|$	$\Delta K_1(m^{-2})$	$\Delta K_2(m^{-2})$
$\pm 7.54 \cdot 10^{-1}$	$\pm 1.632 \cdot 10^{-1}$	$1.21 \cdot 10^{-2}$	$7 \cdot 10^{-2}$	$\Delta K_{KF6} = 5.6 \cdot 10^{-3}$	$\Delta K_{KDEC} = 1.81 \cdot 10^{-3}$
$\pm 8.795 \cdot 10^{-2}$	$\pm 2.058 \cdot 10^{-1}$	$1.73 \cdot 10^{-2}$	$8.58 \cdot 10^{-2}$	$\Delta K_{KFEC} = 4.4 \cdot 10^{-2}$	$\Delta K_{KDEC} = 8.6 \cdot 10^{-3}$

Table 2: Parameters of the coupling compensation using four skew quadrupoles. KSKEWn are the strengths of the couple of skew quadrupoles replacing the two solenoids and KSKEW the strengths of the skew quadrupoles at 3 and 9 o'klock. The ΔK are the variations of the retuning strengths respect to the umperturbed (no coupling) case.

one turn matrix but the values of the driving terms $|C^-|$ and $|C^+|$ seem to be too big according

to the conditions (40) and (41).
Moreover the configuration which forsees KFEC and KDEC as retuning "knobs" does not allow to keep the value β_x at the electron cooling below 10 m (the value given by MAD is 10.7 m) while the (KF6, KDEC) configuration requires a very big value of the current for the new skew quadrupoles which is out of the range of the available power supply.

5.3 Resonance compensation

The sum resonance $Q_x + Q_y = 11$ excited by the electron cooling solenoid is characterized by a driving term C^+ that is close to satisfy the condition (40) even without any compensation. This implies that the simplest way to control coupling is to use the pair of compensator solenoids to reduce the value of C^- until the achievement of the condition (33). The relevant parameters of the compensation are given in tab.3.

KSOLG (rad/m)	$\|C^-\|$	$\|C^+\|$	$\Delta K_1(m^{-2})$	$\Delta K_2(m^{-2})$
$-4.47 \cdot 10^{-1}$	$8 \cdot 10^{-4}$	$3.15 \cdot 10^{-2}$	$\Delta K_{KF6} = 5.2 \cdot 10^{-3}$	$\Delta K_{KDEC} = 1.2 \cdot 10^{-2}$

Table 3: Parameters of the compensation of the driving term of the different resonance using the couple of solenoids. The ΔK are the variations of the retuning strengths respect to the umperturbed (no coupling) case.

In principle a similar method can be followed to compensate exactly both the most close sum and difference resonances ($C^- = C^+ = 0$) making use of four compensators. However in this case the procedure does not lead to an accetable solution because of the incompatibility of the (available) positions of correctors and the constraint on β_x at the electron cooling.

6 The start up solutions

As it has been shown in paragraph 5.1 the best compensation can be achieved (see tab. 1):
- by having at disposal one independent power supply for the corrector solenoids;
- by placing the skew quadrupoles (fed by the same power supply with opposite polarity) as close as possible to QFN14(44).

Both conditions are not satisfied for the AD start up: the two corrector solenoids will be connected in series with the electron cooling one and only the skew quadrupole in section 13 can be placed in the favourable position.
In tab. 4 the two solutions relative to the start up configuration are compiled.
For the first solution care is taken only about the retuning of the machine. This is possible only if the compensator solenoids are used (together with the two normal quadrupole families QF6 and QDEC). The residual values of C^+ and C^- are such that both the sum and difference resonances are significantly "seen" by the beam.

KSOLG (rad/m)	KSKEW (m^{-2})	$\|C^-\|$	$\|C^+\|$	$\Delta K_1(m^{-2})$	$\Delta K_2(m^{-2})$
$-3.834 \cdot 10^{-1}$	-	$1.53 \cdot 10^{-2}$	$4.13 \cdot 10^{-2}$	$\Delta K_{KF6} = 5 \cdot 10^{-3}$	$\Delta K_{KDEC} = 8 \cdot 10^{-3}$
$-3.834 \cdot 10^{-1}$	$\pm 8 \cdot 10^{-2}$	$7.3 \cdot 10^{-3}$	$4.72 \cdot 10^{-2}$	$\Delta K_{KF6} = 5 \cdot 10^{-3}$	$\Delta K_{KDEC} = 10^{-2}$

Table 4: Parameters of the coupling compensation using two solenoids in series with the main one and two skew quadrupoles placed (not symmetrically) in section 13 and 44 and fed in series by one common power supply.

The second solution forsees the additional use of the two skew quadrupoles (fed by the same power supply). In this case the residual effect of the difference resonance is strongly reduced. Comparing the value of k_c fixed by the condition (37) with the ones which allow the best coupling correction (see tab.1), one might conclude that it would be possible to power the main and corrector solenoids by means of a "small shunting" of the current in the corrector solenoids. However the impedance of the three solenoids connected in series and, consequently, the time constant would be increased; moreover the shunting would not allow the flexible use of the corrector apparatus desirable in the long term.
For these reasons it is recommendable to have for next years machine operation an independent power supply for the two corrector solenoids.

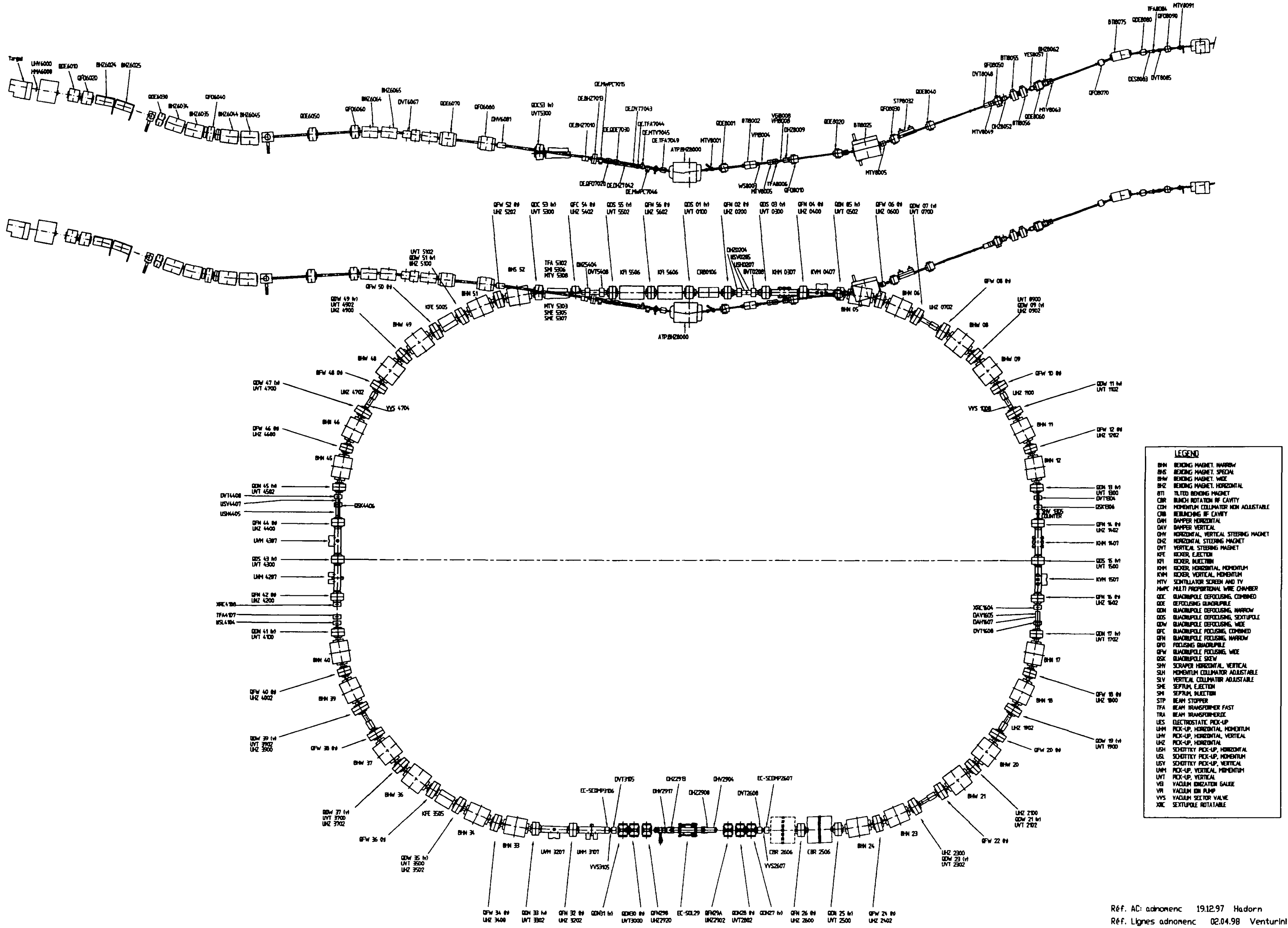
7 Conclusions

A general overview on the source of coupling and on the two different theories to deal with it has been given.
The particular case of AD shows that these two approches are to a large extent consistent and we found several cases for which the compensation of the off-axis terms in the 4×4 one turn matrix leads to a satisfactory compensation of the closest sum and difference resonances.
The general link between the two formalism remains however an open and interesting challange.
The following possibilities have been explored:
- compensation using two independently powered solenoids and two skew quadrupoles placed in different positions in section 13 and 44;
- compensation using two skew quadrupoles instead of two solenoids;
- compensation of the most close difference resonance;
- compensation tacking into account the start up configuration: compensation solenoids connected in series with the main one and skew quadrupoles not simmetrically placed in section 13 and 44.

The correcting tools available allow a good dealing with coupling in the AD machine. Nevertheless to optimize the compensation we forsee [13] the need of one independent power supply for the corrector solenoids.

Acknowledgements

I wish to thank P. Belochitskii, P. Bryant, S. Maury and D. Mohl for stimulating discussions and for their constant availability.



Réf. AC: adnomenc 19.12.97 Hadorn
Réf. Lignes adnomenc 02.04.98 Venturini

Appendix 1

Nominal current for corrector solenoids and skew quadrupoles

Correction with two independent solenoids and two skew quadrupoles

For the assumed current of 400 A the main solenoid generates a field [16] $BSOL = 6.2 \cdot 10^{-2}$ T corrisponding the the strength (p=0.1 GeV/c) $KSOL = 1.859 \cdot 10^{-1}$ rad/m.
With the two skew quadrupoles at 3 and 9 o'klock close to QFN14(44) (see 5.1) two solutions have been found:
1) $|KSKEW| = 8.69 \cdot 10^{-2}$ m^{-2},
2) $|KSKEW| = 1.35 \cdot 10^{-1}$ m^{-2}.
From the relation

$$B\rho[\mathrm{Tm}] = \frac{1}{0.2998} p[\mathrm{GeV/c}] \tag{42}$$

it follows (p=0.1 GeV/c)
1) $|GSKEW| = |KSKEW| \cdot B\rho$=0.02895 T/m,
2) $|GSKEW| = |KSKEW| \cdot B\rho$=0.04503 T/m.
Assuming $L_{eff} = 0.51$ m as effective length of the skew quadrupoles, one gets [14]
1) $|ISKEW| \simeq$ 2A,
2) $|ISKEW| \simeq 2.6$ A.
Fig. (2) shows that during the deceleration from p=0.3 GeV/c to p=0.1 GeV/c the current stays almost constant.

For the same configuration the corresponding values of the strengths of the two correcor solenoids are [15]
1) $|KSOLG|$= 0.328 rad/m,
2) $|KSOLG|$= 0.338 rad/m
that is
1)$BSOLG = B\rho \cdot KSOLG$=0.109 T$\longrightarrow$ ISOLG=162 A
2)$BSOLG = B\rho \cdot KSOLG$=0.113 T$\longrightarrow$ ISOLG=168 A
Fig. (3) shows the current variation during the deceleration.

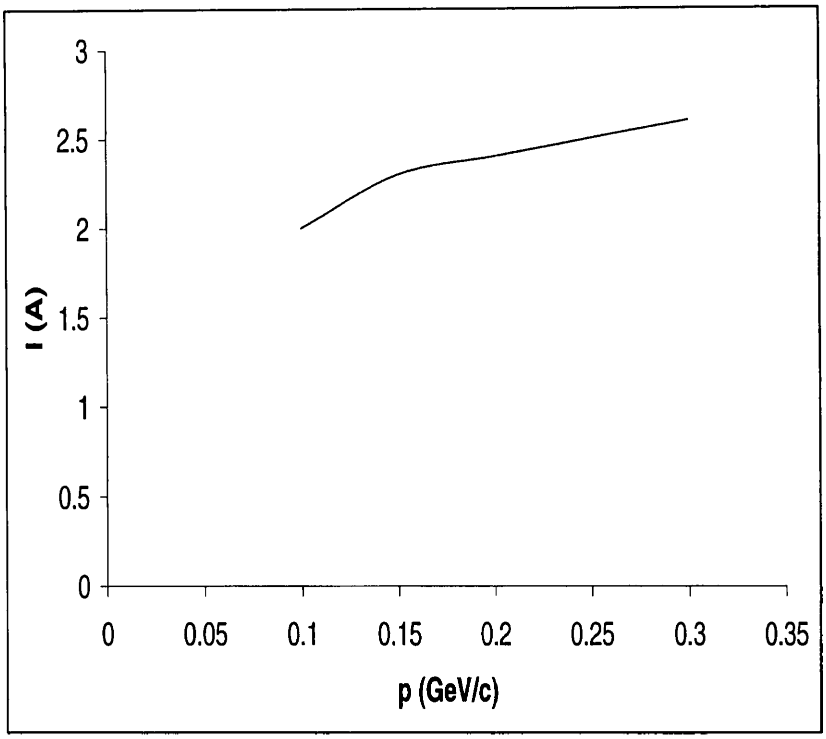


Figure 1: Current variation (absolute value) in the two corrector skew quadrupoles for different momenta (configuration 1).

Correction with four skew quadrupoles

Performing the same calculation one finds:
1)$|KSKEWn| = 7.54 \cdot 10^{-2}\ \mathrm{m}^{-2} \longrightarrow |ISKEWn| \simeq 1.7\ \mathrm{A}$
$|KSKEW| = 1.632 \cdot 10^{-1}\ \mathrm{m}^{-2} \longrightarrow |ISKEW| \simeq 3.8\ \mathrm{A}$
2)$|KSKEWn| = 8.795 \cdot 10^{-2}\ \mathrm{m}^{-2} \longrightarrow |ISKEWn| \simeq 2\ \mathrm{A}$
$|KSKEW| = 2.058 \cdot 10^{-1}\ \mathrm{m}^{-2} \longrightarrow |ISKEW| \simeq 4.7\ \mathrm{A}$.

Correction of the difference resonance driving term

$|KSOLG| = 0.447$ rad/m $\longrightarrow |ISOLG|$=663 A

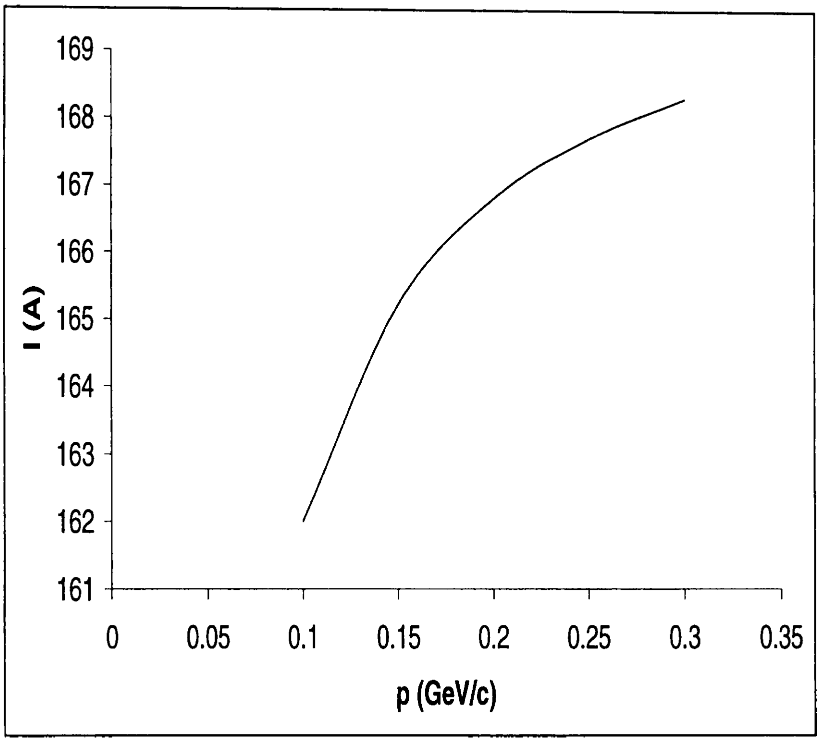


Figure 2: Current variation (absolute value) in the corrector solenoids for different momenta (configuration 1).

Start up configuration

Configuration 2 tab. 4 (varying KDEC and KF6 for retuning):
$|KSKEW| = 7.88 \cdot 10^{-2}\ \mathrm{m}^{-2} \longrightarrow |ISKEW| \simeq 1.8\ \mathrm{A}.$

Fig. (4) shows the current variation in the two skew quadrupoles for different momenta.

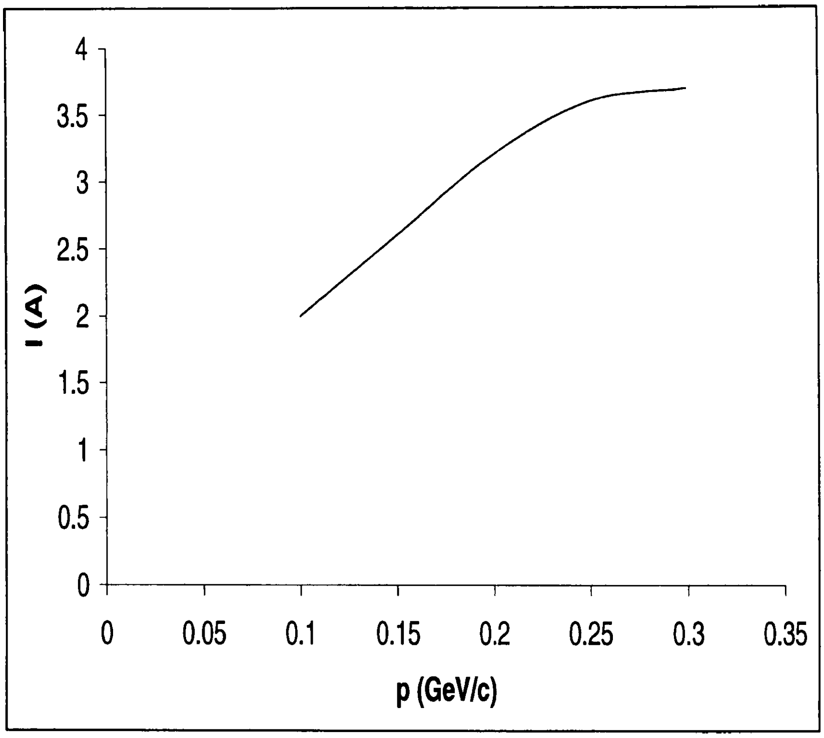


Figure 3: Current variation (absolute value) in the two corrector skew quadrupoles for different momenta (configuration 2 for the start up).

Appendix 2

Current variation in QFW06, QDN27 and QFN29 due to retuning

To fix the current variation needed to change the strength of QFW06, QDN27 and QFN29 so to get the retuning during the coupling compensation, we refer to [16] and [17].
The measurements performed on the QFW06's family for the current range suitable to the AC (1000A<I<2000A) have to be interpolated to find the magnetization curve for I<1000A (low energy part of the AD cycle). Exploiting the numerical expression of the polinomial dependence of the integrated gradient (GL) on I and neglecting any saturation effect the low current part of the magnetization curve can be approximated with a straight line passing by the point (I = 1000 A, $(GL)_{I=1000}$A) and having the same slope of the measured curve in this point.
The following relation has been found out:

$$(GL) = \alpha \cdot I + \beta \tag{43}$$

where $\alpha = 2.084 \cdot 10^{-3}$ T/A and $\beta = 8 \cdot 10^{-3}$ T.
For the umperturbed case (no coupling) the gradient G_{KF6} is (p=0.1GeV/c) [18]:

$$G_{KF6} = KF6 \cdot B\rho = KF6 \frac{1}{0.2998} p = \frac{KF6}{2.998} = 0.1459 T/m \tag{44}$$

and $(GL)^{ump.}_{KF6}$=0.1104T.
Using (43):
$I^{ump.}_{KF6}$=49.1A.
After the retuning for the configuration 1 in tab. 1:
$(GL)_{KF6} = 0.1092\text{T} \longrightarrow I_{KF6} = 48.6\text{A}$.
After the retuning for the configuration 1 in tab. 2:
$(GL)_{KF6} = 0.1091\text{T} \longrightarrow I_{KF6} = 48.5\text{A}$.

The measurements performed on the QDN magnets (including both QDN27 and QFN29) comprise the low current range and this allows to extract directly from them (neglecting saturation) the current values once given the integrated gradients. The values found are:

- QDN27:

$(GL)^{ump.}_{KDEC} = 0.086\text{T} \longrightarrow I^{ump.}_{KDEC} = 29\text{A}$ (umperturbed case);
$(GL)_{KDEC} = 0.084\text{T} \longrightarrow I_{KDEC} = 28.7\text{A}$ (conf.1 tab. 1);
$(GL)_{KDEC} = 0.083\text{T} \longrightarrow I_{KDEC} = 28\text{A}$ (conf.2 tab. 1);

$(GL)_{KDEC} = 0.086\text{T} \longrightarrow I_{KDEC} = 29\text{A}$ (conf.1 tab. 2);
$(GL)_{KDEC} = 0.084\text{T} \longrightarrow I_{KDEC} = 28.3\text{A}$ (conf.2 tab. 2).

- QFN29:

$(GL)^{ump.}_{KFEC} = 0.141\text{T} \longrightarrow I^{ump.}_{KDEC} = 48.7\text{A}$ (umperturbed case);
$(GL)_{KFEC} = 0.132\text{T} \longrightarrow I_{KFEC} = 45.6\text{A}$ (conf.2 tab. 1);
$(GL)_{KFEC} = 0.131\text{T} \longrightarrow I_{KFEC} = 45\text{A}$ (conf.2 tab. 2).

The quadrupoles families QFW06 and QFN29 are connected to the main power supply by the trim 1 and the trim 4 (respectively) [19]. The previous calculations show that after the retuning the two trims still provide a positive current (as prescripted).
The QDN27 family is fed by an independent power supply.

Finally we report separately the forseen current variation for the probable start up configurations.
Configuration 1 in tab. 4 (corrector solenoids and skew quadrupoles switched off):
$(GL)_{QDEC} = 0.084\,\text{T} \longrightarrow I_{KDEC} = 28.3\text{A}$
$(GL)_{KF6} = 0.109\,\text{T} \longrightarrow I_{KF6} = 48.5\text{A}$.

Configuration 2 in tab. 4 (corrector solenoids in series with the main one, varying KF6 and KDEC):
$(GL)_{QDEC} = 0.084\,\text{T} \longrightarrow I_{KDEC} = 28.3\text{A}$
$(GL)_{KF6} = 0.1091\,\text{T} \longrightarrow I_{KF6} = 48.5\text{A}$.

References

[1] R. Talman, *The Mobius accelerator*, CLNS-94-1300 (1994).

[2] G. De Ninno, E. Todesco, *Effect of linear coupling on nonlinear resonances in betatron motion*, Physical review E (1997).

[3] D. Edwards, L. Teng, *Parametrization of the linear coupled motion in periodic systems*, IEEE Trans. Nucl. Sci. (1973).

[4] R.Talman, *Coupled betatron motion and its compensation*,US-CERN School of Particle Accelerators, Capri, Italy (1988).

[5] S. Peggs, *Coupling and decoupling in storage rings*, PAC 1983.

[6] G. Guignard, J. Hagel, *Hamiltonian treatment of betatron coupling*, CERN 92-01 (1992).

[7] P.J. Bryant, *A simple theory for weak betatron coupling*, CERN 94-01 (1994).

[8] G. Guignard, *The general theory of all sum and difference resonances in a three-dimensional magnetic field in a synchrotron*, CERN 76-06 (1976).

[9] G. De Ninno, *Multiple resonance compensation for the betatronic coupling and its equivalence with matrix method*, in preparation.

[10] P. Belochitskii, G. De Ninno, *Coupling due to errors in the AD machine*, in preparation.

[11] H. Grote, F.C. Iseline, *The MAD program, User's Reference Manual*, CERN/SL/90-13 (1990).

[12] P.J. Bryant, *AGILE-lattice program*, in preparation.

[13] G. De Ninno, *Minutes of the meeting on compensation of the linear coupling due to the E-cool solenoid in the AD machine*, AD Note 040 (1998).

[14] D. Cornuet, *Private communication.*

[15] D. Cornuet, G. Patron, *Mesure magnetique des solenoides type 13 du systeme de refroidissement par electrons de LEAR*, AT/MA Note 93-66 (1993).

[16] D. Cornuet, *Private communication.*

[17] Ph. Cartier, E. Chinchio, G. Suberlucq, *Pricipaux parameters des aimants mesures et testes par le groupe PSR. Table des aimants d'ACOL*, not pubblished (1987).

[18] H. Koziol, S. Maury, *Parameter list for the Antiproton Accumulator Complex (AAC)*, CERN/PS 95-15 (1995).

[19] S. Maury *AD power supplies*, PS/AD Note 028 (1998).