Cryogenics 129 (2023) 103628

Contents lists available at ScienceDirect

Cryogenics

journal homepage: www.elsevier.com/locate/cryogenics

Superfluid helium forced flow in the Gorter-Mellink regime

Andrea Vitrano [*,1], Bertrand Baudouy

Université Paris-Saclay, CEA, Département des Accélérateurs, de la Cryogénie et du Magnétisme, Gif-sur-Yvette, 91191, Essonne, France

ARTICLE INFO

ABSTRACT

A dimensional study of the momentum equations of superfluid helium is presented together with a parametric analysis of newly derived dimensionless numbers. The study is performed with a focus on the role of forced flows in the Gorter-Mellink regime. The dimensionless numbers are derived in such a way they become dependent solely on the total fluid velocity, heat flux, and thermophysical properties in order to facilitate their application to engineering problems where the velocity of the single fluid components might be difficult to measure directly. With a similar approach, a novel form of the superfluid Reynolds number is obtained. This form takes into account the velocity of a forced flow and allows to make considerations about the contribution of both forced flow and heat flux to the establishment of the ordinary turbulence in the normal fluid component. It is also presented a formula for a channel critical dimension at which the critical heat flux for the onset of superfluid turbulence causes ordinary turbulence too.

1. Introduction

Superfluid helium (He II) is used as a thermal vector in the cooling system of superconducting magnet technologies because of its extraordinary heat extraction capability. The equivalent thermal conductivity of He II depends strongly on the magnitude of heat currents potentially present. In the heat flux range of magnet cooling applications, the thermal conductivity of He II is several orders of magnitude larger than metals [1], making it a unique coolant. Fluid mechanics studies on He II may provide useful insights for its application fields.

He II can be thought of as a mixture of two fluid components: a normal component that behaves like a classical viscous fluid and carries all the thermal energy; a superfluid component that has no entropy and no viscosity [2]. In a particular He II fluid condition known as the Gorter-Mellink regime [3], the temperature gradient becomes dependent on the cube of the heat flux due to an internal convection mechanism known as counterflow. This regime, which corresponds to the so-called superfluid or quantum turbulent state of He II, is established at heat fluxes higher than a critical value $q_{s,c}$ [4], above which the dissipation is not only due to the viscosity of the normal component but also to the mutual friction between the two He II components. The Gorter-Mellink regime has been extensively studied at zero net mass flow (ZNMF), see the initial work of Vinen [5–8]. In particular, Dimotakis derived a dimensionless number associated with this regime without taking into account the effect of a forced flow [9]. However, He II is often utilized in forced flow applications such as in the refrigeration system of infrared telescopes [10,11]. Moreover, over the last decades, several experiments, whose results were thoroughly summarized by Tough [12], have shown evidence of a second transition at a heat flux greater than $q_{s,c}$ towards another dissipative regime that corresponds to the ordinary turbulent state of the viscous component of He II. As will be explained in the next section, there exists a direct relationship between the heat flux and the velocity of the He II components, which represents an additional reason for investigating the role of forced flow in the two He II turbulent states.

Despite several experimental and numerical studies have been published on He II forced flow (e.g., [13,14]), the dimensional considerations of the topic either apply to ZNMF conditions only or provide dimensionless tools in terms of unfamiliar quantities related to the fluid components rather than He II as a whole, due to ad-hoc experiments that allowed varying independently the normal and superfluid velocities [15,16]. This work aims then to clarify the dimensional relationship between the thermal counterflow and the forced flow velocity and to derive dimensionless numbers in terms of engineering-friendly parameters. In other words, the goal is to study dimensionally the Gorter-Mellink regime in the presence of forced flow to enable future research studies on the identification of distinct He II turbulent fluid regimes. This will be achieved through the non-dimensionalization of the He II

* Corresponding author.
E-mail address: andrea.vitrano@cern.ch (A. Vitrano).
[1] Now at CERN, CH-1211, Geneva 23, Switzerland.

https://doi.org/10.1016/j.cryogenics.2023.103628
Received 6 November 2022; Received in revised form 2 January 2023; Accepted 5 January 2023
Available online 7 January 2023

momentum equations. Before doing so, a background on the fluid dynamics of He II is provided in the next section.

2. Superfluid dynamics

The normal fluid is characterized by the density ρ_n and the superfluid by ρ_s, which are related to the total density ρ of the liquid by

$$\rho = \rho_n + \rho_s. \tag{1}$$

The mass flux density of He II thus can be expressed as

$$\rho \mathbf{v} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s, \tag{2}$$

where $\mathbf{v}$, $\mathbf{v}_n$, and $\mathbf{v}_s$ are the velocities of the fluid, and its normal and superfluid components. As the normal component is the energy carrier of He II, the conductive heat flux q is equal to

$$\mathbf{q} = \rho s T \mathbf{v}_n = \rho_s s T \mathbf{v}_{ns}, \tag{3}$$

where s is the specific entropy, T is the temperature, and $\mathbf{v}_{ns}$ is the relative velocity between the fluid components. Because of the two-motion nature of He II, a ZNMF does not imply that the fluid is static. Instead, from Eq. (2) follows that the two components can still flow in opposite directions giving rise to the thermal counterflow. The counterflow mechanism contributes to the thermo-mechanical effect — a phenomenon for which establishing a temperature gradient in He II causes a pressure difference and vice versa [17]. If the relative velocity $\mathbf{v}_{ns}$ is below a certain critical value (i.e., very low heat flux) [18], the fluid mechanics of He II is well represented by Landau's two-fluid model [19] and He II is in the Landau regime. Above the critical value, quantum vortices arise in the fluid and the superfluid component enters the turbulent regime [18] or Gorter-Mellink regime. As the relative velocity of the two components is related to the heat flux (Eq. (3)), rearranging Eqs. (3) and (1) at ZNMF yields an expression for the critical heat flux for the onset of quantum turbulence:

$$\mathbf{q}_{s,c} = \frac{\rho_s \rho}{\rho_n} s T \mathbf{v}_{s,c}, \tag{4}$$

where $\mathbf{v}_{s,c}$ is the critical superfluid velocity, which was empirically demonstrated to be in relationship with the characteristic dimension of the geometry as [20]

$$v_{s,c} \simeq D^{-\frac{1}{4}}, \tag{5}$$

where D is in cm and $v_{s,c}$ is in cm/s.

The quantum turbulence is caused by a viscous-like mechanism between the superfluid and normal components of He II. This mechanism produces a force called mutual friction force [3]. Since the mutual friction force affects significantly the thermo-fluid dynamics of He II, it is necessary to include it in the two-fluid model to come up with a general system of equations that characterizes He II macroscopically. This system is constituted by the Hall-Vinen-Bekharevich-Khalatnikov (HVBK) equations [21,22]. The HVBK equations can be considered as a generalization of Landau's two-fluid model involving also quantum turbulence. The steady-state incompressible momentum equation for the superfluid reads

$$\left(\mathbf{v}_s \cdot \boldsymbol{\nabla}\right) \mathbf{v}_s = s \boldsymbol{\nabla} T - \frac{1}{\rho} \boldsymbol{\nabla} p + \mathbf{g} + \frac{\rho_n}{2\rho} \boldsymbol{\nabla} v_{ns}^2 + A_{GM} \rho_n v_{ns}^2 \mathbf{v}_{ns}, \tag{6}$$

where A_{GM} is the Gorter-Mellink coefficient, which is roughly proportional to T^3, p is the pressure, and $\mathbf{g}$ is the gravity vector. The terms on the RHS of Eq. (6) represent in order the thermo-mechanical effect, the pressure drop, the acceleration of gravity, a diffusive term deriving from He II chemical potential, and the mutual friction force. By manipulating the HVBK equations it is possible to derive a heat transport equation for superfluid helium [1]:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -\frac{b\mu}{s^2 \rho^2 T d^2} q - \frac{A_{GM} \rho_n}{s^4 \rho_s^3 T^3} q^3, \tag{7}$$

where b is a constant that depends on the geometrical configuration of the helium channel. The terms on the RHS represent respectively the viscous and turbulent contributions to the temperature gradient along the channel. The Landau regime is associated with small geometries and negligible heat fluxes (i.e., negligible turbulent term), whereas the Gorter-Mellink regime is associated with relatively high heat currents (i.e., negligible viscous term). If the viscous term is neglected, the following steady-state heat transport equation is obtained:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -f(T,p) q^n, \tag{8}$$

where $f(T,p)$ is the heat conductivity function and is defined as

$$f(T,p) = \frac{A_{GM} \rho_n}{s^4 \rho_s^3 T^3}. \tag{9}$$

3. Non-dimensionalization of the superfluid momentum equation

3.1. Superfluid dimensionless numbers

The procedure requires all the dimensional variables to be transformed into non-dimensional ones by means of characteristic quantities. The parameters chosen in this study are selected to take advantage of equations valid for superfluid helium, as it will be clear later in this section. Let us consider Eq. (6) and substitute the variables with the following dimensionless parameters:

$$\mathbf{v}_s^* = \frac{\mathbf{v}_s}{v_{s,0}}, \tag{10a}$$

$$\boldsymbol{\nabla}^* = D \boldsymbol{\nabla}, \tag{10b}$$

$$p^* = \frac{p}{\Delta p_0}, \tag{10c}$$

$$T^* = \frac{T}{\Delta T_0}, \tag{10d}$$

$$\mathbf{v}_{ns}^* = \frac{\mathbf{v}_{ns}}{v_{ns,0}}, \tag{10e}$$

$$\mathbf{g}^* = \frac{\mathbf{g}}{g_0}, \tag{10f}$$

where $v_{s,0}$, D, Δp_0, ΔT_0, g_0 and $v_{ns,0}$ are characteristic parameters. In particular, D is the characteristic dimension of the channel filled with helium, Δp_0 is the pressure drop along the channel, ΔT_0 is the temperature difference with respect to the bath, g_0 is the acceleration of gravity. The dimensionless form of Eq. (6) becomes then

$$\left(\mathbf{v}_s^* \cdot \boldsymbol{\nabla}^*\right) \mathbf{v}_s^* = \frac{s \Delta T_0}{v_{s,0}^2} \boldsymbol{\nabla}^* T^* - \frac{\Delta p_0}{\rho v_{s,0}^2} \boldsymbol{\nabla}^* p^* + \frac{g_0 D}{v_{s,0}^2} \mathbf{g}^* + \frac{\rho_n}{2\rho} \frac{v_{ns,0}^2}{v_{s,0}^2} \boldsymbol{\nabla}^* \mathbf{v}_{ns}^{*2} + A_{GM} \rho_n D \frac{v_{ns,0}^3}{v_{s,0}^2} \mathbf{v}_{ns}^{*3}. \tag{11}$$

The velocity $v_{s,0}$ can be related to $v_{ns,0}$ and a characteristic total fluid velocity v_0 through Eq. (2) and the definition of $v_{ns,0}$:

$$v_{s,0} = v_0 - \frac{\rho_n}{\rho} v_{ns,0}. \tag{12}$$

Also, $v_{ns,0}$ can be expressed as a function of a characteristic heat flux q_0 through Eq. (3):

$$v_{s,0} = v_0 - \frac{\rho_n q_0}{\rho \rho_s s T_b}, \tag{13}$$

where T_b is the bath temperature. This trick allows us to determine the dimensionless numbers in terms of parameters that are more easily obtainable (i.e., total velocity of the fluid and heat flux applied). Substituting Eq. (13) into Eq. (11) yields the final form of the non-dimensionalized equation:

$$\left(\mathbf{v}_s^* \cdot \boldsymbol{\nabla}^*\right) \mathbf{v}_s^* = \mathcal{A} \boldsymbol{\nabla}^* T^* - \mathcal{E} \boldsymbol{\nabla}^* p^* + \mathcal{F}^{-2} \mathbf{g}^* + \mathcal{B} \boldsymbol{\nabla}^* \mathbf{v}_{ns}^{*2} + \mathcal{C} \mathbf{v}_{ns}^{*3}, \tag{14}$$

where $\mathcal{A}$, $\mathcal{E}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{F}$ are the dimensionless numbers of the equation. In particular, $\mathcal{E}$ and $\mathcal{F}$ are the equivalent superfluid versions of the Euler and Froude numbers in the classical Navier-Stokes equations, whereas $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are associated with terms that are proper of He II. The dimensionless numbers read as follows:

$$\mathcal{A} = \frac{\Delta T_0 \rho^2 \rho_s^2 s^3 T_b^2}{\left(\rho\rho_s s T_b v_0 - \rho_n q_0\right)^2}, \tag{15a}$$

$$\mathcal{E} = \frac{\Delta p_0 \rho \rho_s^2 s^2 T_b^2}{\left(\rho\rho_s s T_b v_0 - \rho_n q_0\right)^2}, \tag{15b}$$

$$\mathcal{F} = \frac{\rho\rho_s s T_b v_0 - \rho_n q_0}{\rho\rho_s s T_b \sqrt{g_0 D}}, \tag{15c}$$

$$\mathcal{B} = \frac{\rho\rho_n q_0^2}{2\left(\rho\rho_s s T_b v_0 - \rho_n q_0\right)^2}, \tag{15d}$$

$$\mathcal{C} = \frac{A_{GM}\rho_n \rho^2 q_0^3 D}{\rho_s s T_b \left(\rho\rho_s s T_b v_0 - \rho_n q_0\right)^2}. \tag{15e}$$

It is interesting to notice that, in absence of heat currents, $\mathcal{E}$ and $\mathcal{F}$ become their respective numbers for ordinary fluids ($Eu = \Delta p_0/\rho {v_0}^2$, $Fr = v_0/\sqrt{g_0 D}$). The condition $\rho_n q_0 = 0$ nullifies the dimensionless numbers that originate directly from the relative motion of the He II components (i.e., $\mathcal{B}$ and $\mathcal{C}$). On the contrary, $\mathcal{A}$ becomes $\mathcal{A}\Big|_{\rho_n q_0 = 0} = s\Delta T_0/{v_0}^2$, which looks much like the Euler number. This similarity provides an additional insight into the fountain effect [23] and indicates that $\mathcal{A}$ represents the relationship between the thermo-mechanical force and the inertial force of the superfluid stream.

At ZNMF, the superfluid component may still move because of the counterflow mechanism, which is confirmed by setting $v_0 = 0$ in the dimensionless numbers. In particular, it might be interesting to highlight the relationship found between $\mathcal{C}$ and the dimensionless number of Dimotakis [9], who non-dimensionalized a general momentum equation in pure counterflow (i.e., ZNMF). Dimotakis obtained a number associated with the mutual friction term equal to $Di = \rho_s A_{GM} v_{ns,0} D$, which can be written as well as

$$Di = \alpha(1-\alpha)\mathcal{C}\Big|_{v_0=0}, \tag{16}$$

where α is the superfluid density fraction and $\mathcal{C}$ is calculated at ZNMF. Computing the roots of α for $\mathcal{C}\Big|_{v_0=0} = Di$ reveals that there exists no real value of α that allows this condition. This is due to the nature of the two numbers: Di was derived by merging the momentum equations of both fluid components of He II, whereas $\mathcal{C}$ is herein derived from the superfluid component equation only.

3.2. Parametric analysis

It is now possible to compare the dimensionless numbers among each other by varying the characteristic parameters. Since some thermophysical properties appear in Eqs. (15), the dimensionless numbers must depend on the pressure as well. However, since no major difference has been observed in their behavior in the range of pressures of helium cooling applications (i.e., between the λ-point and atmospheric pressure), the pressure is simply set to the saturated vapour one and the properties are evaluated at T_b. In the Gorter-Mellink regime the mutual friction force is expected to be one of the dominant terms. For this reason, the following considerations are made with respect to Eq. (15e).

Fig. 1 shows the relative importance of $\mathcal{E}$ with respect to $\mathcal{C}$ as a function of two characteristic parameters. The range of colors conveys information about how many orders of magnitude a dimensionless number is higher (or lower) than the other. This is obtained by calculating the common logarithm of the ratio between the two dimensionless numbers under investigation. If a ratio contains more than two characteristic parameters, the other ones are kept constant at values specified in the caption of the figure. Fig. 1 shows the relevance of the pressure drop term in the momentum equation as a function of the characteristic heat flux q_0 and velocity v_0. The characteristic pressure drop Δp_0 is expressed as a function of v_0 through an empirical equation derived from data collected by Fuzier [24]. Fuzier et al. conducted He II forced flow experiments in a 1 m long tube with an internal diameter of 1 cm [25]. They proved the importance of the pressure drop term in the He II heat transport for non-negligible flow velocities and a 100 kW/m^2 heat flux. Fig. 1 confirms this outcome with regards to the superfluid mass transport as well, that is, the role played by the pressure drop in the superfluid momentum equation.

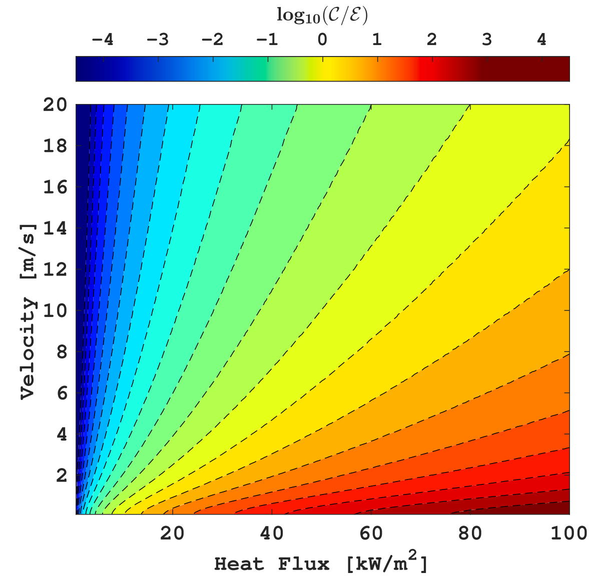

Fig. 1. Relative magnitude between $\mathcal{C}$ and $\mathcal{E}$ as a function of the total velocity v_0 and heat flux q_0 at 1.9 K and $D = 1$ cm. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

It is easy to evaluate the entity of the number associated with the thermo-mechanical effect by using the definition of the heat conductivity function (Eq. (9)). Combining the $\mathcal{A}$ and $\mathcal{C}$ numbers yields then

$$\mathcal{A} = \frac{\Delta T_0}{f(T,p) q_0^3 D}\mathcal{C}. \tag{17}$$

If we consider again Eq. (7), it is clear from Eq (17) that $\mathcal{A}$ and $\mathcal{C}$ tend to equal each other for high heat fluxes, that is, when the viscous contribution can be neglected in Eq. (7). This evidence reveals that, for sufficiently high heat fluxes, the thermo-mechanical effect counts as much as the mutual friction force in the dynamics of the superfluid component. This result confirms the validity of the main assumption of the single fluid model proposed by Kitamura [26].

Since both the $\mathcal{C}$ and $\mathcal{F}$ numbers depend on the characteristic length D, their ratio is not affected by this parameter. Fig. 2 shows their relation as a function of the temperature and heat flux. It is clear that, apart from very low heat fluxes, the acceleration of gravity plays a little role in the superfluid dynamics. It also appears that the $\mathcal{F}$ number is most significant at around 1.876 K. The presence of peak values is due to the shape of the heat conductivity function. However, the temperature at which the maximum occurs is modified by the other parameters appearing in the numbers ratio. It is important to notice how the mutual friction force is by far the most dominant term in the vicinity of the lambda temperature, which is clearly due to the combination of both the steep rise of the Gorter-Mellink coefficient A_{GM} and the diminishing ρ_s as the temperature approaches T_λ.

The energy diffusion term associated with the $\mathcal{B}$ number is often neglected in the two-fluid model [3,27,1,28], as its contribution is considered to be a small part in the total effect deriving from the relative motion of the He II fluid components. This is confirmed by Fig. 3, which

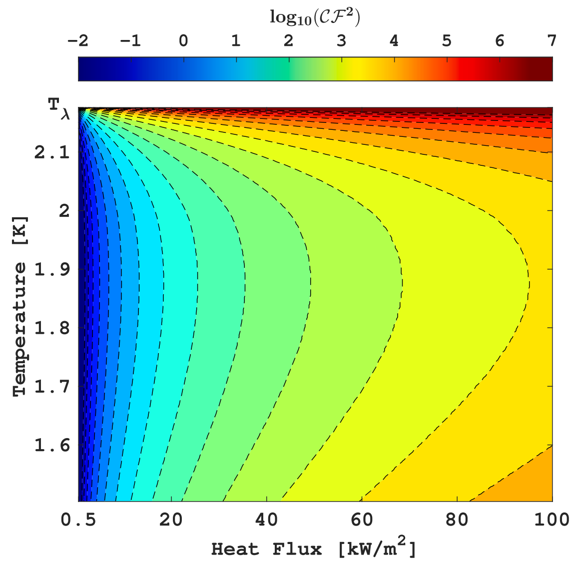

Fig. 2. Relative magnitude between $\mathcal{C}$ and $\mathcal{F}$ as a function of the temperature T_b and heat flux q_0.

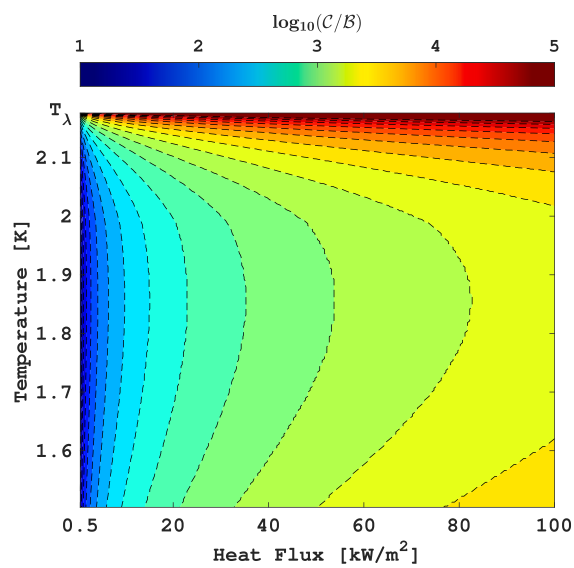

Fig. 3. Relative magnitude between $\mathcal{C}$ and $\mathcal{B}$ as a function of the temperature T_b and heat flux q_0 for $D = 1$ cm.

shows that the contribution of the mutual friction grows with the heat flux value and distinctly dominates. Obviously, since $\mathcal{B}$ does not depend on the characteristic length, the mutual friction becomes less influential for small geometries. The $\mathcal{B}$ number appears to be most significant at around 1.855 K, which is independent of D.

4. Ordinary and quantum turbulence

A similar approach can be applied to the normal component momentum equation, which carries an additional dissipative term associated with the viscosity of He II. The normal component velocity can be expressed in terms of the heat flux and total velocity of the fluid as

$$v_{n,0} = v_0 + \frac{q_0}{\rho s T_b}. \tag{18}$$

A novel version of the modified Reynolds number first proposed by Staas et al. [29] can be derived then:

$$\mathcal{R} = \frac{D\left(\rho s T_b v_0 + q_0\right)}{\mu s T_b}, \tag{19}$$

where μ is the dynamic viscosity. $\mathcal{R}$ takes into account the effect of a forced flow on the He II fluid regime. It follows that ordinary turbulence can be achieved through a combination of both the total fluid flow and thermal counterflow. Similarly to Eqs. (15b) and (15c), in absence of heat currents the counterflow mechanism ceases and $\mathcal{R}$ takes the form of the ordinary Reynolds number ($Re = D\rho v_0/\mu$). On the other hand, at ZNMF ($v_0 = 0$), Eq. (3) can be applied and $\mathcal{R}$ becomes the modified Reynolds number by Staas ($Re_s = D\rho v_n/\mu$). Since the normal fluid velocity v_n is proportional to the heat flux applied (Eq. (3)), it follows that ordinary turbulence can be triggered simply by enhancing sufficiently the thermal counterflow. It is worth investigating whether there is any condition at which this critical heat flux matches the one for the onset of quantum turbulence. As well as in ordinary turbulence, also in He II the dimensions of a channel are expected to affect the fluid regime transition. Therefore, there must exist a relationship between the characteristic dimension of a channel and the critical heat flux for the onset of the normal fluid turbulence, as discussed below.

By utilizing Eq. (4) as heat flux, $\mathcal{R}$ becomes

$$\mathcal{R}_{s,c} = \frac{\rho D\left(\rho_n v_0 + \rho_s v_{s,c}\right)}{\mu \rho_n}, \tag{20}$$

which is the superfluid Reynolds number computed at the application of the critical heat flux for the onset of quantum turbulence. Equating Eq. (20) to the critical Reynolds number $Re_{s,c}$ for the onset of ordinary turbulence yields an expression for the velocity at which ordinary turbulence is achieved in the presence of counterflow:

$$v_{0,c} = \frac{\mu Re_{s,c}\rho_n - \rho\rho_s D v_{s,c}}{\rho\rho_n D}. \tag{21}$$

Finally, by substituting Eq. (5) into Eq. (21) and setting it equal to zero, it is possible to obtain an expression for a critical characteristic dimension at which the critical heat flux for the onset of quantum turbulence at ZNMF is sufficient to trigger ordinary turbulence too:

$$D_c \simeq a\left(\frac{1-\alpha}{\alpha} Re_{s,c}\nu\right)^{\frac{4}{3}}, \tag{22}$$

where ν is the kinematic viscosity, and a is equal to $10^{-10/3}$ s$^{4/3}$/m$^{5/3}$ and arises because of the units of measure of Eq. (5). Fig. 4 shows how D_c varies with the temperature below the λ-point for $Re_{s,c} = 1200$, the generally accepted value of critical Reynolds number for flow in a tube from the comprehensive experimental study by Staas [29]. The thermophysical properties in Fig. 4 are computed at the saturated vapor pressure. D_c rises towards the λ-point as the superfluid density fraction tends to zero.

5. Conclusions

In this work, a dimensional study of the He II momentum equations was presented to emphasize the role of a forced flow in the Gorter-Mellink regime. A non-dimensionalization of the equations was performed such that novel dimensionless numbers were derived as a function of macroscopic quantities only. In particular, the numbers depend on the total fluid velocity, heat flux, and thermophysical properties of He II, without considering the velocity of the single fluid components. A parametric analysis of these numbers was then carried out to understand the weight of the various terms in the superfluid momentum equation. Comparative figures showed that the most significant terms are the pressure drop, the mutual friction force, and the thermo-mechanical effect, confirming the assumptions of other authors in the attempt of simplifying the equations for analytical and numerical models [26,28]. The other terms (i.e., diffusive and gravitational terms) should not be neglected in the numerical modeling of He II only in cases where heat currents are little or absent. In the case of pure counterflow or weak forced flows, the pressure drop term can be neglected too. Utilizing an experimental correlation between the pressure drop

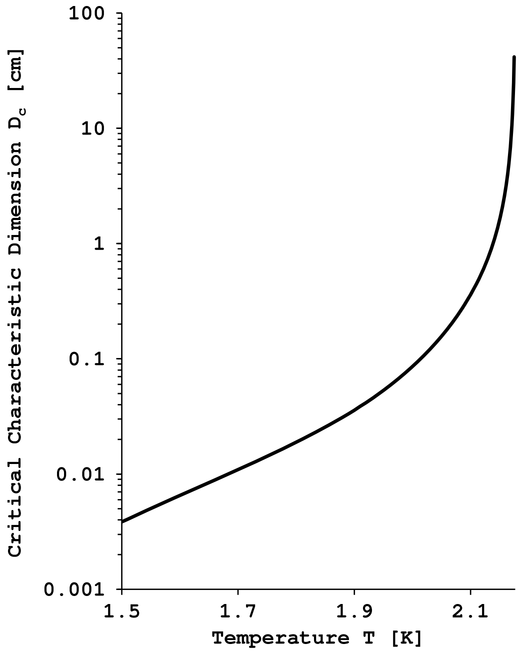

Fig. 4. Critical characteristic dimension D_c as a function of the temperature at the saturated vapor pressure.

and the total velocity allowed a direct comparison between the pressure drop term and the mutual friction force, which revealed a range of velocities and heat fluxes where the two terms have similar orders of magnitude. This evidence supports Fuzier's explanation of the discrepancy found between their numerical model and experimental data for a specific range of velocities [25], which was correctly attributed to a region where the pressure and temperature gradients are comparable in magnitude in the He II heat transport.

Following a similar approach, a superfluid Reynolds number was derived to take into account the effect on the He II fluid regime of both the thermal counterflow and forced flow. The novel Reynolds number represents an alternate form for non-negligible forced flow velocities of the one derived by Staas [29]. By manipulating the number and exploiting analytical and empirical equations it was possible to obtain a simple formula that defines a critical characteristic dimension of a channel at which the critical heat flux for the onset of quantum turbulence at ZNMF initiates the normal fluid turbulence.

CRediT authorship contribution statement

Andrea Vitrano: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing – original draft. **Bertrand Baudouy:** Conceptualization, Funding acquisition, Project administration, Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgement

This research was mainly supported by EASITrain – European Advanced Superconductivity Innovation and Training. This Marie Skłodowska-Curie Action (MSCA) Innovative Training Networks (ITN) received funding from the European Union's H2020 Framework Programme under grant agreement no. 764879.

References

[1] Van Sciver SW. Helium cryogenics. New York: Springer; 2012.
[2] Tisza L. Transport phenomena in helium II. Nature 1938;141(3577):913.
[3] Gorter C, Mellink J. On the irreversible processes in liquid helium II. Physica 1949;15(3):285–304.
[4] Sciacca M, Jou D, Mongiovi MS. Effective thermal conductivity of helium II: from Landau to Gorter–Mellink regimes. Z. Angew. Math. Phys. 2015;66(4):1835–51.
[5] Vinen WF. Mutual friction in a heat current in liquid helium II i. experiments on steady heat currents. Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 1957;240(1220):114–27.
[6] Vinen WF. Mutual friction in a heat current in liquid helium II. ii. experiments on transient effects. Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 1957;240(1220):128–43.
[7] Vinen WF. Mutual friction in a heat current in liquid helium II iii. theory of the mutual friction. Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 1957;242(1231):493–515.
[8] Vinen WF. Mutual friction in a heat current in liquid helium II. iv. critical heat currents in wide channels. Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 1958;243(1234):400–13.
[9] Dimotakis PE. Gorter-mellink scale, and critical velocities in liquid-helium-II counterflow. Phys. Rev. A 1974;10(5):1721.
[10] Urbach AR, Mason PV. IRAS cryogenic system flight performance report. In: Advances in cryogenic engineering. Advances in cryogenic engineering, vol. 29. 1984. p. 651–8.
[11] Werner MW, Murphy JP, Witteborn FC, Wiltsee CB. The SIRTF mission. In: Lear JW, Monfils A, Russak SL, Seeley JS, editors. Instrumentation for optical remote sensing from space. International society for optics and photonics, vol. 0589. SPIE; 1986. p. 210–21.
[12] Tough JT. Superfluid turbulence. Prog. Low Temp. Phys. 1982;8:133–219.
[13] Rousset B, Claudet G, Gauthier A, Seyfert P, Martinez A, Lebrun P, et al. Pressure drop and transient heat transport in forced flow single phase helium II at high Reynolds numbers. Cryogenics 1994;34:317–20.
[14] Fuzier S, Baudouy B, Van Sciver S. Steady-state pressure drop and heat transfer in He II forced flow at high Reynolds number. Cryogenics 2001;41(5–6):453–8.
[15] Ashton RA, Opatowsky LB, Tough JT. Turbulence in pure superfluid flow. Phys. Rev. Lett. 1981;46(10):658.
[16] Courts SS, Tough JT. Transition to superfluid turbulence in two-fluid flow of He II. Phys. Rev. B 1988;38(1):74.
[17] London F. On the bose-einstein condensation. Phys. Rev. 1938;54(11):947–54.
[18] Feynman R. Chapter II application of quantum mechanics to liquid helium. Progress in low temperature physics, vol. 1. Elsevier; 1955. p. 17–53.
[19] Landau LD. Theory of the superfluidity of helium II. Phys. Rev. 1941;60(4):356–8.
[20] Van Alphen W, Van Haasteren G, De Bruyn Ouboter R, Taconis K. The dependence of the critical velocity of the superfluid on channel diameter and film thickness. Phys. Lett. 1966;20(5):474–5.
[21] Hall HE, Vinen WF. The rotation of liquid helium II ii. the theory of mutual friction in uniformly rotating helium II. Proc. R. Soc. Lond. Ser. A, Math. Phys. Sci. 1956;238(1213):215–34.
[22] Bekarevich IL, Khalatnikov IM. Phenomenological derivation of the equations of vortex motion in He II. J. Exp. Theor. Phys. 1961;13:643.
[23] Keller W, Hammel E. Heat conduction and fountain pressure in liquid He II. Ann. Phys. 1960;10(2):202–31.
[24] Fuzier S. Heat transfer and pressure drop in forced flow helium II at high Reynolds numbers. PhD Thesis. 2004.
[25] Fuzier S, Van Sciver S. Experimental measurements and modeling of transient heat transfer in forced flow of He II at high velocities. Cryogenics 2008;48(3–4):130–7.
[26] Kitamura T, Shiramizu K, Fujimoto N, Rao Y, Fukuda K. A numerical model on transient, two-dimensional flow and heat transfer in He II. Cryogenics 1997;37(1):1–9.
[27] Wilks J. The Properties of Liquid and Solid Helium. International Series of Monographs on Physics. Clarendon P.; 1967.
[28] Soulaine C, Quintard M, Allain H, Baudouy B, Van Weelderen R. A PISO-like algorithm to simulate superfluid helium flow with the two-fluid model. Comput. Phys. Commun. 2015;187:20–8.
[29] Staas F, Taconis K, Van Alphen W. Experiments on laminar and turbulent flow of He II in wide capillaries. Physica 1961;27(10):893–923.