Z_2-odd Polonyi field in twin Higgs model

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Abstract: We consider a supersymmetric mirror Twin Higgs model in a gravity-mediated supersymmetry-breaking scenario. We point out that the Polonyi field can be odd under the Z_2 symmetry exchanging the Standard Model with the mirror sector while gaugino masses are generated at tree-level. We discuss the dynamics of the Polonyi field during and after inflation and show that the Polonyi problem is absent. The Polonyi field couples to the two sectors with opposite signs, which may serve as origin of the Z_2-breaking of the Higgs potential in Twin Higgs models. We also estimate the Z_2-breaking in soft masses of supersymmetric particles.

Contents

1 Introduction

In mirror Twin Higgs (TH) theories, the exact copy of gauge groups and particle contents of the SM sector are introduced, and two sectors so obtained enjoy Z_2 exchanging symmetry. Due to Z_2, there arises an accidental global SU(4) symmetry of the mass terms of the Higgses. Further assuming that the Higgs quartic terms are also approximately SU(4) symmetric, the SM-like Higgs boson can be understood as the pseudo Nambu-Goldstone boson (pNGB) of the spontaneously broken global SU(4) by the twin Higgs [1, 2]. As such, the quadratic sensitivity of the SM-like Higgs mass squared to the UV cut-off scale is relaxed, and so is the little hierarchy problem of the electroweak scale. The two main

challenges of the TH theories concern 1) the SU(4) invariant quartic coupling and 2) the big hierarchy between the twin electroweak (EW) scale and the fundamental scale.

These two challenges may be resolved by extending TH theories by supersymmetry (SUSY). The twin EW scale can be set by the soft mass scale, and the SU(4) invariant quartic can be obtained by an F-term potential from a new singlet field [3–6] or a D-term potential from a new gauge interaction [7–9]. The SUSY TH models are indeed able to explain the EW scale with a mild tuning. For example, in the D-term model with an U(1) gauge symmetry [7], the required tuning can be as low as $\sim 20\%$. Another noticeable thing in SUSY TH models is that the tree-level SM-like Higgs mass is larger than the MSSM one by about a factor $\sqrt{2}$, and the observed SM-like Higgs boson mass can be obtained without radiative corrections from a large stop mass, which also reduces fine-tuning. As a further bonus, the lightest SUSY particle, which may be a super-partner of a mirror particle [10, 11], is a dark matter candidate [12–14].

On the other hand, the simplest scheme of the mediation of SUSY breaking, namely, gravity mediation, suffers from the infamous Polonyi problem [15]. In gravity mediation, gaugino masses of the order of scalar masses require a singlet SUSY-breaking field called the Polonyi field. Although the Polonyi field S may sit at a minimum of the potential during inflation thanks to a Hubble-induced potential, that minimum is in general displaced from the minimum after inflation by as large as $\mathcal{O}(M_P)$, where $M_P \simeq 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass. The resultant oscillation of S around the minimum is dangerous as it (or its decay products) overcloses the universe afterward. The problem is fundamentally attributed to the lack of any charge of S and hence the absence of the symmetry enhanced point of the S field space. A solution to the Polonyi problem by a large coupling of the Polonyi field with the inflaton [16–18] and that by a coupling with a pseudo-flat direction [19] have been considered in the literature.

Motivated by the advantages of SUSY TH models and the cosmological danger lurking in the gravity mediation, we consider the possibility where S transforms as $S \to -S$ under the Z_2 symmetry exchanging the SM and mirror sectors. Such a transformation rule of S clearly pins down the origin of the field space of S, providing a complete solution to the Polonyi problem. Moreover, in this set-up, any neutral (up to Z_2) operator $\mathcal{O}$ in the SM and $\mathcal{O}'$ in the mirror SM can couple to S via $S(\mathcal{O} - \mathcal{O}')$. $\mathcal{O}$ includes gauge kinetic terms, so tree-level gaugino masses are obtained despite the Z_2-odd charge of S. As we will see, the spontaneous Z_2 breaking by the F term of S, together with R symmetry breaking, can contribute to the phenomenologically required Z_2-breaking terms in the Higgs potential either at tree-level or by radiative corrections.

The outline of the paper is as follows. In section 2, we briefly review the Higgs sector in the SUSY TH models. Section 3 is dedicated to discussion for the generation of gaugino masses, trilinear scalar couplings, scalar masses, and the μ and b terms in the presence of the Z_2 odd field S. We show that the Z_2 breaking in the Higgs potential can be sufficiently suppressed. In section 4, we compute the SM-like Higgs mass. Finally in section 5, we present a concrete model for the SUSY breaking sector and study the early-universe dynamics of S.

We use the same symbol for a chiral superfield and its scalar component. We also use primes to denote the fields in the mirror sector. Finally, the R-charge of an operator $\mathcal{O}$ is denoted as $R[\mathcal{O}]$.

2 Review of supersymmetric twin Higgs

In this section, we briefly review SUSY TH models.

As in generic TH models, the Z_2 symmetry exchanging the SM particles with their mirror partners is introduced. The Z_2 symmetry is extended to SUSY particles, so the mirror squarks, sleptons, gauginos, and higgsinos are also introduced.

The Higgs sector at the minimal contains H_u, H_d, H'_u, and H'_d, and they have quartic and quadratic terms. The quartic terms arise from the F and D terms. To be concrete, we only introduce D term potentials from the SM and mirror electroweak symmetry,

$$V_{D,\text{EW}} = \left(\frac{g_2^2 + g_Y^2}{8} \right) \left[\left(|H_u|^2 - |H_d|^2 \right)^2 + \left(|H'_u|^2 - |H'_d|^2 \right)^2 \right], \tag{2.1}$$

and that from a new gauge symmetry under which both the SM and mirror particles are charged. The form of this potential depends on models [7–9]. To be concrete, we assume a gauge symmetry with an opposite charge of H_d to H_u. The new gauge symmetry is broken at a scale above the SM and mirror EW symmetry breaking scale. If this occurs via a supersymmetric potential, the D term potential of the new gauge interaction decouples. A non-zero D term remains if the gauge symmetry breaking involves a SUSY-breaking potential, which is parameterized by ϵ,

$$V_{D,\text{new}} = \left(\frac{\lambda_X}{2} \right)^2 \left(|H_u|^2 - |H_d|^2 + |H'_u|^2 - |H'_d|^2 \right)^2, \quad \lambda_X^2 \equiv (1 - \epsilon^2) g_X^2 / 2, \tag{2.2}$$

where g_X is the new gauge coupling constant. The D term potential from the EW symmetry violates the SU(4) symmetry but preserves the Z_2 symmetry, while the one from the new gauge interaction preserves both SU(4) and Z_2.

Quadratic terms arise from a supersymmetric mass μ and soft supersymmetry breaking,

$$V_2 = \left(|\mu^2| + m_{H_u}^2 \right) |H_u|^2 + \left(|\mu^2| + m_{H_d}^2 \right) |H_d|^2 + \left(|\mu'^2| + m_{H'_u}^2 \right) |H'_u|^2 + \left(|\mu'^2| + m_{H'_d}^2 \right) |H'_d|^2$$
$$+ \left(b H_u H_d + \text{h.c.} \right) + \left(b' H'_u H'_d + \text{h.c.} \right). \tag{2.3}$$

As we will see in the next section, Z_2-breaking masses arise at tree-level or from quantum corrections. Without loss of generality, we assume Z_2 breaking such that $v^2 = |\langle H_u \rangle|^2 + |\langle H_d \rangle|^2 < v'^2 = |\langle H'_u \rangle|^2 + |\langle H'_d \rangle|^2$. In order for the potential to be bounded from below, $2|b'| < m_{H'_u}^2 + m_{H'_d}^2 + 2|\mu'|^2$ is required.

We work in the decoupling limit of heavy Higgses so that the effective theory around the mirror and SM EW scale is given by that of the SM-like Higgs H and its partner H', which may be obtained by the following replacement;

$$H_u = H \sin\beta, \quad H_d = H \cos\beta, \quad H'_u = H' \sin\beta', \quad H'_d = H' \cos\beta', \tag{2.4}$$

where $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ and $\tan\beta' = \langle H'_u \rangle / \langle H'_d \rangle$ are given by

$$\sin 2\beta = \frac{2b}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}, \quad \sin 2\beta' = \frac{2b'}{m_{H'_u}^2 + m_{H'_d}^2 + 2|\mu'|^2}. \quad (2.5)$$

The potential of H and H' is

$$\begin{aligned} V(H, H') = {} & m^2(|H|^2 + |H'|^2) + \bar{\lambda}(|H|^2 + |H'|^2)^2 \\ & + \bar{\kappa}(|H|^4 + |H'|^4) \\ & + \Delta m_H^2 |H|^2 + \bar{\rho}|H|^4 \,, \end{aligned} \quad (2.6)$$

where

$$\bar{\lambda} = \left(\frac{\lambda_X}{2} \right)^2 \cos 2\beta \cos 2\beta', \quad (2.7)$$

$$\bar{\rho} = \left[\left(\frac{\lambda_X}{2} \right)^2 + \left(\frac{g_2^2 + g_Y^2}{8} \right) \right] (\cos^2 2\beta - \cos^2 2\beta') \,, \quad (2.8)$$

$$\bar{\kappa} = \left(\frac{\lambda_X}{2} \right)^2 \cos 2\beta' (\cos 2\beta' - \cos 2\beta) + \left(\frac{g_2^2 + g_Y^2}{8} \right) \cos^2 2\beta' \,, \quad (2.9)$$

$$m^2 = |\mu'|^2 + m_{H'_u}^2 \sin^2 \beta' + m_{H'_d}^2 \cos^2 \beta' - b' \sin 2\beta' \,, \quad (2.10)$$

$$\begin{aligned} \Delta m_H^2 = {} & \left(|\mu|^2 + m_{H_u}^2 \sin^2 \beta + m_{H_d}^2 \cos^2 \beta - b \sin 2\beta \right) \\ & - \left(|\mu'|^2 + m_{H'_u}^2 \sin^2 \beta' + m_{H'_d}^2 \cos^2 \beta' - b' \sin 2\beta' \right). \end{aligned} \quad (2.11)$$

In eq. (2.6), the first line respects both Z_2 and the global SU(4), the second line breaks SU(4) explicitly while respecting Z_2, and the last line breaks Z_2 and SU(4). $\Delta m_H^2 |H|^2$ and $\bar{\rho}|H|^4$ are soft and hard Z_2 breaking, respectively. In addition to these tree-level potentials, quantum corrections from the top Yukawa are important in determining the SM-like Higgs mass. We will discuss it in section 4.

For $m^2 \gg \Delta m_H^2$ and $\bar{\lambda} \gg \bar{\kappa}, \bar{\rho}$, the potential is approximately SU(4) symmetric. The approximate SU(4) symmetry is broken to SU(3) by the condensation of H', and the SM-like Higgs is understood as four pseudo-Nambu Goldstone bosons. (Three are eaten by W' and Z'.) The EW scale is thus doubly protected by SUSY and the SU(4) symmetry. See [20, 21] for other models with double protection based on SUSY and a global symmetry.

The tree-level mass is enhanced in comparison with the MSSM. To see this, let us use the nonlinear realization of the Higgs h as a pNGB of the broken SU(4);

$$H = f \sin \frac{h}{\sqrt{2}f}, \quad H' = f \cos \frac{h}{\sqrt{2}f} \,, \quad (2.12)$$

where $f^2 = v^2 + v'^2$ is the SU(4)-breaking scale. For $\bar{\lambda} \gg \bar{\kappa}, \bar{\rho}$, the pNGB mass reads

$$m_h^2 \simeq 4v^2 (2\bar{\kappa} + \bar{\rho}) \left(1 - \frac{v^2}{f^2} \right) + \mathcal{O}(\bar{\kappa}/\bar{\lambda}) \,, \quad (2.13)$$

which is proportional to explicit SU(4)-breaking quartic couplings. This is larger than the MSSM tree-level prediction $4\bar{\kappa}v^2$, so a large stop mass is not required to explain the

observed Higgs mass. In particular, for $f \gg v$, $\bar{\rho} \ll \bar{\kappa}$, and a large $\tan\beta$, the tree-level Higgs mass is already around 125 GeV.

The Z_2 breaking in the quadratic term Δm_H^2 is determined from v and v'. In the limit $\bar{\kappa} \gg \bar{\rho}$ and $f \gg v$, combined with eq. (2.13), eq. (2.11) reduces to

$$\Delta m_H^2 \simeq \frac{1}{4} \left(\frac{f}{v}\right)^2 m_h^2 \simeq (200\text{GeV})^2 \times \left(\frac{f/v}{3}\right)^2 \simeq \frac{0.07}{\bar{\lambda}}(-m^2) \equiv \Delta m_{H,\text{req}}^2. \qquad (2.14)$$

SUSY TH models may have two types of tuning: 1) the tuning of Δm_H^2 to obtain the hierarchy $v < f$ and 2) the tuning to obtain $f < m_{\text{SUSY}}$. The former is given by

$$\Delta_{v/f} = \frac{\partial \log(v^2/f^2)}{\partial \Delta m_H^2} = \frac{f^2}{2v^2} - 1. \qquad (2.15)$$

Avoiding the tuning worse than 10% requires f to be smaller than $5v$. The latter is typically dominated by the quantum correction from the stop mass and is given by

$$\Delta_f \equiv \frac{\log f^2}{\log m_{\tilde{t}}^2} \simeq \frac{3}{8\pi^2} \frac{y_t^2}{\bar{\lambda}} \frac{m_{\tilde{t}}^2}{f^2}. \qquad (2.16)$$

The total degree of fine-tuning is given by the product $\Delta_{v/f} \times \Delta_f$,

$$\Delta_{\text{TH}} \equiv \Delta_{v/f} \times \Delta_f \simeq \frac{3}{8\pi^2} \frac{y_t^2}{\bar{\lambda}} \frac{m_{\tilde{t}}^2}{2v^2}. \qquad (2.17)$$

In comparison with the fine-tuning in the MSSM, $(3y_t^2 m_{\tilde{t}}^2/8\pi^2 \lambda_{\text{SM}} v^2)$, the fine-tuning in SUSY TH models is improved by a factor of $2\bar{\lambda}/\lambda_{\text{SM}} \simeq 15\bar{\lambda}$. Together with the large tree-level Higgs mass, which remove the necessity of a large stop mass, the EW scale can be obtained with a mild tuning of $O(1\text{-}10)$ %.

3 Soft masses from Z_2-odd Polonyi field

We assume that the Polonyi field S is odd under the Z_2 exchange symmetry in the TH mechanism. In this section, we show how the SM and mirror sector fields can couple to S and obtain soft masses.

We assume that S is the dominant source of supersymmetry breaking, so that its F-term is given by

$$F_S = \sqrt{3} m_{3/2} M_P \equiv m_{3/2} M_*, \qquad (3.1)$$

where $m_{3/2}$ is the gravitino mass and $M_P = 2.4 \times 10^{18}$GeV is the reduced Planck mass. Here we define $M_* = \sqrt{3} M_P$ in order to remove the factors of $\sqrt{3}$ from the formulae for soft masses shown below.

We write down the coupling between S and other fields in terms of dimension-less coefficients and M_*. If the cutoff scale M_{cut} is around M_P, we expect that dimension-less coefficients are $\mathcal{O}(1)$. When M_{cut} is below M_P, dimension-less coefficients are expected to be $\mathcal{O}\left((M_P/M_{\text{cut}})^{D-2}\right)$ for the Kahler potential and $\mathcal{O}\left((M_P/M_{\text{cut}})^{D-3}\right)$ for the superpotential and gauge kinetic terms, where D is the dimension of the operator.

Since F_S is Z_2 odd, the couplings of S to SM and mirror fields seem to lead to large Z_2-breaking soft masses. However, a linear combination of the Z_2 symmetry and a Z_{4R} subgroup of R symmetry is unbroken by non-zero F_S, and the apparent Z_2 breaking can be removed by a discrete R rotation. Here we assign a vanishing R charge to the scalar component of S. Physical Z_2-breaking arises only by picking-up R symmetry breaking, which at the minimal is given by the non-zero VEV of the superpotential, namely, the gravitino mass. The effect of the R breaking can be expressed by the F term of the conformal compensator ϕ,

$$\phi = 1 + F_\phi \theta^2, \quad F_\phi = m_{3/2}. \tag{3.2}$$

As we will see, supergravity effects including anomaly mediation generate physical Z_2 breaking. A non-zero VEV of the scalar component of S also breaks R symmetry. However, as we will see in section 5.1, it is much smaller than the Planck scale in supersymmetry breaking models without the Polonyi problem and introduces negligible R symmetry breaking. In the following, we take $\langle S \rangle \ll M_P$ that will be justified in section 5.

3.1 Gaugino masses

Usually, in gravity mediation, tree-level gaugino masses of order scalar masses require that the Polonyi field be singlet under any symmetry. This is because the kinetic terms of gauge multiplets are singlet. In supersymmetric TH mechanism, the kinetic terms transform into their Z_2 partner. Therefore, S can couple to gauge multiplets via

$$W \supset k_i g_i^2 \frac{S}{M_*} \text{Tr}[-W_i^\alpha W_{\alpha i} + W_i^{'\alpha} W_{\alpha i}'], \tag{3.3}$$

where g_i are gauge coupling constants, W_i^α ($i = 1, 2, 3$) are the field strength superfields of the SM gauge fields with a spinor index α, and k_i are dimensionless coupling constants.

Eq. (3.3) provides Z_2-odd gaugino masses and their values at the TeV scale are $M_3 \simeq 1.3 k_3 m_{3/2}$, $M_2 \simeq 0.4 k_2 m_{3/2}$, and $M_1 \simeq 0.2 k_1 m_{3/2}$. The anomaly mediation [22, 23] gives Z_2-even gaugino masses. Adding them together, we obtain

$$M_i = -b_i \frac{\alpha_i}{4\pi} m_{3/2} - k_i g_i^2 m_{3/2}, \quad M_i' = -b_i' \frac{\alpha_i'}{4\pi} m_{3/2} + k_i g_i^2 m_{3/2}, \tag{3.4}$$

where $b_i = b_i' = (33/5, 1, -3)$ and $\alpha_i \equiv g_i^2/(4\pi)$. These b_is are the first beta function coefficients in the MSSM, which need properly modified in the extensions including additional particle contents depending on models.

As we anticipated, gaugino masses have physical Z_2 breaking only if both contributions are present. When $M_{\text{cut}} \sim M_P$, since $k_i = \mathcal{O}(1)$, the mass splitting between the MSSM and mirror gaugino is $O(0.1\text{-}1)$ %, so gaugino mass terms remain nearly degenerate. When $M_{\text{cut}} \ll M_P$, $k_i \sim M_P/M_{\text{cut}} \gg 1$ and even smaller splitting is expected. As we will see, this splitting can cause the splitting in the Higgs mass squared radiatively.

Couplings of S to charged fields Q in the Kähler potential,

$$K \supset \frac{S}{M_*}(Q^\dagger Q - Q'^\dagger Q'), \tag{3.5}$$

also generate one-loop suppressed gaugino masses [24]. These, however, are also Z_2-odd and negligible in comparison with the tree-level Z_2-odd one.

After SM and mirror EW symmetry breaking, $v' > v$ introduces further mass splitting for the bino and wino. This dominates over the radiative ones discussed above, and the twin bino and wino are lighter than the MSSM bino and wino. The twin bino ($\tilde{B}'$) LSP is studied in [10]. Because of $v' > v$, the fermions in the mirror sector is heavier than in the MSSM sector, which helps avoid the chirality suppression of the $\tilde{B}'$ annihilation into a pair of fermions in the mirror sector. This makes $\tilde{B}'$ annihilation cross section larger than in the MSSM sector, which can help avoid the overabundance of $\tilde{B}'$ dark matter and achieve $\Omega_{\tilde{B}'} h^2 \simeq 0.12$ [10].

In D-term SUSY TH models, the new gauge multiplet whose D-term is responsible for the SU(4) invariant quartic coupling should be Z_2 even so that it can couple to both the Higgs and Twin Higgs. The coupling of it to S is forbidden by the Z_2 symmetry, and the gaugino mass is absent at tree-level. Potential large corrections to the soft masses of Higgses due to the large gauge coupling constant are automatically suppressed.

3.2 A terms

A terms are also obtained at tree-level. For example, the following superpotential

$$W \supset d_u \frac{S}{M_*} y_u \left(Q H_u u^c - Q' H_u' u'^c \right), \tag{3.6}$$

with Q (u^c) being SU(2)$_L$ doublet (singlet) and Q' (u'^c) SU(2)$_L'$ doublet (singlet), gives rise to Z_2-odd A terms. Here y_u is the up-type yukawa coupling and d_u is a dimensionless coupling constant. The Kähler potential of the form in (3.5) also gives Z_2-odd A terms of the same order. On the other hand, the anomaly mediation [22, 23] gives Z_2-even A terms. When combined, A-terms for the two sectors can be written as

$$A^u = y_u d_u m_{3/2} - y_u (\gamma_Q + \gamma_{H_u} + \gamma_{u^c}) m_{3/2} \equiv A_S^u + A_{\mathrm{AMSB}}^u, \tag{3.7}$$

$$A'^u = -y_u d_u m_{3/2} - y_u (\gamma_{Q'} + \gamma_{H_u'} + \gamma_{u'^c}) m_{3/2} \equiv -A_S^u + A_{\mathrm{AMSB}}^u, \tag{3.8}$$

where γ_X is the anomalous dimension of the field X. When $M_{\mathrm{cut}} \sim M_P$, we expect $d_u, d_d = \mathcal{O}(1)$ and the splitting in the A terms is of $O(1)\%$. For $M_{\mathrm{cut}} \ll M_P$, $d_u, d_d \sim M_P / M_{\mathrm{cut}} \gg 1$ and the splitting would be smaller than $\mathcal{O}(1)\%$.

3.3 Sfermion masses

Sfermion masses $m_{\tilde{f}}$ are $O(m_{3/2} M_P / M_{\mathrm{cut}})$. At tree-level, there is no Z_2-breaking soft masses, since $|F_S|^2$ is Z_2 even. $F_S F_\phi^\dagger$ is Z_2-odd, but the Kähler potential $\phi^\dagger S Q Q^\dagger$ is forbidden by the formal conformal symmetry at tree-level. Such terms arise at loop-level and give Z_2-breaking soft masses. For example, quantum corrections from gaugino masses and/or A terms generate Z_2-breaking soft scalar masses.

Since $m_{\tilde{f}}^2$ receives quantum corrections from gaugino masses and A terms (see appendix. A), the splitting in these quantities between two sectors ($\Delta |M_i|^2$ and $\Delta |A^f|^2$) contributes to the RGE of the splitting in the sfermion mass $\Delta m_{\tilde{f}}^2 \equiv m_{\tilde{f}}^2 - m_{\tilde{f}'}^2$. Figure 1
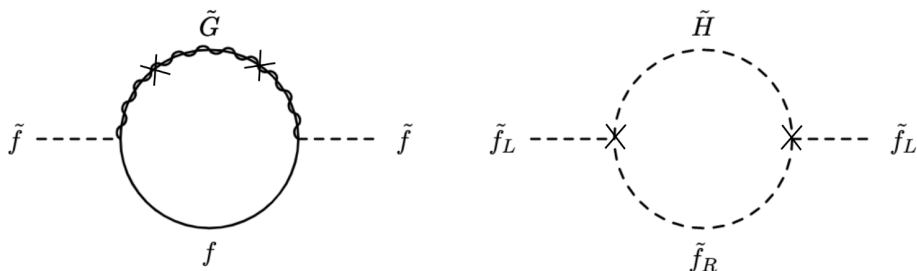
Figure 1. The one loop diagrams that result in difference in $\delta m_{\tilde{f}}$ and $\delta m_{\tilde{f}'}$. The crosses represent gaugino masses (left panel) or scalar trilinear couplings (right panel).

shows the one-loop diagrams contributing to these splittings. The interference between Z_2-even and odd contributions to gaugino masses and the trilinear couplings A^f in eqs. (3.4) and (3.8) respectively has the opposite sign for the two sectors and this leads to

$$\Delta|M_i|^2 = 4\left(b_i\frac{\alpha_i}{4\pi}m_{3/2}\right)(k_ig_i^2m_{3/2}), \quad \Delta|A^f|^2 = 4A_S^f A_{\text{AMSB}}^f. \tag{3.9}$$

By substituting eq. (3.9) into eq. (A.1) and solving RGEs, we estimate the sfermion mass splittings $\Delta m_{\tilde{f}}^2$ between the two sectors. We find that $\Delta m_{\tilde{f}}^2$ is dominated by ΔM_i^2 and

$$\Delta m_{\tilde{f}}^2 \simeq \begin{cases} (8\times 10^{-2})\times m_{3/2}\times M_3[1\text{TeV}] & \text{for } \tilde{Q}_L, \tilde{q}_R, \\ -(8\times 10^{-3})\times m_{3/2}\times M_2[1\text{TeV}] & \text{for } \tilde{L}_L, \\ -(5\times 10^{-2})\times m_{3/2}\times M_1[1\text{TeV}] & \text{for } \tilde{e}_R, \end{cases} \tag{3.10}$$

where we write $\Delta m_{\tilde{f}}^2$ in terms of the gravitino mass and tree-level gaugino masses in eq. (3.3) evaluated at the TeV scale. The sign difference in the sfermion mass splitting for squarks and sleptons is ascribed to $b_3 < 0$ and $b_2, b_1 > 0$.

The twin stau is a natural dark matter candidate. Ref. [11] studied the tree-level mass splitting between the MSSM and twin staus induced by $v' > v$, and found that the twin stau may be lighter than the MSSM stau if $\tan\beta$ is large and/or the twin tau yukawa is larger than the SM tau yukawa. The radiative mass splitting in eq. (3.10) can help realize the twin stau LSP if $m_{3/2}M_1 < 0$.

3.4 Higgs mass parameters

In this subsection, we study how the soft masses and the μ-term of Higgses are generated. We will see that the opposite signs of S couplings to the SM and mirror operators give a Z_2-breaking Higgs potential. We then compute the Z_2-breaking mass in the potential of the SM-like Higgs and the twin Higgs after integrating out heavy Higgses.

3.4.1 Tree-level mass terms

We begin with the tree-level contributions to the μ-term, b-term, and soft mass squared of the Higgses. To be concrete, we consider $R[H_uH_d]=0$, so that the μ and b terms can be naturally obtained from R and supersymmetry breaking.[1]

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[1]There can be another contribution to μ and b terms from the direct coupling of the SUSY-breaking sector to H_uH_d in the superpotential [25, 26]. As is shown in appendix. B, appropriate size of b terms may be obtained while μ terms cannot.

First, we study the tree-level sources of μ-term and b-term from the Kähler potential. Given a Kähler potential K, one can obtain the leading contribution by expanding $-3e^{-K/(3M_P^2)}$ below,

$$\mathcal{L} \supset -3M_P^2 \int d^2\theta d^2\bar{\theta} \; \phi^\dagger \phi e^{-K/(3M_P^2)} \,. \tag{3.11}$$

The following operators in Kähler potential including $H_u H_d$ are allowed by the Z_2 and R symmetry,

$$-3M_P^2 e^{-K/(3M_P^2)} \supset c_2(H_u H_d + H'_u H'_d) + \frac{r_3 S^\dagger + r_{3*} S}{M_*}(H_u H_d - H'_u H'_d)$$
$$+ r_4 \frac{S^\dagger S}{M_*^2}(H_u H_d + H'_u H'_d) + \text{h.c.} + \dots \,, \tag{3.12}$$

where c's and r's are the dimensionless coefficients of each operator and each subscript denotes the mass dimension of the associated operators. We use c for operators consisting of H_u and H_d only whereas r's are used for the couplings of S to Higgses.

After scaling out ϕ, using $\phi = 1 + F_\phi \theta^2$, and integrating over $\bar{\theta}^2$ on $\phi^\dagger$ and $S^\dagger$ and over θ^2 on $H_u H_d$, we obtain μ-parameters

$$\mu = c_2 F_\phi + r_3 \frac{F_S}{M_*} \,, \quad \mu' = c_2 F_\phi - r_3 \frac{F_S}{M_*} \,. \tag{3.13}$$

Integrating over $\theta^2 \bar{\theta}^2$ on ϕ, $\phi^\dagger$, S, and $S^\dagger$ gives b-parameters

$$b = -c_2 F_\phi^2 - r_3 F_\phi \frac{F_S}{M_*} + r_{3*} F_\phi \frac{F_S}{M_*} + r_4 \left(\frac{F_S}{M_*}\right)^2 \,,$$
$$b' = -c_2 F_\phi^2 + r_3 F_\phi \frac{F_S}{M_*} - r_{3*} F_\phi \frac{F_S}{M_*} + r_4 \left(\frac{F_S}{M_*}\right)^2 \,. \tag{3.14}$$

Note that the terms linearly proportional to F_S are Z_2-odd. Using $F_\phi \simeq F_S/M_* \simeq m_{3/2}$, μ and b-parameters are given by

$$\mu = (c_2 + r_3)m_{3/2} \,, \quad \mu' = (c_2 - r_3)m_{3/2} \,,$$

$$b = (-c_2 - r_3 + r_{3*} + r_4)m_{3/2}^2 \,, \quad b' = (-c_2 + r_3 - r_{3*} + r_4)m_{3/2}^2 \,. \tag{3.15}$$

Physical Z_2 breaking arises from the coexistence of Z_2-even contributions proportional to c_2 and r_4, and Z_2-odd contributions proportional to r_3 and r_{3*}.

The tree-level Higgs soft mass squared comes from the following Kähler potential,

$$-3M_P^2 e^{-K/(3M_P^2)} \supset SS^\dagger + H_u H_u^\dagger + H'_u H_u'^\dagger + r_4^u \left(\frac{S^\dagger S}{M_*^2} H_u H_u^\dagger + \frac{S^\dagger S}{M_*^2} H'_u H_u'^\dagger\right) \,. \tag{3.16}$$

Here we use the superscript u for the dimensionless coefficients of couplings to the up-type Higgs. For the down-type Higgs, we use the superscript d. Terms such as $SH_u H_u^\dagger$ can be eliminated by S-dependent rotation of H_u and $H_u^\dagger$. After scaling out ϕ, one can see that there are no terms proportional to $S^\dagger \phi^n H_u H_u^\dagger$ without extra S. Therefore, a Z_2-odd soft
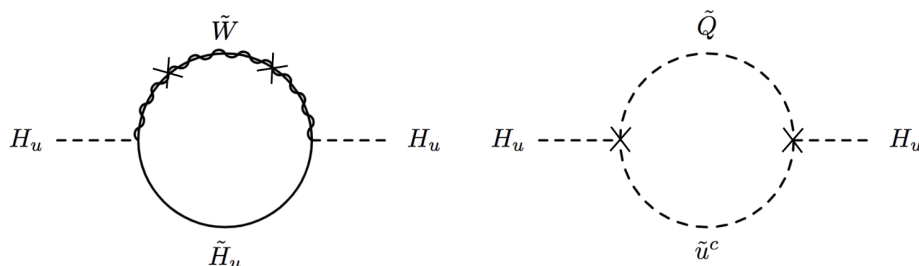
Figure 2. The one loop diagrams that result in difference in $\delta m^2_{H_u}$ and $\delta m^2_{H'_u}$. The crosses represent the wino mass (left panel) or scalar trilinear couplings (right panel).

mass term proportional to $F^\dagger_S F_\phi$ is suppressed by $\langle S \rangle / M_P$ and $m^2_{H_u} \simeq m^2_{H'_u}$ should hold at the tree-level. After integrating over $\theta^2 \bar\theta^2$, we obtain

$$m^2_{H_u} \simeq m^2_{H'_u} \simeq r^u_4 \left(\frac{F_S}{M_*} \right)^2 \equiv r^u \left(\frac{F_S}{M_*} \right)^2 . \tag{3.17}$$

The same applies for H_d and H'_d as well with the coefficients r^u's replaced with r^d's;

$$m^2_{H_u} \simeq m^2_{H'_u} \simeq r^u m^2_{3/2} , \quad m^2_{H_d} \simeq m^2_{H'_d} \simeq r^d m^2_{3/2} . \tag{3.18}$$

Note that μ and b terms generically have tree-level Z_2 breaking, which may induce too large Δm^2_H. We consider the following two cases where Δm^2_H can be naturally suppressed:

- Case I: $|r^u| \sim |r^d| = O(1) \gg c_2, r_3, r_{3*}, r_4$ at the TeV scale.

- Case II: $|r^u| \sim |r^d| \sim r_4 \gg r_3, r_{3*} \gg c_2$ at the TeV scale.

Case I is naturally realized when $M_{\rm cut} \sim M_P$ and couplings involving $H_u H_d$ are suppressed, for example by an approximate symmetry under which $H_u H_d$ is charged. The Z_2-even and Z_2-odd contributions to μ and b are comparable with each others, but since μ and b are smaller than $m^2_{H_u}$ and $m^2_{H_d}$, Δm^2_H is suppressed. For this, it suffices to have a hierarchy of parameters only around 0.3, since Δm^2_H has to be only smaller than $\mathcal{O}(0.1) m^2_H$ (see eq. (2.14)) and is proportional to μ^2 and b^2.

Case II is naturally realized when $M_{\rm cut} < M_P$. We then expect $c_2 = O(1)$, $r_3, r_{3*} = O(M_*/M_{\rm cut})$, and $r_4, r_u, r_d = O((M_*/M_{\rm cut})^2)$ at the UV scale, and the hierarchy $|r^u| \sim |r^d| \sim r_4 \gg r_3, r_{3*} \gg c_2$ is achieved. Z_2-odd and even contributions to μ and b dominate respectively and the Z_2 breaking Δm^2_H is suppressed. We also use re-scaled couplings $\tilde r = r \times (M_{\rm cut}/M_*)^{D_r - 2}$, where D_r is the dimension of the corresponding operators. For example, $\tilde r_4 = r_4 \times (M_{\rm cut}/M_*)^2$. The natural values of the rescaled couplings are $O(1)$.

3.4.2 Quantum corrections

Next, we investigate Z_2-breaking quantum corrections to the Higgs mass parameters. We first examine the one-loop radiative corrections $\delta m^2_{H_u}$ and $\delta m^2_{H_d}$ from the gauge and

Yukawa interactions. The splitting in wino masses in eq. (3.4) induces $\delta m^2_{H_u} - \delta m^2_{H'_u} \neq 0$ from gauge interactions as shown in the left panel of figure 2. The cross denotes the wino-mass insertion. Using the RGE for $m^2_{H_u}$ in appendix. A, we obtain

$$\Delta m^2_{H_u,\text{gauge}} \equiv \delta m^2_{H_u} - \delta m^2_{H'_u} \simeq \frac{(4 \times 6)}{16\pi^2} \times \int_{\log[1\text{TeV}]}^{\log[M_{\text{cut}}]} dt g_2^2 \left(b_2 \frac{\alpha_2}{4\pi} m_{3/2} \right) (k_2 g_2^2 m_{3/2})$$

$$= (8 \times 10^{-3}) \times m_{3/2} \times M_2[1\text{TeV}] = (90 \text{ GeV})^2 \frac{m_{3/2}}{\text{TeV}} \frac{M_2[1\text{TeV}]}{\text{TeV}}, \qquad (3.19)$$

where $t = d\log\mu$. The similar diagram for H_d gives the similar $\Delta m^2_{H_d,\text{gauge}} \equiv \delta m^2_{H_d} - \delta m^2_{H'_d}$. $\Delta m^2_{H_u,\text{gauge}}$ and $\Delta m^2_{H_d,\text{gauge}}$ are smaller than Δm^2_H in eq. (2.14) and do not introduce too much Z_2 breaking.

The splitting of the trilinear coupling in eq. (3.6) induces $\delta m^2_{H_u} - \delta m^2_{H'_u} \neq 0$ from the Yukawa interaction via the diagram shown in the right panel of figure 2, where the cross denotes trilinear scalar coupling. Using the RGE for $m^2_{H_u}$ in appendix. A.1 and the anomalous dimensions in appendix. A.3, we obtain

$$\Delta m^2_{H_u,\text{Yuk}} \simeq -\frac{(4 \times 6)}{16\pi^2} \times \int_{\log[1\text{TeV}]}^{\log[M_{\text{cut}}]} dt' \ A_S^t(t') A_{\text{AMSB}}^t(t')$$

$$\simeq -\frac{(4 \times 6)}{16\pi^2} \times m_{3/2} \int_{\log[1\text{TeV}]}^{\log[M_{\text{cut}}]} dt' \ A_S^t(t') y_t(t') (\gamma_{Q_3} + \gamma_{H_u} + \gamma_{u_3^c}). \qquad (3.20)$$

$A_S^t(t)$ in general depends on the UV boundary condition of it and the gaugino mass. To be concrete, we assume that $A_S^t(M_{\text{cut}}) = 0$ and dA_S^t/dt is dominantly determined by the one-loop correction from the top Yukawa, gauge interaction, and the gaugino mass; see appendix. A.2. We then obtain

$$\Delta m^2_{H_u,\text{Yuk}} \simeq -0.01 \times m_{3/2} \times M_3[1\text{TeV}] \simeq -(200\text{GeV})^2 \frac{m_{3/2}}{\text{TeV}} \frac{M_3[4\text{TeV}]}{4\text{TeV}}. \qquad (3.21)$$

$\Delta m^2_{H_u,\text{Yuk}}$ is typically larger than $\Delta m^2_{H_u,\text{gauge}}$ and may be as large as Δm^2_H in eq. (2.14) to provide an appropriate size of Z_2 breaking.

There are radiative corrections to b and b' parameters as well. The one-loop corrections δb and $\delta b'$ are similar to what is shown in the left panel of figure 2 with the external line for $H_u^\dagger$ replaced with H_d and a single mass insertion for each of the wino and Higgsino propagator instead of two mass insertions in the Higgsino propagator. If $\text{sign}[b] = +(-)\text{sign}[b']$ at tree-level, quantum corrections such that $\delta b - \delta b'(\delta b + \delta b') \neq 0$ contribute to physical Z_2 breaking. For Case II, the assumed hierarchy $r_4 \gg r_3, r_{3*}, c_2$ ensures that $\text{sign}[b] = \text{sign}[b']$. For Case I, both signs are possible. To be concrete, we discuss the case with $\text{sign}[b] = \text{sign}[b']$ and compute $\delta b - \delta b'$.

In Case I, the correction comes from the tree-level Z_2-even μ term proportional to c_2 and the tree-level Z_2-odd wino mass. Solving the RGE of b and b' in eq. (A.2), we find $\delta b - \delta b'$ at the TeV scale to be

$$\Delta m^2_{H,b} \equiv \delta b - \delta b' \simeq -(400\,\text{GeV})^2 \frac{\mu[1\text{TeV}]}{0.3\text{TeV}} \frac{M_2[1\text{TeV}]}{\text{TeV}}, \qquad (3.22)$$

where we assume that the μ term is dominated by the Z_2-even contribution. The contribution of $\Delta m_{H,b}^2$ to Δm_H^2 may be suppressed by $\sin 2\beta < 1$. However, as we will see in section 4.2, the consistency of the SM-like Higgs mass with a TeV scale stop mass requires $\tan \beta \sim 2-3$ and $\sin 2\beta \sim 0.7$ does not provide suppression. The resultant Δm_H^2 is comparable to the required one in (2.14).

In Case II, although μ terms are dominated by the Z_2-odd terms, the dominant contribution to $\delta b - \delta b'$ comes from the interference between the tree-level Z_2-even contribution to μ proportional to c_2 and the tree-level Z_2-odd contribution to the gaugino mass. At the TeV scale it is

$$\Delta m_{H,b}^2 \equiv \delta b - \delta b' \simeq -(240\text{GeV})^2 \left(\frac{c_2}{0.1 r_3} \right) \frac{\mu[1\text{TeV}]}{\text{TeV}} \frac{M_2[1\text{TeV}]}{\text{TeV}} , \tag{3.23}$$

which is as large as the required one in (2.14) if $c_2 \sim 0.1 r_3$, corresponding to $M_{\text{cut}} \sim 0.1 M_*$.

The above quantum corrections to $b - b'$ may be understood as corrections to $r_3 - r_{3*}$ and suggest that the assumed hierarchy for Case I and II is natural. Other types of hierarchy are not radiatively stable. For example, the hierarchy $|r^u| \sim |r^d| \sim c_2 = O(1) \gg r_3, r_{3*}, r_4$ could suppress the Z_2-breaking μ and b terms, but the correction in eq. (3.22) makes this hierarchy radiatively unstable.

4 Higgs potential and Higgs mass

In this section, we compute the potential of the Higgs and Twin Higgs in the decoupling limit and compute the mass of the SM-like Higgs.

4.1 The difference of $\tan \beta$

In this subsection, we compute the difference between $\tan \beta$ and $\tan \beta'$ and its effect on the Higgs mass terms in the decoupling limit.

Case I. For the Higgs potential to be bounded from below, $|2b'| < m_{H'_u}^2 + m_{H'_d}^2 + 2|\mu'|^2$ is required. In terms of the parameters introduced in section 3.4.1,

$$2|c_2 - r_4 - r_3 + r_{3*}| < 2(c_2 - r_3)^2 + r^u + r^d . \tag{4.1}$$

Without loss of generality, we assume $c_2 - r_4 > 0$. We also assume $|r_3 - r_{3*}| < c_2 - r_4$ for simplicity. $b, b' < 0$ for this case, so we flip the sign of $H_u H_d$ to take $b, b' > 0$. Then from eq. (2.5), $\sin 2\beta$ and $\sin 2\beta'$ are given by

$$\sin 2\beta = \frac{2(c_2 - r_4) + 2(r_3 - r_{3*})}{2(c_2 + r_3)^2 + r^u + r^d}, \quad \sin 2\beta' = \frac{2(c_2 - r_4) - 2(r_3 - r_{3*})}{2(c_2 - r_3)^2 + r^u + r^d} . \tag{4.2}$$

Z_2 breaking in the Higgs potential gives a small difference between $\tan\beta$ and $\tan\beta'$. Using the relation $\tan \beta \simeq (2/\sin 2\beta) - (\sin 2\beta/2)$ and denoting the leading quantum corrections to μ, b and soft masses squared of Higgs as $\delta \mu$, δb, $\delta m_{H_u}^2$ and $\delta m_{H_d}^2$, we find $\Delta \tan \beta$ at leading order is given by

$$\begin{aligned}
\Delta \tan \beta \simeq & [(m_{H_u}^2 + m_{H_d}^2)(b' - b + \delta b' - \delta b) + 2\mu^2 \delta b' - 2\mu'^2 \delta b \\
& + b'\{\delta m_{H_u}^2 + \delta m_{H_d}^2 + 2(\mu^2 + \delta \mu^2)\} - b\{\delta m_{H'_u}^2 + \delta m_{H'_d}^2 + 2(\mu'^2 + \delta \mu'^2)\}] \\
& \times (bb' + b'\delta b + b\delta b')^{-1} .
\end{aligned} \tag{4.3}$$

Taking into account the assumed hierarchy among parameters, this can be approximated as

$$\Delta \tan \beta \approx \frac{(r^u+r^d)[2(r_{3*}-r_3)-0.3k_2(c_2+r_3)]-0.6k_2(c_2+r_3)(c_2^2+r_3^2)}{(c_2-r_4)^2-(r_3-r_{3*})^2}$$

$$+\frac{(c_2-r_4)(\mathcal{O}(10^{-2})k_3+8c_2r_3)-(r_3-r_{3*})[4(c_2^2+r_3^2)]}{(c_2-r_4)^2-(r_3-r_{3*})^2}$$

$$\approx \frac{(r^u+r^d)(2(r_{3*}-r_3)-0.3k_2(c_2+r_3))+8(c_2-r_4)c_2r_3}{(c_2-r_4)^2}, \tag{4.4}$$

where we used $|r_3-r_{3*}|<|c_2-r_4|$ in the second equality.

Let us assess the effect of $\Delta\tan\beta$ on Z_2 breaking in the Higgs mass terms. In the decoupling limit, the quadratic terms of the Higgs and Twin Higgs are

$$V(H^\cdot H')_{\text{quadratic}} = \left[(c_2+r_3)^2+r^d\cos^2\beta+r^u\sin^2\beta-(c_2-r_4+r_3-r_{3*})\sin 2\beta\right]m_{3/2}^2|H|^2$$

$$+\left[(c_2-r_3)^2+r^d\cos^2\beta'+r^u\sin^2\beta'-(c_2-r_4-r_3+r_{3*})\sin 2\beta'\right]m_{3/2}^2|H'|^2. \tag{4.5}$$

The Z_2-breaking mass of the Higgs from $\Delta\tan\beta$ is, to the leading order in $(\Delta\tan\beta)/\tan\beta$,

$$\Delta m_{H,\Delta\beta}^2 \simeq \left(r^u-r^d+(c_2-r_4)\tan\beta\right)\frac{2\Delta\tan\beta}{\tan^3\beta}m_{3/2}^2 \tag{4.6}$$

$$\simeq \frac{r^u-r^d+(c_2-r_4)\tan\beta}{(c_2-r_3)^2+r^d\cos^2\beta'+r^u\sin^2\beta'-(c_2-r_4)\sin 2\beta'}\frac{2\Delta\tan\beta}{\tan^3\beta}m_{H'}^2.$$

As we will see in section 4.2, the observed Higgs mass and the stop mass of $\mathcal{O}(1)$ TeV requires that $\tan\beta = 2\text{-}3$. $\Delta\tan\beta$ should not be larger than $O(1)$ in order to suppress $\Delta m_{H,\Delta\beta}^2$. For $c_2, r_4 < r^{u,d}$, to suppress the term proportional to $r_{3*}-r_3$ in eq. (4.4) requires that $r_{3*}-r_3$ be somewhat smaller than c_2-r_4.

Case II. The analysis for Case II is parallel to that of Case I. Without loss of generality, we take $r_4 > 0$. The Higgs potential is bounded from below if

$$2r_4 < 2r_3^2+r^u+r^d. \tag{4.7}$$

$\sin 2\beta$ and $\sin 2\beta'$ at the leading order are

$$\sin 2\beta, \ \sin 2\beta' \simeq \frac{2r_4}{r^u+r^d+2r_3^2} = \frac{2\tilde{r}_4}{\tilde{r}^u+\tilde{r}^d+2\tilde{r}_3^2}, \tag{4.8}$$

where we used $r^u = \tilde{r}^u(M_*/M_{\text{cut}})^2$, $r^d = \tilde{r}^d(M_*/M_{\text{cut}})^2$, $r_3 = \tilde{r}_3(M_*/M_{\text{cut}})$ and $r_4 = \tilde{r}_4(M_*/M_{\text{cut}})^2$ for the last equality.

$\Delta\tan\beta$ is given by the generic formula in eq. (4.3). Taking into account the hierarchy of the parameters,

$$\Delta\tan\beta \approx \frac{2(r^u+r^d)(r_3-r_{3*})-(4\times 10^{-3})r_3^3-(c_2-r_4)(\mathcal{O}(10^{-2})k_3+8c_2r_3)}{r_4^2}$$

$$\approx \mathcal{O}(10^{-2})\frac{k_3}{r_4}+\frac{2(r^u+r^d)(r_3-r_{3*})}{r_4^2}$$

$$= \mathcal{O}(10^{-2})\frac{M_{\text{cut}}}{M_*}\frac{\tilde{k}_3}{\tilde{r}_4}+\frac{M_{\text{cut}}}{M_*}\frac{2(\tilde{r}^u+\tilde{r}^d)(\tilde{r}_3-\tilde{r}_{3*})}{\tilde{r}_4^2} \tag{4.9}$$

For $\mathcal{O}(1)$ couplings with tildes, the second term dominates and $\Delta\tan\beta = \mathcal{O}(M_{\text{cut}}/M_*)\ll 1$.

In the decoupling limit, the quadratic terms of the Higgs and Twin Higgs are

$$
\begin{aligned}
V(H, H')_{\text{quadratic}} = {} & \left[r_3^2 + r^d \cos^2 \beta + r^u \sin^2 \beta - r_4 \sin 2\beta \right] m_{3/2}^2 |H|^2 \\
& + \left[r_3^2 + r^d \cos^2 \beta' + r^u \sin^2 \beta' - r_4 \sin 2\beta' \right] m_{3/2}^2 |H'|^2 \,,
\end{aligned} \tag{4.10}
$$

where we have included only the Z_2 breaking through $\beta \neq \beta'$. Z_2-breaking mass is, to the leading order in $(\Delta \tan \beta)/\tan \beta$,

$$
\begin{aligned}
\Delta m_{H, \Delta\beta}^2 \simeq {} & (r^u - r^d + r_4 \tan \beta) \frac{2\Delta \tan \beta}{\tan^3 \beta} m_{3/2}^2 \\
\simeq {} & \frac{r^u - r^d + r_4 \tan \beta}{r^d \cos^2 \beta' + r^u \sin^2 \beta' - r_4 \sin 2\beta'} \frac{2\Delta \tan \beta}{\tan^3 \beta} m_{H'}^2 \,.
\end{aligned} \tag{4.11}
$$

For $|r^u| \sim |r^d| \sim r_4$ and $\tan\beta = 2\text{-}3$, $\Delta m_{H\Delta\beta}^2$ may be as large as $\Delta m_{H,\text{req}}^2$ in eq. (2.14) if $\Delta \tan \beta = \mathcal{O}(M_{\text{cut}}/M_*)$ is not too much smaller than 1.

4.2 The SM-like Higgs mass

In this subsection, we compute the SM-like Higgs mass and determine the preferred value of the stop mass and $\tan \beta$.

The Higgs potential is given by eq. (2.6) plus quantum corrections around the soft mass scale, which are dominated by the top Yukawa coupling. The one loop correction is given by

$$
\begin{aligned}
V_{\text{SU(4)}}^{\text{top}} \simeq {} & \frac{3}{16\pi^2} (\hat{g}_t^2 |H|^2 + m_{\tilde{t}}^2)^2 \left[\log \left(\frac{\hat{g}_t^2 |H|^2 + m_{\tilde{t}}^2}{\mu_R^2} \right) - \frac{3}{2} \right] - \frac{3}{16\pi^2} \hat{g}_t^4 |H|^4 \left[\log \left(\frac{\hat{g}_t^2 |H|^2}{\mu_R^2} \right) - \frac{3}{2} \right] \\
& + \frac{3}{16\pi^2} (\hat{g}_t'^2 |H'|^2 + m_{\tilde{t}}^2)^2 \left[\log \left(\frac{\hat{g}_t'^2 |H'|^2 + m_{\tilde{t}}^2}{\mu_R^2} \right) - \frac{3}{2} \right] - \frac{3}{16\pi^2} \hat{g}_t'^4 |H'|^4 \left[\log \left(\frac{\hat{g}_t'^2 |H'|^2}{\mu_R^2} \right) - \frac{3}{2} \right] \,,
\end{aligned} \tag{4.12}
$$

where μ_R is a renormalization scale, $\hat{g}_t \equiv y_t$, and $\hat{g}_t' \equiv y_t \sin \beta' / \sin \beta$ with y_t the top Yukawa coupling. We assumed identical left-handed and right-handed stop masses $m_{\tilde{t}}$ for simplicity.

In figure 3, we show the contours of the SM-like Higgs mass computed from the potentials in eqs. (2.6) and (4.12). In the computation, we take $v = 174\text{GeV}$, $f/v = 3$, $\lambda_X = 2$, and $\mu_R = 1\text{TeV}$. Each solid and dashed line corresponds to $\Delta \tan \beta = -1$ and 10^{-2} respectively. We find that the predicted Higgs mass is insensitive to $\Delta \tan \beta$ as long as $\Delta \tan \beta \lesssim 1$. The Higgs mass decreases as f/v or λ_X decrease. To satisfy the experimental lower bound on the stop mass of 1 TeV [27–29], $\tan \beta \lesssim 3$ is required.

In figure 4, we show the contours of $\tan \beta$ and the tree-level Δm_H^2 in the (c_2, r^d) and $(\tilde{r}_4, \tilde{r}^d)$ planes. Here we normalize Δm_H^2 by the required value $\Delta m_{H,\text{req}}^2$ in eq. (2.14). The upper and lower panels correspond to case I and II, respectively. In the orange shaded regions, stable mirror EW symmetry-breaking vacua do not exist, i.e., $2b' > m_{H_u'}^2 + m_{H_d'}^2 + 2|\mu'|^2$ or $b'^2 < (|\mu'|^2 + m_{H_u'}^2)(|\mu'|^2 + m_{H_d'}^2)$.

For Case I, as an example, we took $r^u = -0.7$. The gray dot-dashed and black dashed lines give $\tan \beta = 2$ and 3, respectively. For $r_3, r_{3*} < c_2, r_4 = \mathcal{O}(0.1) < r^{u,d} = \mathcal{O}(1)$, $|\Delta \tan \beta| =$
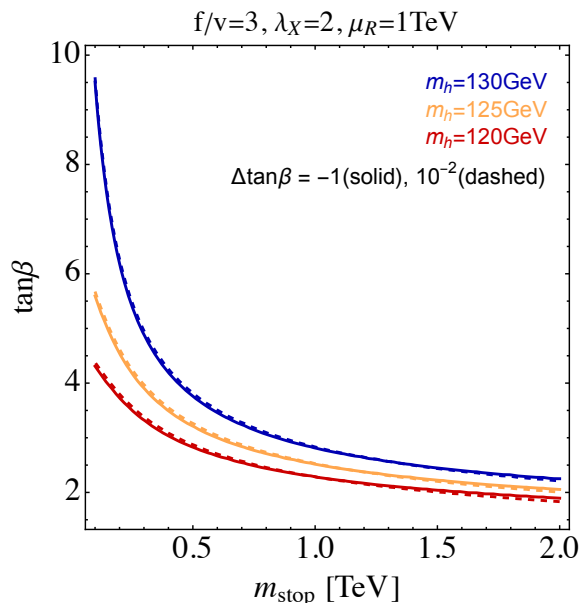
Figure 3. The SM-like Higgs mass as a function of m_{stop} and $\tan\beta$. For the solid and dashed lines, $\Delta\tan\beta \equiv \tan\beta - \tan\beta' = -1$ and 10^{-2} are assumed, respectively.

$\mathcal{O}(0.1)$ according to eq. (4.4). For the TeV scale m_{stop}, the observed Higgs mass is explained for $\tan\beta = 2-3$, which can be achieved when $m_{H_u}^2 + m_{H_d}^2 \sim b$, i.e., $r^u + r^d \sim c_2 - r_4$. Along with $c_2 = 0.1 - 0.3$ for suppressing $\Delta m_{H,b}^2$ in eq. (3.22), this requirement results in the parameter space shown in two upper panels in figure 4. The comparison of the two panels shows that a smaller r_4 requires a smaller c_2 and more fine-tuned cancellation between r^u and r^d for maintaining $\tan\beta \sim 3$ and the sufficient suppression of $\Delta\tan\beta$.

For Case II, as an example, we take $(\tilde{r}^u, \tilde{r}_3, \tilde{r}_{3*}, c_2, M_*/M_{\text{cut}}) = (-1, 1, 0.5, 0, 10)$ and $(-1, 1, 0.5, 0, 20)$. As c_2 increases, the orange shaded region becomes broader and the viable parameter space becomes narrower. On the other hand, as M_*/M_{cut} increases, the Z_2 breaking in the Higgs mass terms becomes smaller and the contours of Δm_H^2 moves to the left, so that the tree-level Z_2 breaking cannot provide the required Δm_H^2 starting from $M_*/M_{\text{cut}} \sim 40$. Also, for a given soft mass scale, as M_*/M_{cut} increases, $m_{3/2}$ becomes smaller, and Δm_H^2 from quantum corrections becomes smaller. Therefore, obtaining the required amount of Δm_H^2 from tree-level or one-loop level Z_2 breaking requires that $m_{3/2} > \mathcal{O}(100)\text{GeV}$.
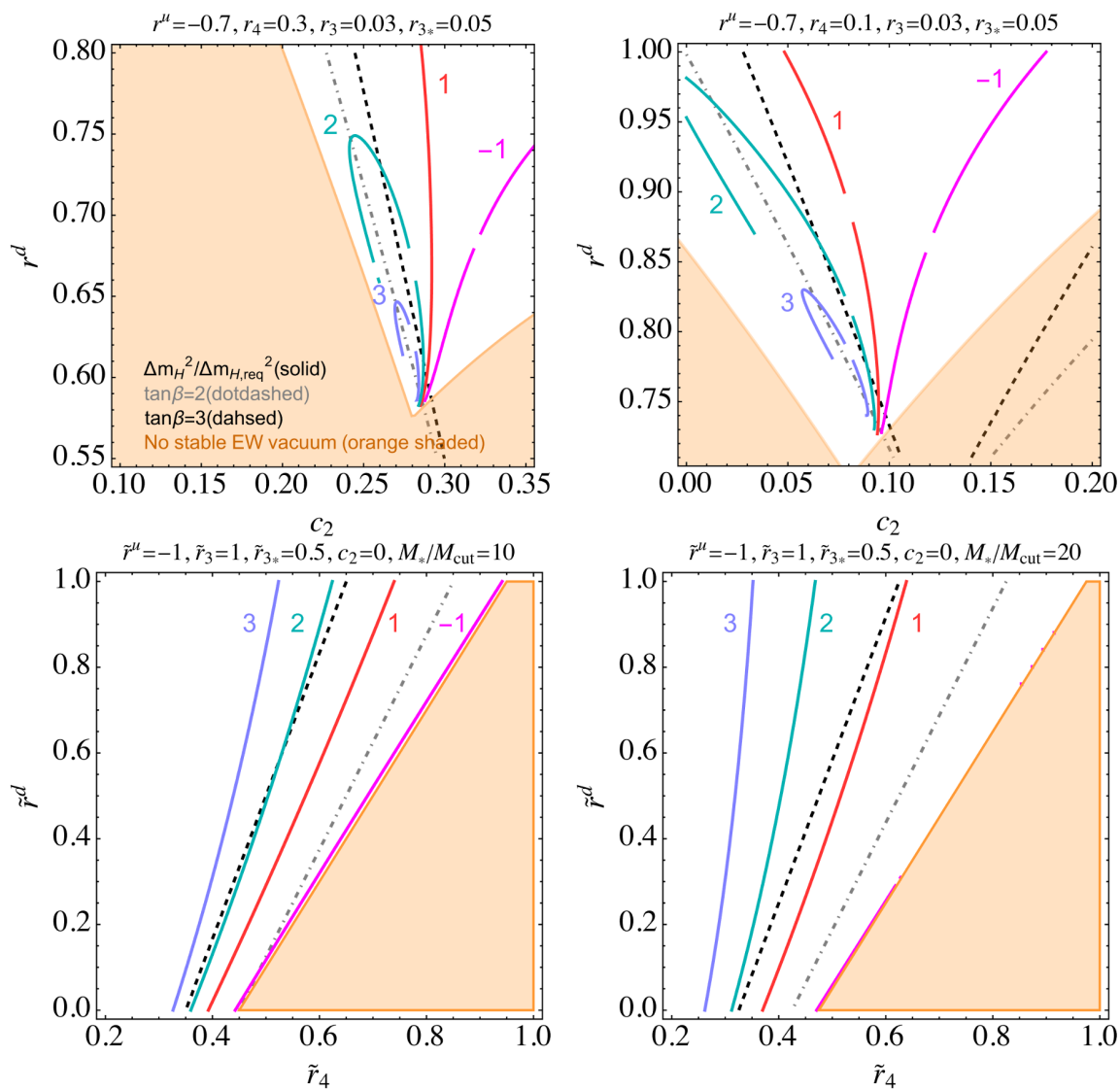
Figure 4. Contours of $\tan\beta$ and the tree-level Z_2-breaking Higgs mass term Δm_H^2. There is no stable EW vacuum in the orange shaded region. Upper and lower panels correspond to Case I and II, respectively.

	Q_i	S_{ij}	H_u	H'_u	H_d	H'_d
Sp(1)	$\square$	-	-	-	-	-
Z_{4R}	$+1$	0	x	x	$-x$	$-x$
Z_4	$+1$	$+2$	$H_u \leftrightarrow H'_u$		$H_d \leftrightarrow H'_d$	

Table 1. Charge assignment for the chiral superfields.

5 UV completion and early universe cosmology of Polonyi field

In this section, we present a concrete UV completion where the Z_2-odd Polonyi field obtains a large enough mass around the origin. We then discuss the cosmological evolution of the Polonyi field and show how the model can successfully address the cosmological problem which it suffers from.[2]

5.1 SUSY-breaking sector

We construct Planck scale mediated SUSY-breaking scenarios where the SUSY-breaking occurs in a hidden sector. To obtain gaugino masses as large as scalar masses, the SUSY-breaking field should be a gauge-singlet. To be concrete, we analyze the Izawa-Yanagida-Intriligator-Thomas model [35, 36], but similar discussion is applicable to generic O'Raifeartaigh-type SUSY-breaking models with gauge-singlet SUSY-breaking fields.[3]

We consider Sp(1) gauge theory with $N_F = 4$ matter fields Q_i with i being the flavor index running from 1 to 4. We also introduce singlet fields S_{ij} with anti-symmetric indices. As we will see, a linear combination of S_{ij} is a SUSY-breaking field. We may impose an anomaly-free discrete R-symmetry Z_{4R} and discrete flavor symmetry Z_4. For the vanishing mixed anomaly of $Z_{4R} - [\mathrm{SU}(2)_L]^2$ and $Z_{4R} - [\mathrm{SU}(3)_c]^2$ within the MSSM, $R[H_u H_d] = 0$ modulo 4 is required [41]. The same applies for the mirror sector. Therefore, we choose $R[H_u H_d] = R[H'_u H'_d] = 0$. In table 1, we show the charge assignment for the SUSY-breaking sector and (H_u, H_d) and (H'_u, H'_d). Under Z_4, S_{ij} is transformed into $-S_{ij}$ and the MSSM

---

[2]Given the cross quartic coupling in eq. (2.6), the SM Higgs-mediated scattering keeps the visible and mirror sectors in the thermal equilibrium until the temperature $T_D = \mathcal{O}(1)\mathrm{GeV}$ is reached [4]. This makes states in the mirror sector lighter than T_D contribute to the extra effective neutrino species ΔN_{eff} which is large as $5 - 6$ [30], which is not consistent with the constraint from CMB, $\Delta N_{\mathrm{eff}} \lesssim 0.1$ [31]. Hard Z_2-breaking in the yukawa coupling can raise the twin fermions masses and the twin QCD phase transition temperature and reduce ΔN_{eff} to an acceptable value [32, 33]. A heavy particle decay-induced entropy production after the decoupling of the two sectors, i.e. $T < T_D$, but before the BBN era, can also resolve this problem. As an example, the right-handed (s)neutrino can serve as the heavy particle to inject the entropy dominantly to the visible sector [30, 34].

[3]As R-breaking operators may pose a challenge to the stability for the SUSY-breaking vacuum [37], it is important for the model to have a discrete R-symmetry. However, it may be violated by the gravitational instanton effects although it remains uncertain if the path integral needs to include all the gravitational instantons [38]. For example, the Eguchi-Hanson instanton [39] can break R-symmetry down to Z_{2R} [40]. For our purpose, it suffices to assume the discrete Z_{4R} as an approximate symmetry, rather than an exact gauge symmetry. We assume that the gravitational instanton effects can be exponentially suppressed so that it does not introduce too a large R-violation.

sector is transformed into the mirror sector, so the Z_4 symmetry may be identified with the exchange symmetry in the TH model.

The superpotential of the SUSY-breaking sector above the dynamical scale Λ_* is

$$W \supset \lambda_{ijk\ell} S_{ij} Q_k Q_\ell \,, \tag{5.1}$$

where $\lambda_{ijk\ell}$ is a dimensionless coupling constant. For simplicity, we take $\lambda_{ijk\ell} = \lambda \delta_{ik} \delta_{j\ell}$, which preserves global $SU(4)$ symmetry. Below the dynamical scale, there arises a deformed moduli constraint $\mathrm{Pf}[Q_i Q_j] = \Lambda_*^4$ [42]. The SUSY-breaking sector can be described by the effective theory of the meson fields and the quantum moduli constraint can be equivalently rewritten as $\mathrm{Pf}[M_{ij}] = \Lambda_*^2/(4\pi)^2$, where six meson fields are defined via $M_{ij} \equiv Q_i Q_j/(4\pi\Lambda_*)$. The mesons are sixplet of the global $SU(4) \simeq SO(6)$, and the global rotation allows us to take a basis where $\langle M_1 \rangle = \Lambda_*/(4\pi)$ and $\langle M_a \rangle = 0$ ($a = 2-6$). We denote the corresponding basis of S_{ij} fields as S_a. We denote M_1 and S_1 simply as M and S from this point forward.

After integrating out M with the constraint $MM + M_a M_a = \Lambda_*^2/(16\pi^2)$, the SUSY-breaking sector can be effectively described by [43]

$$
\begin{aligned}
K_{\mathrm{eff}} &= S^\dagger S + S_a^\dagger S_a + M_a^\dagger M_a + \dots \,, \\
W_{\mathrm{eff}} &= \frac{\lambda}{4\pi} (\Lambda_* S \sqrt{\Lambda_*^2/(16\pi^2) - M_a M_a} + \Lambda_* S_a M_a) \,.
\end{aligned}
\tag{5.2}
$$

Here the factors of 4πs in W_{eff} are determined in accordance with the naive dimensional analysis [44, 45].

The F terms of S and S_a are

$$
\begin{aligned}
\frac{\partial W_{\mathrm{eff}}}{\partial S} &= \frac{\lambda}{4\pi} \Lambda_* \sqrt{\Lambda_*^2/(16\pi^2) - M_a M_a} \,, \\
\frac{\partial W_{\mathrm{eff}}}{\partial S_a} &= \frac{\lambda}{4\pi} \Lambda_* M_a \,.
\end{aligned}
\tag{5.3}
$$

We see that $\partial W_{\mathrm{eff}}/\partial S = 0$ and $\partial W_{\mathrm{eff}}/\partial S_a = 0$ cannot be satisfied simultaneously, which implies SUSY-breaking. At the minimum of the potential, $\mathrm{Im} M_a = 0$, while $\mathrm{Re} M_a$ is not fixed. They correspond to the Nambu-Goldstone bosons associated with $SO(6)/SO(5)$. They can be lifted by breaking $SO(6)$ symmetry with different Yukawa couplings in eq. (5.1) without disturbing the SUSY-breaking dynamics [43]. In particular, by taking the Yukawa coupling of S to be smaller than that of S_a, it is ensured that $\langle M \rangle = \Lambda_*/(4\pi)$ and $\langle M_a \rangle = 0$. From the F-term of S, $F_S = \lambda \Lambda_*^2/(4\pi)^2$, and the vanishing vacuum constant, the gravitino mass is given by

$$m_{3/2} = \frac{\lambda}{\sqrt{3} M_P} \left(\frac{\Lambda_*}{4\pi} \right)^2 \,. \tag{5.4}$$

The F-term conditions of M_a give

$$\frac{\partial W_{\mathrm{eff}}}{\partial M_a} \simeq \frac{\lambda}{4\pi} \frac{\Lambda_* S M_a}{\sqrt{(\Lambda_*^2/16\pi^2) - M_a M_a}} - \frac{\lambda}{4\pi} \Lambda_* S_a = 0 \quad \Rightarrow \quad S_a \simeq \frac{S M_a}{\sqrt{(\Lambda_*^2/16\pi^2) - M_a M_a}} \,. \tag{5.5}$$

So each S_a is fixed for a given M_a. For our choice of the basis with $\langle M \rangle = \Lambda_*/(4\pi)$ and $\langle M_a \rangle = 0$ ($a = 2-6$), we have $\langle S_a \rangle = 0$.

S is massless at the tree-level [46]. Once we integrate out heavy fields at the energy scale below Λ_*, we obtain

$$W_{\text{eff}} \supset \lambda \left(\frac{\Lambda_*}{4\pi}\right)^2 S\,. \tag{5.6}$$

This form of W_{eff} applies to all range of S; for $\lambda S > \Lambda_*$, the theory becomes a pure gauge theory with a S-dependent gauge coupling and gaugino condensation induces the effective superpotential in eq. (5.6).

Let us discuss the potential of S given by one-loop corrections. For $\lambda S \ll \Lambda_*$, the Kähler potential of S is given by [43, 47]

$$K_{\text{eff}} \supset |S|^2 - \frac{\eta \lambda^2}{4} \frac{|S|^4}{\Lambda_*^2} + \cdots\,, \tag{5.7}$$

where η is a constant that is expected to be $\mathcal{O}(1)$. From eqs. (5.6) and (5.7), the potential of S is

$$V(S) \simeq \left(\frac{\lambda \Lambda_*^2}{(4\pi)^2}\right)^2 \left(1 + \eta \lambda^2 \frac{|S|^2}{\Lambda_*^2}\right) \quad (\text{for } \lambda S \ll \Lambda_*)\,. \tag{5.8}$$

Near the origin of S field space, the mass of S is

$$m_S^2 \simeq \frac{\eta \lambda^3}{(4\pi)^2} \sqrt{3} M_P m_{3/2}\,. \tag{5.9}$$

Note that there is no linear term of S in the Kähler potential and no tilt of the potential that could destabilize S from around the origin. This is in contrast to the MSSM case, where a linear term arises at quantum level even if Z_2 symmetry is imposed to the tree-level potential, and the Polonyi problem is reintroduced [47].

The condensation of QQ breaks $Z_4 \times Z_4$ down to Z_{4R}, under which S has a charge of 2. Z_{4R} is further broken down to Z_{2R} by the non-zero gravitino mass. There is no residual symmetry under which S is charged and we expect a non-zero vev of S. Indeed, the effective superpotential in eq. (5.6) and the supergravity effect induce a tadpole term of S, $V(S) \supset \lambda m_{3/2} \Lambda_*^2 S/(4\pi)^2$. The balance between the tadpole term and the mass term in eq. (5.8) gives

$$\langle S \rangle \simeq \frac{(4\pi)^2}{2\lambda^3 \eta} m_{3/2}\,, \tag{5.10}$$

which is much smaller than M_P. Also, as long as $\lambda > \Lambda_*/M_P$, $\lambda\langle S \rangle$ is indeed smaller than Λ_* and the above computation is consistent.

The non-zero vev of S may result in a non-zero F term of the meson field M. To see that this is possible, we introduce a Lagrange multiplier X for the deformed moduli constraint, which may be understood as a glueball field,

$$W \supset 4\pi X \left(M^2 - \frac{\Lambda_*^2}{16\pi^2}\right)\,. \tag{5.11}$$

The F term of M is

$$-\langle F_M^* \rangle = \frac{\lambda}{4\pi} \Lambda_* \langle S \rangle + 8\pi \langle X \rangle \langle M \rangle = \frac{(4\pi)}{2\lambda^2 \eta} \Lambda_* m_{3/2} + 2\Lambda_* \langle X \rangle\,. \tag{5.12}$$

It is in principle possible that the two terms cancel with each other, but we expect that the F term is of the same order as the first term. This has an implication to the possibility where the μ term of the Higgs comes from the meson condensation as discussed in appendix B.

5.2 Cosmological evolution of the Polonyi field

The gravitino mass in the range $m_{3/2} > \mathcal{O}(0.1-1)\text{TeV}$ gives the lower bound on the SUSY-breaking scale via eq. (5.4),

$$m_{3/2} > \mathcal{O}(0.1-1)\text{TeV} \quad \rightarrow \quad \Lambda_* \gtrsim \mathcal{O}(10^{10})\text{GeV}. \tag{5.13}$$

We note that there can be two sources of domain wall problem: the spontaneous breaking of Z_4 and Z_{4R} when the Sp(1) gauge theory becomes strongly coupled and the $\langle QQ \rangle$ condensation forms. To avoid this problem, we require

$$\Lambda_* \gtrsim H_{\text{inf}}, \tag{5.14}$$

where H_{inf} is the Hubble expansion rate during the inflation.[4]

The mass of the Polonyi field is given by eq. (5.9). Let us first discuss the case with $H_{\text{inf}} > m_S$. During the inflation, S field is fixed at the minimum of Hubble-induced potential. We assume that the Hubble induced mass is positive so that the origin, $S = 0$, is the minimum. Note that this is possible because of the Z_2-odd nature of S.

After inflation ends, the Hubble-induced mass decreases and when H becomes as small as m_S in eq. (5.9), S field starts the coherent oscillation around the minimum of $V(S)$ given by eq. (5.10). The amplitude of the oscillation is comparable to $\langle S \rangle = \mathcal{O}((4\pi)^2 m_{3/2})$, which is much smaller than M_P. Therefore, the initial energy density of S field ($\rho_{S,\text{ini}}$) gets reduced by a factor $(m_{3/2}^2 M_P^2)/(m_S^2 \langle S \rangle^2) \simeq (M_P/m_{3/2}) \times (4\pi)^{-2}$ in our model in comparison with the case with neutral S.

The condensate of the Polonyi field will mainly decay to a pair of gravitinos, and the BBN constrains the number density $n_{3/2}$ of gravitinos normalized by the entropy density s, $Y_{3/2} \equiv n_{3/2}/s$. For $m_{3/2} = \mathcal{O}(0.1-1)\text{TeV}$, $m_S = \mathcal{O}(10^9)\text{GeV}$, and the decay rate of S is $\Gamma_S \simeq m_S^5/(300 m_{3/2}^2 M_P^2) = \mathcal{O}(100)\text{GeV}$ [47]. Unless the reheating temperature of the universe is above $\sqrt{\mathcal{O}(100)\text{GeV} M_P} \simeq 10^{10}\text{GeV}$, the Polonyi field decays before the completion of reheating. Note that n_S/ρ_{inf} is conserved after the beginning of the Polonyi oscillation but before the completion of reheating, where ρ_{inf} is the energy density of the inflaton. Using this and $m_S \sim H$ at the beginning of the oscillation, we obtain

$$Y_{3/2} = \frac{n_{3/2}}{s} \simeq \frac{2n_S}{\frac{4\rho}{3T_{\text{rh}}}} = \frac{T_{\text{rh}} \langle S \rangle^2}{2m_S M_P^2} \sim (4\pi)^5 \frac{T_{\text{rh}} m_{3/2}^{3/2}}{M_P^{5/2}} \sim 10^{-27} \times \left(\frac{T_{\text{rh}}}{10^9 \text{GeV}} \right) \left(\frac{m_{3/2}}{1\text{TeV}} \right)^{3/2}. \tag{5.15}$$

This is well-below the BBN constraint $Y_{3/2} < 10^{-16}$ [48] and thus the Polonyi problem is completely resolved in our model.

For $H_{\text{inf}} < m_S$, the oscillation amplitude of S is even smaller and the Polonyi problem is also absent.

---

[4]One may refer to [25] for a potential gravitational wave signature of the model with the discrete R-symmetry and the strong dynamics for the dynamical SUSY-breaking.

6 Summary and discussion

The SUSY TH theories are appealing frameworks with double protection of the electroweak scale by SUSY and an approximate global symmetry. However, as in the MSSM, if SUSY TH models are embedded into the simplest mediation scheme of SUSY breaking, namely, gravity mediation, they encounter the possibility of too large energy density of the Polonyi field S. In this work, we considered the case in which S transforms as $S \to -S$ under the discrete Z_2 symmetry exchanging the SM and mirror sectors. The Polonyi field can couple to gauge multiplets to give tree-level gaugino masses of the order of scalar masses. Because of the Z_2-odd nature, the symmetry enhanced point of the S field space, namely, the origin, is clearly identified, which results in great reduction of the amplitude of the oscillations of the Polonyi field in the early universe and the Polonyi problem is resolved in our setup.

The SUSY breaking by S spontaneously breaks the Z_2 symmetry and induces Z_2-breaking soft masses and the μ term. This can explain the required Z_2-breaking in the Higgs potential. Also, the degeneracy of the masses of supersymmetric particles with their twin partners is resolved. As discussed in section 3, the model predicts the splitting of masses of the two sector by quantum corrections of $\mathcal{O}(0.1-1)\%$, $\mathcal{O}(10)\%$, $\mathcal{O}(1)\%$ times $(M_{\rm cut}/M_*)$ for gauginos, squarks, and sleptons, respectively. We leave the study of the phenomenological implication of the mass splitting for future work.

In this paper, we focused on a Z_2-odd Polonyi field in supersymmetric Twin Higgs models motivated by the little hierarchy problem. A similar idea can also work for another scenarios with Z_2 symmetry that exchanges gauge fields with their partners. For example, mirror Z_2 symmetry is also motivated from the heavy QCD axion scenario that relaxes the quality problem [49–52]. The Polonyi field may be also Z_2-odd in such a scenario. Another possible scenario is a solution to the strong CP problem based on a UV $\mathrm{SU}(3)_c \times \mathrm{SU}(3)'_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2))L' \times \mathrm{U}(1)(\times \mathrm{U}(1)')$ gauge symmetry and a parity symmetry [53]. In this case, however, S should transform as $S \to S^\dagger$ under the parity symmetry and the real part of S is not Z_2-odd. We need to add an extra Z_2 symmetry under which the real part of S is odd. We leave the survey of various possibilities of a Z_2-odd Polonyi field to future work.

Acknowledgments

A Useful equations for computing soft masses

The following formulae are taken from [54].

A.1 RGEs for sfermion and Higgs masses

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m^2_{\tilde{Q}_L} &\supset -\frac{32}{3} g_3^2 |M_3|^2 + 2|A^q|^2 \,, \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{u}_R} &\supset -\frac{32}{3} g_3^2 |M_3|^2 + 4|A^q|^2 \,, \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{d}_R} &\supset -\frac{32}{3} g_3^2 |M_3|^2 + 4|A^q|^2 \,, \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{L}_L} &\supset -6 g_2^2 |M_2|^2 + 2|A^\ell|^2 \,, \\
16\pi^2 \frac{d}{dt} m^2_{\tilde{e}_R} &\supset -\frac{24}{5} g_1^2 |M_1|^2 + 4|A^\ell|^2 \,, \\
16\pi^2 \frac{d}{dt} m^2_{H_u} &\supset -6 g_2^2 |M_2|^2 + 6|A^t|^2 + 6|y_t|^2 (m^2_{H_u} + m^2_t + m^2_{t^c}) \,, \\
16\pi^2 \frac{d}{dt} m^2_{H_d} &\supset -6 g_2^2 |M_2|^2 + 6|A^b|^2 + 6|y_b|^2 (m^2_{H_d} + m^2_b + m^2_{b^c}) \,,
\end{aligned}
\tag{A.1}
$$

where $t = d\log\mu$ and each subscript for $m^2_{\tilde{f}}$ denotes quark and lepton doublets and singlets.

A.2 RGEs for A, μ and b parameters

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} A^t &\supset A^t \left(18 y_t^* y_t - \frac{16}{3} g_3^2 - 3 g_2^2\right) + y_t \frac{32}{3} g_3^2 M_3 \,, \\
16\pi^2 \frac{d}{dt} \mu &\supset \mu \left[3 y_t^* y_t - 3 g_2^2 - \frac{3}{5} g_1^2\right] \,, \\
16\pi^2 \frac{d}{dt} b &\supset b(3 y_t^* y_t - 3 g_2^2) + 6\mu g_2^2 M_2 \,,
\end{aligned}
\tag{A.2}
$$

A.3 Anomalous dimensions

$$
\begin{aligned}
\gamma_{H_u} &= \frac{1}{16\pi^2} \left[3 y_t^* y_t - \frac{3}{2} g_2^2 - \frac{3}{10} g_1^2\right] \,, \\
\gamma_{Q_3} &= \frac{1}{16\pi^2} \left[y_t^* y_t + y_b^* y_b - \frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2\right] \,, \\
\gamma_{u_3^c} &= \frac{1}{16\pi^2} \left[2 y_t^* y_t - \frac{8}{3} g_3^2 - \frac{8}{15} g_1^2\right] \,.
\end{aligned}
\tag{A.3}
$$

B μ and b terms from the hidden strong dynamics

The charge assignment specified in table. 1 allows the following superpotential terms,

$$
W \supset \frac{h_{ij}}{M_P} Q_i Q_j \left(H_u H_d - H_u' H_d'\right) \,,
\tag{B.1}
$$

where h_{ij} is a dimensionless coupling constant. In the confined phase of Sp(1), thanks to the deformed moduli constraint $\mathrm{Pf}[Q_i Q_j] = \Lambda_*^4$, eq. (B.1) leads to

$$
W_{\mathrm{eff}} \supset \frac{h}{M_P} \left(\frac{\Lambda_*}{4\pi}\right)^2 \left(H_u H_d - H_u' H_d'\right) \,,
\tag{B.2}
$$

where we suppressed the flavor indices of the coupling constant for simplicity. From eqs. (B.2) and (5.4), one can read the following contribution to μ term from the coupling of Sp(1) quark fields to the operator $H_u H_d$

$$\mu = \frac{\sqrt{3}h}{\lambda} m_{3/2} \, . \tag{B.3}$$

In terms of the meson field M, eq. (B.2) reads

$$W_{\text{eff}} \supset \frac{h\Lambda_*}{4\pi M_P} M \left(H_u H_d - H'_u H'_d \right) \, , \tag{B.4}$$

The F term of M is expected to be non-zero. Using eq. (5.12), we find

$$b = -b' \sim h \frac{(4\pi)^2}{\lambda^3} m_{3/2}^2 . \tag{B.5}$$

For perturbative λ, if $h = \mathcal{O}(1)$, $|b| \gg m_{3/2}^2$ and a stable electroweak vacuum does not exist, so we need $h \ll 1$. This provides another way to suppress the Z_2 breaking in the Higgs potential while $\tan\beta = 2 - 3$. The Z_2-odd b terms from the meson field may be comparable to $m_{H_u}^2$ and $m_{H_d}^2$ to ensure $\tan\beta = 2 - 3$. The μ term generated from the meson field is too small, but we may obtain μ from R symmetry breaking, as in eq. (3.15). It is generically Z_2-breaking, but as long as $\mu^2 \ll m_{H_u}^2, m_{H_d}^2$, the Z_2-breaking is suppressed.

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