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BEAM DETERIORATION BY MULTIPLE SCATTERING ON REST GAS

Niels Madsen

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Beam deterioration by multiple scattering on rest gas

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1 Introduction

As shown in earlier calculations of Gröbner [1] the main cause for need for better vacuum in the AD as compared to the AC comes from the beam heating which is induced by multiple scattering of beam particles on the residual gas in the vacuum chamber. This is an overview of the calculations necessary to estimate the influence of the average ring pressure on the beam emittance.

2 Vacuum Description

As the cross section of the interaction between antiprotons and rest gas depends on the nature of the rest gas, the composition of the rest gas needs to be taken into account. Relevant numbers are given in table 1.

Species	Ion Gauge Cal.	RGA Cal.	Composition uncal.	Pressure N_2 eq. [Pa]	Pressure Abs [Pa]	Density [m^{-3}]
H_2	2.50	1.54	178	$4.48{\cdot}10^{-7}$	$1.12{\cdot}10^{-6}$	$2.70{\cdot}10^{14}$
CH_4	0.78	0.71	10	$5.45{\cdot}10^{-8}$	$4.25{\cdot}10^{-8}$	$1.03{\cdot}10^{13}$
H_2O	1.00	1.00	40	$1.55{\cdot}10^{-7}$	$1.54{\cdot}10^{-7}$	$3.74{\cdot}10^{13}$
CO	0.83	1.00	35	$1.36{\cdot}10^{-7}$	$1.12{\cdot}10^{-7}$	$2.71{\cdot}10^{13}$
N_2	1.00	1.00	0	0	0	0
Ar	0.69	0.83	0	0	0	0
CO_2	0.72	1.54	3	$7.54{\cdot}10^{-9}$	$5.43{\cdot}10^{-9}$	$1.31{\cdot}10^{12}$

Table 1: Residual gas description for the AC, $P_{IG,N_2} = 8.00{\cdot}10^{-7}$Pa (=$6.0{\cdot}10^{-9}$ Torr)

Where the calibrations of the Ion Gauge and the rest gas analyzer (RGA) are given in terms sensitivity to N_2 divided by the sensitivity to the gas in question. The pressure returned by a standard Ion Gauge is the Nitrogen equivalent pressure (P_{IG,N_2}).

3 Multiple Scattering

3.1 The cross section

Following the line of thinking in [2] we assume pure coulomb scattering. Thus we can use the Rutherford cross section. For small scattering angles this can be written :

$$\frac{d\sigma}{d\theta}(\theta) = 8\pi Z^2 r_e^2 \left(\frac{m_e c^2}{\beta c p}\right)^2 \theta^{-3} \tag{1}$$

where Z is the charge of the scattering atoms (the incoming particle is assumed to be a proton), r_e is the clasical electron radius, m_e the electron mass, p the momentum of the scattered proton and β the velocity in units of light speed. This formula is only valid in a limited angular range. We'll assume it's zero outside this range.

The minimum angle θ_{min} is defined by the screening effect of the electron shells of the atom :

$$\theta_{min} = \alpha Z^{1/3} \frac{m_e c^2}{\beta c p} \tag{2}$$

the maximum is given by

$$\theta_{max} = \frac{280}{A^{1/3}} \frac{m_e c^2}{\beta c p} \tag{3}$$

where α is the fine structure constant.

3.2 In free space

If a beam of particles moves through a residual gas at some velocity v, the mean square angular deflection rises linearly with time

$$\theta^2_{rms}(t) = n_a \beta c t \int_0^\pi \theta^2 \frac{d\sigma}{d\theta}(\theta) d\theta \tag{4}$$

this expression is equivalent to diffusion in the two dimensional space of the projected scattering angles θ_z and θ_x, which are the two perpendicular components which spans the plane perpendicular to the beam direction. Thus each of these scattering angles increases with half the mean square of θ

$$\theta^2_{z,rms} = \frac{1}{2}\theta^2_{rms} \tag{5}$$

3.3 In a focusing field

In the focusing field of a storage ring an angular deflection will be 'converted' to an increase in the betatron amplitude according to the beta function at the position of the event. To ease calculations we average the betafunction over the circumference. We will furthermore assume that the two transverse dimensions are uncoupled.

As seen on figure 1 the angular spread induced by multiple scattering will lead to an increased beam size (and emittance). On average the increase in the beam emittance and size by scattering is given by :

$$\epsilon = \theta^2_{z,rms} \cdot \beta \quad ; \quad \epsilon = \frac{\sigma^2_{z,rms}}{\beta} \tag{6}$$

Thus for a storage ring we obtain the following expression for the time development of the beam emittance in each transverse plane :

$$\begin{aligned}
\epsilon_z(t) &= \epsilon_{z0} + \frac{1}{2}\theta^2_{rms}(t) \cdot \beta_z \\
&= \epsilon_0 + \frac{1}{2}\beta_z n_a \beta c t \int_0^\pi \theta^2 \frac{d\sigma}{d\theta}(\theta) d\theta \\
&= \epsilon_0 + \frac{1}{2}\beta_z n_a \beta c t \int_{\theta_{min}}^{\theta_{max}} \theta^2 8\pi Z^2 r_e^2 \left(\frac{m_e c^2}{\beta c p}\right)^2 \theta^{-3} d\theta
\end{aligned}$$

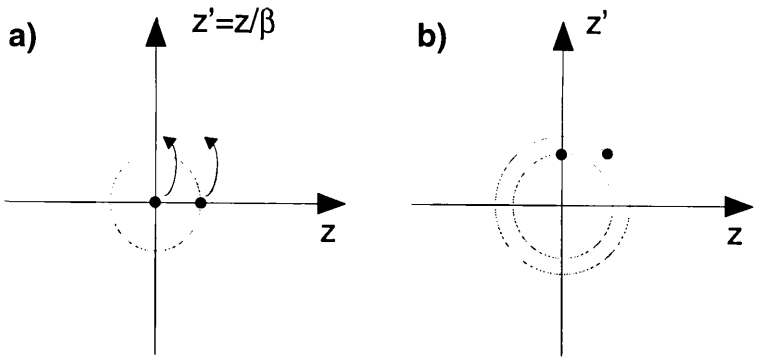

Figure 1: Transverse Phase Space. Two different particles gets the same angular kick by collision with rest gas, this results in an increase in the rms spread of the beam at this position.

$$
\begin{aligned}
&= \epsilon_0 + 4\pi\beta_z n_a r_e^2 \frac{(m_e c^2)^2}{\beta c p^2} Z^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) t \\
&= \epsilon_0 + 4\pi\beta_z n_a r_e^2 \frac{(m_e c^2)^2}{\beta c p^2} Z^2 \ln\left(\frac{\alpha^{-1} \cdot 280}{(A \cdot Z)^{1/3}}\right) t
\end{aligned} \tag{7}
$$

it is evident from this formula that the emittance growth depends on the residual gas composition. This we have already discussed earlier, and in table 1 was listed the composition for the AC. To incorporate this dependence we introduce the gas factor G :

$$
G = Z^2 \ln\left(\frac{\alpha^{-1} \cdot 280}{(A \cdot Z)^{1/3}}\right) \tag{8}
$$

as the densities of the various composits are not equal it is often easier to define a compensated molecular density of the gas (it is an approximation to add contributions from atoms in molecules as if they were free, but it's a good approximation), defined below

$$
n_{MS} = \sum_i n_i \cdot G_i \tag{9}
$$

Using the values from table 1 we obtain for the AC vacuum conditions a multiple scattering density of $n_{MS} = 5.81 \cdot 10^{16}$ m^{-3}.

This simplifies the expression for the emittance growth to

$$
\epsilon_z(t) = \epsilon_0 + 4\pi\beta_z n_{MS} r_e^2 \frac{(m_e c^2)^2}{\beta c p^2} t \tag{10}
$$

Now the emittance ordinarily used in discussions on the AD is the 2 sigma emittance ($\epsilon_2 = (2\sigma)^2/\beta$), thus the above expression should be multiplied with a factor of 4 to compensate :

$$
\epsilon_{2,z}(t) = \epsilon_{2,0} + 16\pi\beta_z n_{MS} r_e^2 \frac{(m_e c^2)^2}{\beta c p^2} t \tag{11}
$$

inserting in this equation, using an average $\bar{\beta_z}$=7.0m (AD), we obtain :

$$
\epsilon_{2,z}(t) = 12.3\text{mm mrad} + \frac{1.270 \cdot 10^{-2}\text{mm mrad}}{\beta (p[GeV/c])^2} t[s] \tag{12}
$$

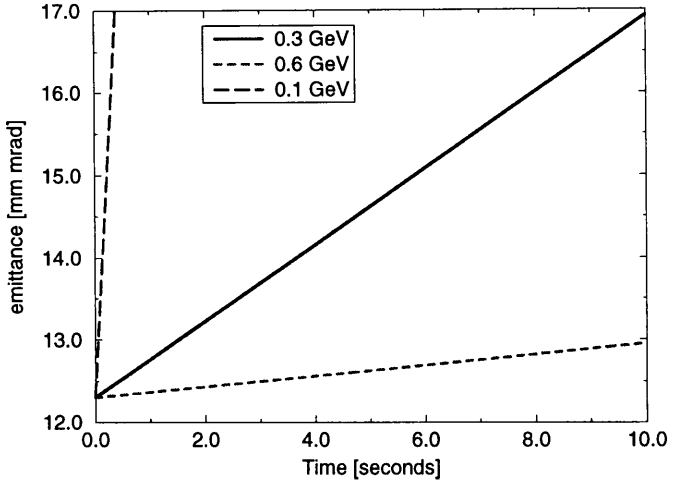

Figure 2: Emittance growth for 3 different beam energies for the AD (vacuum conditions of AC, table 1) with an average focusing function $\bar{\beta}_z$=7.0m)

which is illustrated on figure 2.

With this calculation in hand, we can also estimate the equilibrium emittance as a function of the energy of the beam. We estimate a cooling time of τ_c=1.0s, which gives us the equilibrium emittance behavior illustrated in figure 3 (using $\epsilon_{2,eq.} = \frac{d\epsilon_2}{dt}\tau_{cool}$).

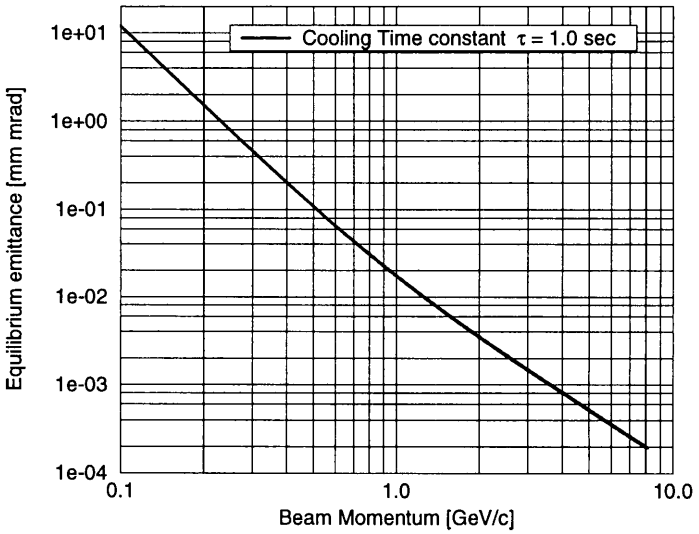

Figure 3: AD equilibrium emittance estimate with cooling.

A final relevant observation is that the beam can't be allowed to expand forever, as the vacuum chamber defines a finite available aperture. According to [1] the available acceptance is $\epsilon_{accep} = 150\ \pi$ mm mrad, and the beam will start to be lost when the beam halo hits the chamber, which is defined as the 6 sigma emittance.

A halo of 6 sigma with area $\epsilon_{accep} = 150\ \pi$ mm mrad corresponds to a 2 sigma emittance of $\epsilon_{2,accep} = 16.7$ mm mrad. We thus have the following expression for the time it takes before the beam starts being scraped :

$$t_{scraped}(p) = \frac{\epsilon_{2,accep} - \epsilon_{init}}{1.270 \cdot 10^{-2}\text{mm mrad}} \cdot \beta(p[GeV/c])^2 \tag{13}$$

inserting numbers we obtain

$$t_{scraped}(p) = 343.8 \cdot \beta(p[GeV/c])^2 \tag{14}$$

illustrated on figure 4

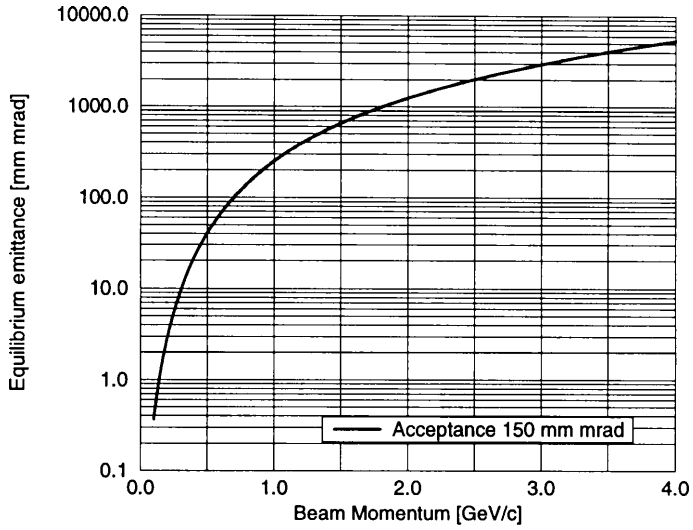

Figure 4: The time it takes in the AD (with AC vacuum) before the multiple scattering makes the beam emittance large enough that losses start occuring.

4 Conclusions

It is evident from the discussion and the equations presented that better vacuum is needed. At the lowest energy (0.1 GeV/c) the beam would start hitting the vacuum chamber severely after only 0.4 seconds, which isn't time enough to the manipulations necessary at this time in the injection cycle (the cycle estimates that we are at this energy for about 1 second). Furthermore the equilibrium emittance needed by the experiments is $\epsilon = 1\ \pi$ mm mrad. Thus a 20 fold improvement of the vacuum to $4{\cdot}10^{-8}$Pa ($3{\cdot}10^{-10}$Torr) has been proposed and has to be implemented. Assuming a linear scaling of the gas composition this will increase the time at 0.1 GeV/c to approx. 8 seconds which should be enough. It is however necessary to be aware that is it critical that the vacuum is good at places with large beta functions as these have the worst influence on the beam.

References

[1] O. Gröbner and M. Brouet, in "Upgrading the AC vacuum system for the Antiproton Decelerator (AD)", Vac. Tec. Note 97-06, LHC Div., CERN, 1997

[2] E. Fischer, in "Residual gas scattering...", ISR-VAC/67-16, 1967