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DIFFERENCE AMPLIFIERS FOR THE BEAM OBSERVATION SYSTEM

Summary

This report sets out to show the underlying principles for the design of difference amplifiers.

The requirements demanded of a difference amplifier for the Beam Observation System are evaluated.

The constructed amplifiers are described.

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1. General considerations of difference amplifiers

Difference amplifiers are generally used for the amplification of a voltage difference between two terminals which carry also a common signal.

In the following this voltage difference will be called the antiphase input signal and the common signal will be called the in-phase input signal.

The difference amplifier must be constructed in such a way as to cancel the in-phase inputs so that these do therefore not appear in the output.

However, as pure in-phase inputs may give rise to antiphase outputs and vice versa, the expression for the output voltage, when the input voltage is:

$$e_{in} = e_o + e_i \quad (1) \qquad \text{where } e_i = \text{inphase}, \quad e_o = \text{antiphase input}$$

can be written as:

$$E_{out} = E_o + E_i = A.e_o + B.e_i + C.e_i + D.e_o \qquad (2)$$

where
- A = amplitude of E_o generated by e_o
- B = amplitude of E_o generated by e_i
- C = amplitude of E_i generated by e_i
- D = amplitude of E_i generated by e_o

The extend to which our difference amplifier serves its purpose can now be expressed by three factors (ref. 4).

1) The discrimination factor F , which is equal to the ratio between the amplification which a pure anti-phase signal and a pure in-phase signal undergo.

2) The rejection factor H , which is equal to the ratio between an inphase signal and an antiphase signal that have to be applied to the input in order to produce the same anti-phase voltage at the output. This factor is also called common made rejection, transmission factor.

3) A third factor G , which is less important and nameless[4].

These three factors are related to A, B, C and D from equation (2), by the following definitions:

$$F = A/C \qquad (3)$$

$$H = \frac{A}{B} \qquad (4)$$

$$G = A/D \qquad (5)$$

Equation (2) can now be written as:

$$E_{out} = A \cdot e_o + \frac{A}{H} \cdot e_i + \frac{A}{F} \cdot e_i + \frac{A}{G} \cdot e_o \qquad (6)$$

If, now, for some reason or other the values of F are not high enough, difference amplifiers or stages can be cascaded.

For two difference stages in cascade the values for the overall F, H and G factors are given as :

$$F_{tot} = H_1G_2 \frac{1 + \frac{1}{G_1H_1}}{1 + \frac{H_1G_2}{F_1F_2}} \qquad (7)$$

$$H_{tot} = H_1 \frac{1 + \frac{1}{G_1H_1}}{1 + \frac{H_1}{F_1H_2}} \qquad (8)$$

$$G_{tot} = G_2 \frac{1 + \frac{1}{G_1H_2}}{1 + \frac{G_2}{G_1F_2}} \qquad (9)$$

and if we assume that $\frac{1}{G_1H_1}$ and $\frac{1}{G_1H_2}$ resp. $\ll 1$, equations 7, 8 and 9 can be rewritten as :

$$F_{tot} = F_1 \cdot F_2 \cdot \frac{H_1G_2}{F_1 \cdot F_2 + H_1G_2}$$ and assuming that $H_1G_2 \gg F_1 \cdot F_2$

$$F_{tot} \approx F_1 \cdot F_2 \qquad (10)$$

$$H_{tot} = H_1 \frac{F_1H_2}{F_1H_2 + H_1}$$ and assuming that $F_1H_2 > H_1$

$$H_{tot} \approx H_1 \qquad (11)$$

$$G_{tot} = \frac{G_1 G_2 F_2}{G_1F_2 + G_2} \qquad (12)$$

2. Requirements to be met by difference amplifiers for the Beam Observation System

Difference amplifiers form an integral part in the handling of the signals obtained from the accelerated proton beam by means of Pick-up electrodes. It may be remembered that the radial and vertical pick-up electrodes have a differential response to the beam position[11]. The general expressions for the two voltages which are induced on the two parts of the electrode are :

$$e_i = K_1 \; f_1(n) \; g_1(+ R) \qquad (13)$$

$$e_2 = K_2 \; f_2(n) \; g_2(- R) \qquad (14)$$

where

K_1, K_2	=	constants comprising capacitances, losses, form factors etc...
$f_1(n)$, $f_2(n)$	=	functions of the intensity of the circulating beam
$g_1(+ R) g_2(- R)$	=	functions of beam displacement.

The electrodes and the impedance transformer have to be constructed in such a way that K, f(n) and the function ($\pm$ R) are equal for the two signals[13]. Hence, each of the two voltages can be regarded as having an in-phase component (due to f(n),) and an anti-phase component due to g(R).

The input voltage for the difference amplifier as provided by the electrode and associated electronics satisfies our equation 1 perfectly.

Further relations which exist between the anti-phase and the in-phase components are that for a beam displacement of $\pm$ 1 cm from zero one gets

$$e_i \approx 12\, e_o \qquad (15)$$

and for a beam exactly in the middle of the electrode

$$e_o \approx 0 \qquad (16).$$

Disregarding the term in expression (6) which contains $\frac{A}{G}$ the output voltage of the different amplifiers, allowing for (15), is :

$$E_{out} = A \cdot e_o + 12 \cdot e_o \cdot A\left(\frac{1}{H} + \frac{1}{F}\right) \qquad (17)$$

If we now require the circuit to be of 1% precision, i.e. precise enough to measure 1 cm with an accuracy of 0,1 mm, the error component has to fulfil :

$$10^{-2} \cdot A \cdot e_o \geq 12 \cdot e_o \cdot A\left(\frac{1}{H} + \frac{1}{F}\right)$$

which means that

$$\frac{1}{H} + \frac{1}{F} \leq 83 \cdot 10^{-5} \qquad (18)$$

As may be seen in the Chapter Performances, the values for F and H are at 10 mc/s :

$$F = 1.800, \quad H = 30.000$$

and we find :

$$\frac{1}{H} + \frac{1}{F} \approx \frac{1}{F} \approx 55 \cdot 10^{-5}$$

which satisfies our equation (18).

3. Circuit discussion

The circuit which is most suitable for difference amplifiers is the so-called "long-tailed pair" (LTP). Although other types of difference amplifier circuits exist[12)] the L.T.P. offers the best chances for operating over a larger frequency band.

In Appendix 1 the expressions for F and H have been computed.

These are :

$$F = 1 + 2\ gmRk \tag{20}$$

and

$$H = \frac{2F}{\frac{\Delta Ri}{Ri} - \frac{\Delta \mu}{\mu} - \frac{\Delta R}{R}} \tag{21}$$

Inspection shows readily that F increases with increasing Rk. This sometimes leads to designs with extremely high cathode resistances and constant current tube cathode circuits.

However, we see from (21) that a high value for Rk does not necessarily mean that H rises to a high value, although it is sometimes assumed that a high value for Rk is a cure for everything[14),2),9),3)]. Moreover it was not at all possible to use a very high value for Rk as the bandwidth needed goes up to some 20 Mhz. As we wanted to maintain a good discrimination factor over the frequency band we had to limit ourselves to a cathode resistance of 1 kΩ.

The (hot)-cathode/earth capacitances are of the order of 10 pF together and this brings the 3 dB point of the discrimination factor to some 15 M

A test circuit was constructed using E 810 F tubes, because of the high slope of these tubes. This would permit a discrimination factor of 100 in one L.T.P. with Rk = 10 kΩ. However, the tubes were found to be too noisy owing to the high slope and this scheme had to be abandoned.

The tube, which is used in the final circuit, is the D.3 A. This tube has a nominal slope of 35 mA/V and this means that, with Rk = 1 k a discrimination factor of 70 of this tube can be achieved.

The tolerances of this tube, according to publications as well as measurements are :

$$gm \quad 35 \pm 5 \text{ mA/V}$$

$$Ri \quad 120 \pm 20 \text{ k}\Omega$$

and as gm and Ri vary over some 30 % one may safely assumed that μ. also varies to the same extent.

The minimum value of F is 60 and, without any precautions for balancing the circuit, the minimum value of H (with an anode load of 1% resistances) is 200 (assuming that all Δ values go in the most unfavourable direction).

As can be seen from (18) these values are by no means sufficient, and therefore two L.T.P. circuits in cascade were used and a means of balancing the circuit was adopted. However, in the first instance, cascading two L.T.P. circuits will only yield a better F (equation 10 for $H_1G_2 > F_1F_2$).

The total transmission factor H of a cascade tends however to be smaller than the transmission factor of the first stage (equation 11), unless $F_1H_2 > H_1$. This makes it essential to apply some sort of balance control in the second stage, and also to prevent H_2 from being too small to fulfil the assumption for equation 11.

As evaluated in the Appendix, the expression for H is :

$$H = \frac{2F}{\frac{\Delta Ri}{Ri} - \frac{\Delta \mu}{\mu} - \frac{\Delta R}{R}} \qquad (21)$$

where, as has been mentioned already, the Δ quantities may be positive or negative.

In order to raise H to a suitable value, it is clear that the denominator of (21) must be made as small as possible. This is sometimes done with a variable anode load ($\frac{\Delta R}{R}$ - controlled) but the fact that we had to maintain a proper frequency response already prohibits this type of balance, which for other reasons too is not a good solution.

The balance control is made in the following way. If a tube, with parameters μ and Ri is shunted by a resistance R, it can be shown that the assembly parameters are modified to :

$$\mu' = \frac{\mu \cdot R}{Ri + R} \tag{22}$$

and

$$Ri' = \frac{Ri \cdot R}{Ri + R} \tag{23}$$

while the slope remains unchanged

In a L.T.P. circuit with a potentiometer R connected between the anodes and the slider of this potentiometer connected to the cathodes, the potentiometer can be adjusted in such a way as to cancel the differences in tube parameters.

Let the potentiometer be devided (by the slider) into values αR and βR

R = total pot resistance

$\beta = 1 - \alpha$

The tubes are then shunted by αR and $\beta \cdot R$ resp. and the modified μ expressions are :

$$\mu'_1 = \frac{\mu_1 \cdot \alpha \cdot R}{Ri + \alpha R} \quad \text{and} \quad \mu'_2 = \frac{\mu_2 \cdot \beta R}{Ri + \beta R} \tag{24}$$

For a potentiometer setting of

$$\frac{\mu_1}{\mu_2} = \frac{\beta}{\alpha} \tag{25}$$

these two modified amplification factors are equal.

Where it has been assumed that
$Ri \gg \alpha R$ and $Ri \gg \beta R$.

If we now write for $\mu_1 = \mu_2 + \Delta\mu$ (and for $\mu_2 \Longrightarrow \mu$) equation (25) can be transformed into

$$\frac{\Delta\mu}{\mu} = \frac{\beta}{\alpha} - 1 \tag{26}$$

and this shows that the fraction $\frac{\Delta\mu}{\mu}$ can be set to any required value.

What happens now if the balance is adjusted as the following :

The fraction $\frac{\Delta Ri}{Ri}$ is modified also by the shunt resistances, but it certainly is not a minimum for the same values for β and α from (26). This is of no importance as the balancing action tends to cancel the denominator of H (21) out, or in other words, β and α are adjusted so that

$$\frac{\Delta\mu}{\mu} \qquad \frac{\Delta R}{R} + \frac{\Delta Ri}{Ri} \tag{27}$$

This type of balance control has been adopted for both of the L.T.P. circuits.

The gain control in the second L.T.P. circuit is performed by means of a of a variable bias applied to both of the tubes.

The whole circuit of the difference amplifier consists of two L.T.P. circuits in cascade and two cathode followers of conventional construction.

High frequency heater decoupling was necessary and the conventional decoupling methods have been applied wherever necessary.

For the proper functioning of the Beam Observation System the sum of the two electrode voltages is wanted. This part does not belong to the difference amplifier as such, but it has been mounted on the same chassis, and therefore appears on the circuit diagram (see[13]).

It was necessary to make the lay-out as symmetrical as possible. The practical values for F and H do not mechanically remain at the high initial values at high frequency input. This is caused by capacitive feed through, and reduction of Z_k.

However, up to some 15 mc/s the achieved values for H and F agree with the required figures set forth in equation (18).

4. Performances

Antiphase gain	min. 7 x		max. 17 x
Frequency response of antiphase gain			150 Hz - 20 Mhz (3dB)
Discrimination factor F	3.600	(100 khz)	
	3.500	(1 Mhz)	
	2.300	(5 Mhz)	
	1.000	(10 Mhz)	
	600	(20 Mhz)	
Transmission factor H	50.000	(100 khz)	
	40.000	(1 Mhz)	
	30.000	(10 mhz)	
	7.000	(20 mhz)	
Max. output voltage	3 Vpp	across 75 Ω	
Max. inphase input voltage	≈ 5 Vpp		
Input impedance	150 Ω // 20 pF		
Output impedance	~ 30 Ω		
Power consumption	210 V DC - 130 mA	, stabilised	
	6.3 AC - 2.3 A.		

These specifications show average values. For typical difference amplifiers one may find smaller values for F and H at the high frequency end. However, the majority of the constructed amplifiers agree very well with these quoted results.

5. Maintenance instructions

The amplifiers need little attention while they are in use. For over 18 months 48 of these units have been in constant use in the Beam Observation stations without excessive maintenance to be done.

The balance needs a check every 6 months or so.

Tube exchanges

After every tube exchange the balance and the gain have to be readjusted.

To adjust the balance, it is sufficient to apply a 5 $mc/_s$ input signal to both inputs in parallel (via coaxial T piece) and to adjust the balance control for minimum output signal.

The gain must be adjusted according to the procedure described in lit. ref.[13].

6. Circuit Diagram Key

R_{1}	150 ohms	1%	$\frac{1}{2}$ W	
R_{2}	150 ohms	1%	$\frac{1}{2}$ W	
R_{3}	220 k.ohms		$\frac{1}{4}$ W	
R_{4}	220 k.ohms		$\frac{1}{4}$ W	
R_{5}	33 ohms		1/8 W	
R_{6}	33 ohms		1/8 W	
R_{7}	270 ohms	1%	$\frac{1}{4}$ W	
R_{8}	270 ohms	1%	1/8 W	
R_{9}	2.2 k.ohm		2 W	
R_{10}	2.2 k.ohms		2 W	
R_{11}	33 ohms		1/8 W	
R_{12}	33 ohms		1/8 W	
R_{13}	470 ohms		$\frac{1}{4}$ W	
R_{14}	470 ohms		$\frac{1}{4}$ W	
R_{15}	10 k.ohms		$\frac{1}{4}$ W	pot
R_{16}	62 k.ohms		$\frac{1}{4}$ W	
R_{17}	150 k.ohms		$\frac{1}{4}$ W	
R_{18}	120 k.ohms		$\frac{1}{4}$ W	
R_{19}	50 k.ohms		$\frac{1}{4}$ W	pot
R_{20}	150 k.ohms		$\frac{1}{4}$ W	
R_{21}	100 ohms		$\frac{1}{4}$ W	
R_{22}	220 k.ohms		$\frac{1}{4}$ W	
R_{23}	220 k.ohms		$\frac{1}{4}$ W	
R_{24}	33 ohms		1/8 W	
R_{25}	33 ohms		1/8 W	
R_{26}	270 ohms	1%	$\frac{1}{4}$W	
R_{27}	270 ohms	1%	$\frac{1}{4}$W	
R_{28}	2.2 k.ohms		2 W	
R_{29}	2.2 k.ohms		2 W	
R_{30}	33 ohms		1/8 W	
R_{31}	33 ohms		1/8 W	
R_{32}	470 ohms		$\frac{1}{4}$ W	
R_{33}	470 ohms		$\frac{1}{4}$ W	
R_{34}	25 k.ohms		$\frac{1}{4}$ W	pot
R_{35}	220 k.ohms		$\frac{1}{4}$ W	
R_{36}	220 k.ohms		$\frac{1}{4}$ W	
R_{37}	33 ohms		1/8 W	
R_{38}	33 ohms		1/8 W	
R_{39}	1 k.ohm		$\frac{1}{2}$ W	
R_{40}	1 k.ohm		$\frac{1}{2}$ W	
R_{41}	1 k.ohm		2 W	
R_{42}	1 k.ohm		2 W	
R_{43}	100 ohms		$\frac{1}{4}$ W	
R_{44}	100 ohms		3 W	
R_{45}	220 k.ohms		$\frac{1}{2}$ W	
R_{46}	220 k.ohms		$\frac{1}{2}$ W	
R_{47}	33 ohms		1/8 W	
R_{48}	1 k.ohm		$\frac{1}{2}$ W	
R_{49}	33 ohms		1/8 W	
R_{50}	33 ohms		1/8 W	
R_{51}	33 ohms		1/8 W	
R_{52}	3.3 k.ohms		3 W	

C_{1}	0.1 µF	
C_{2}	0.1 µF	
C_{3}	10 nF	F.T.
C_{4}	10 nF	F.T.
C_{5}	2 nF	
C_{6}	2 nF	
C_{7}	10 nF	F.T.
C_{8}	10 nF	F.T.
C_{9}	0.47 µF	Mylar
C_{10}	10 nF	F.T.
C_{11}	0.1 µF	
C_{12}	0.1 µF	
C_{13}	0.1 µF	
C_{14}	0.1 µF	
C_{15}	10 nF	F.T.
C_{16}	10 nF	F.T.
C_{17}	2 nF	
C_{18}	2 nF	
C_{19}	10 nF	F.T.
C_{20}	10 nF	F.T.
C_{21}	0.47 µF	Mylar
C_{22}	10 nF	F.T.
C_{23}	0.1 µF	
C_{24}	0.1 µF	
C_{25}	0.1 µF	
C_{26}	0.1 µF	
C_{27}	20 nF	F.T.
C_{28}	10 nF	F.T.
C_{29}	50 µF	350 V
C_{30}	-	
C_{31}	10 nF	F.T.
C_{32}	0.1 µF	
C_{33}	4 µF	300 V
C_{34}	0.1 µF	
C_{35}	0.1 µF	
C_{36}	10 nF	F.T.
C_{37}	10 nF	F.T.

Tube complement: 6 x D 3 A; 1 x E 88 CC

connectors all B.N.C.

Ferroscube beads F = 5659065 - 4B

F.T. = FEED THROUGH CAPACITOR.

7. References and Acknowledgement

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K. Gase

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APPENDIX

I. DERIVATION OF DISCRIMINATION AND TRANSMISSION FACTORS

The L.T.P. circuit is drawn in its simplest form in fig. 1

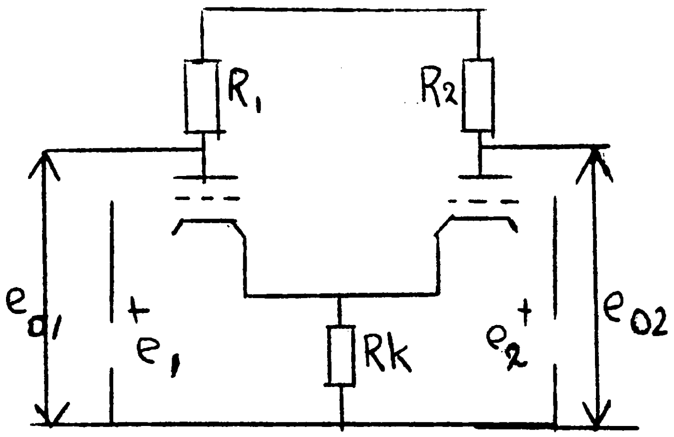


Fig. 1

The loop equations for this circuit are :

$$i_1 \cdot R_1 + Va_1 + Rk(i_1 + i_2) = 0 \qquad (1)$$

$$i_2 \cdot R_2 + Va_2 + Rk(i_1 + i_2) = 0 \qquad (2)$$

$$V_{g_1} + Rk(i_1 + i_2) = e_1 \qquad (3)$$

$$V_{g_2} + Rk(i_1 + i_2) = e_2 \qquad (4)$$

The general expression for the currents i_1 respectively i_2 is :

$$i \cdot Ri = \mu \cdot Vg + Va \qquad (5)$$

Substitution of (1) and (3) yield :

$$i_1 = \frac{\mu_1 e_1 - i_2 \cdot Rk(\mu_1 + 1)}{Ri_1 + R_1 + Rk\ (u_1 + 1)} \qquad (6)$$

and substitution of (2) and (4) result for i_2 in :

$$i_2 = \frac{\mu_2 e_2 - i_1 \cdot Rk(\mu_2 + 1)}{Ri_2 + R_2 + Rk(\mu_2 + 1)} \qquad (7)$$

and solving (6) and (7) for i_1 and i_2 gives :

$$i_1 = \frac{\mu_1 e_1 \left(Ri_2 + R_2 + Rk\,(\mu_2 + 1)\right) - \mu_2 \cdot e_2 \cdot Rk(\mu_1 + 1)}{\left(Ri_1 + R_1 + Rk(\mu_1 + 1)\right)\left(Ri_2 + R_2 + Rk(\mu_2 + 1)\right) - Rk^2(\mu_1 + 1)(\mu_2 + 1)} \qquad (8)$$

$$i_2 = \frac{\mu_2 e_2 (Ri_1 + R_1 + Rk(\mu_1 + 1)) - \mu_1 e_1 Rk(\mu_2 + 1)}{\left(Ri_1 + R_1 + Rk(\mu_1 + 1)\right)\left(Ri_2 + R_2 + Rk(\mu_2 + 1)\right) - Rk^2(\mu_1 + 1)(\mu_2 + 1)}$$

For the computation of the discrimination factor, we take one half of the circuit in consideration. Further we assume that

$$\mu_1 = \mu_2 = \mu$$

$$Ri_1 = Ri_2 = Ri$$

$$R_1 = R_2 = R$$

The output voltage e_{o1} can be written now as :

$$e_o = \frac{- R \left(\mu \cdot e_1 (Ri + R + Rk(\mu + 1) - \mu \cdot e_2\, Rk(\mu + 1)\right)}{D} \qquad (10)$$

where $D =$ denominator of 8 resp. 9.

Now we write for the input voltages :

$$\left.\begin{aligned} e_1 &= e_i - e_o \\ e_2 &= e_i + e_o \end{aligned}\right) \qquad (11)$$

where e_i = inphase input component

e_o = antiphase input component.

Now we assume $e_o = 0$ which means that a pure inphase input is applied to the circuit.

The gain, given by (10) for this input condition is :

$$\frac{e_{out}}{e_i} = - \frac{\mu(Ri + R)\ R}{D} \qquad (12)$$

Assuming, however, that $e_i = 0$, which means that a pure antiphase input is applied to the circuit, we find for the gain :

$$\frac{e_{out}}{e_o} = - \frac{(2\mu Rk(\mu + 1) + \mu(Ri + R)\ \ R}{D} \qquad (13)$$

From the definition of F we see that with (12) and (13) the expression for F is :

$$F = \frac{2\mu Rk(\mu+1) + \mu(Ri + R)}{\mu(Ri + R)} = 1 + \frac{2Rk(\mu + 1)}{R + Ri} \qquad (14)$$

and assuming that $\mu \gg 1$, and $Ri \gg R$ we find finally for F :

$$F = 1 + 2\,gm \cdot Rk \tag{15}$$

II. DERIVATION OF THE TRANSMISSION FACTOR

For the derivation of the transmission factor of a L.T.P. circuit we go out from equations (8) and (9). However, we assume that

$$e_1 = e_2 = e$$

and $\mu_1 \gg 1, \mu_2 \gg 1$

and now equations (8) and (9) may be simplified to

$$i_1 = \frac{e \cdot \mu_1 (Ri_2 + R_2)}{D} \tag{16}$$

$$i_2 = \frac{e \cdot \mu_2 (Ri_1 + R_1)}{D} \tag{17}$$

and the output voltage e_{out} is equal to

$$e_{out} = L_1 R_1 - L_2 R_2 = e \, \frac{\mu_1 R_1 (Ri_2 + R_2) - \mu_2 R_2 (Ri_1 + R_1)}{D} \tag{18}$$

Now we assume that the following equations are valid :

$$\left.\begin{aligned} \mu_1 &= \mu \\ \mu_2 &= \mu + \Delta\mu \\ Ri_1 &= Ri \\ Ri_2 &= Ri + \Delta Ri \\ R_1 &= R \\ R_2 &= R + \Delta R \end{aligned}\right\rbrace \qquad (19)$$

and substitution of (19) in (18) gives :

$$e_{out} = e \; \frac{\mu \cdot R\,(R + Ri)\left\{\frac{\Delta Ri + \Delta R}{Ri + R} - \frac{\Delta\mu}{\mu} + \frac{\Delta R}{R}\right\}}{D} \qquad (20)$$

The gain for an antiphase input signal, where $e_1 = e$ and $e_2 = -e$, without taking the Δ values into account is :

$$e_{out} = e \; \frac{2\,\mu\,(R \cdot Ri + R \cdot Ra + 2R \cdot Rk \cdot \mu)}{D} \qquad (21)$$

and the ratio of (20) and (21) yield for H :

$$H = \frac{2\,\mu \cdot R(Ri + Ra + 2\,\mu \cdot Rk)}{\mu \cdot R(R + Ri)\left\{\frac{\Delta Ri + \Delta R}{Ri + R} - \frac{\Delta\mu}{\mu} + \frac{\Delta R}{R}\right\}} \qquad (22)$$

and assuming again $Ri > R$ we get, with the aid of equation (15)

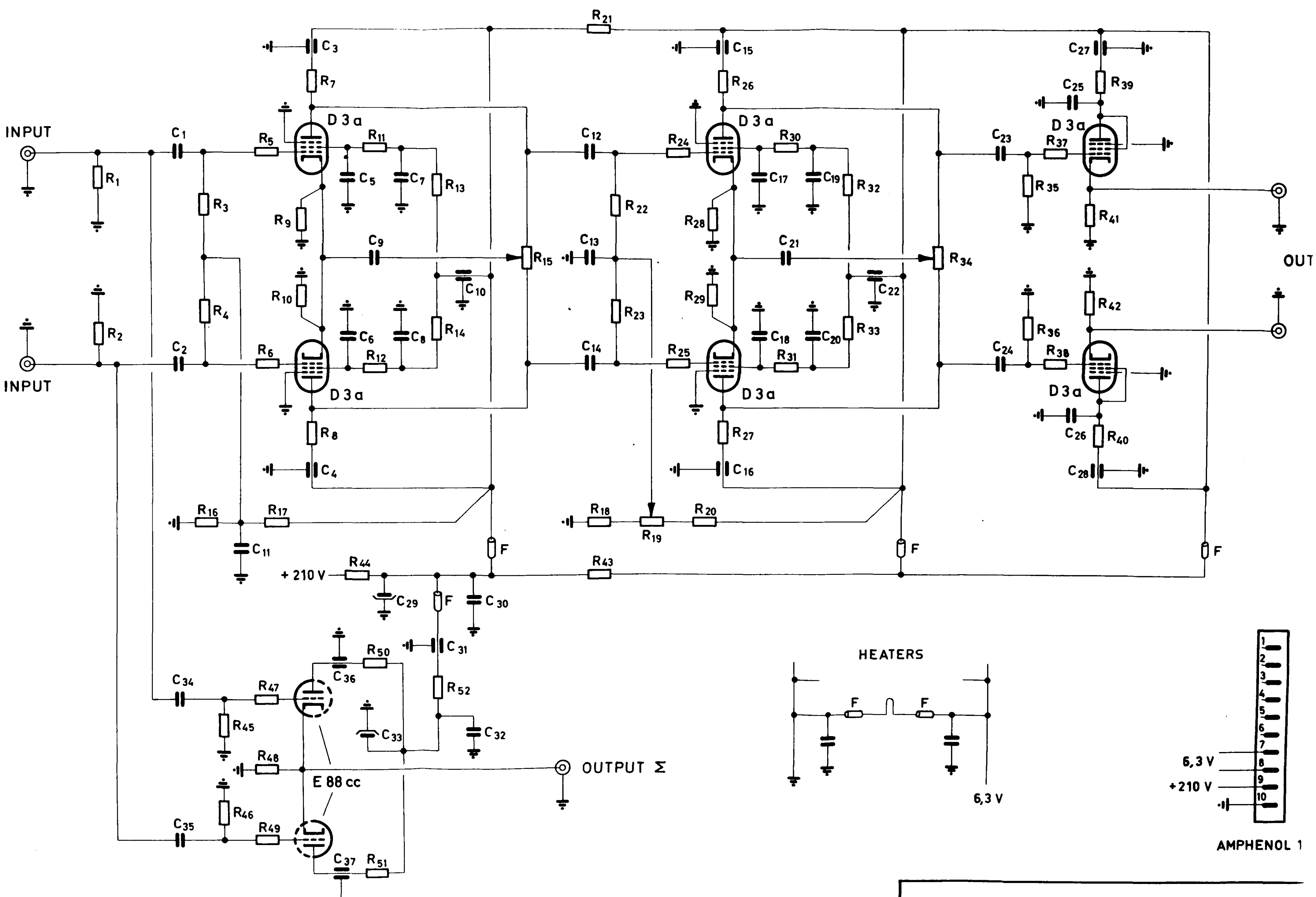
$$H = \frac{2F}{\frac{\Delta Ri}{Ri} + \frac{\Delta R}{Ri} - \frac{\mu}{\mu} - \frac{\Delta R}{R}} \quad (23)$$

and if the simplification that $\frac{R}{Ri}$ represents a second order factor is true we find finally :

$$H = \frac{2F}{\frac{\Delta Ri}{Ri} - \frac{\Delta \mu}{\mu} - \frac{\Delta R}{R}} \quad (24)$$

Similar, but no equal, expressions have been derived by Zaalberg v. Zelst[5), Parnum[8) and Andrews[2).

However, as we could always assume that $Ri > Rk \cdot \mu$ and $Ri > R$, which they could not, our expression is rather simple. One should however bear in mind that equation (24) is only valid for $Ri > (R + \mu \cdot Rk)$.



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