P.H. STANDLEY

MPS/Int. RF 64 - 5
2.4.1964

ANALYSIS OF PU ELECTRODE SIGNAL

Summary

The signal for the observation of the circulating proton beam in the PS as derived from an electrostatic PU electrode, is transmitted to the observation points via an impedance transformer of cathode-follower type which matches the high impedance of the electrode to a 75 ohms coaxial cable.

Since during the machine cycle the proton beam is bunched rather tightly, high voltages can be influenced on the electrode which may lead to overloading of the cathode-follower.

In this report the excitation signal for the cathode-follower is analysed and methods for reducing its amplitudes are considered.

Assuming a triangular distribution of charge within a proton bunch simple equations have been written for the voltage appearing at the PU electrode. These equations are valid once the bunch has assumed its final structure, i.e. above about 4 GeV.

Fig. 1 shows the basic construction of a PU station.

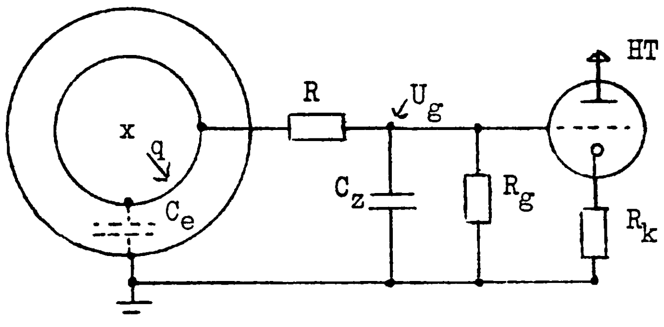


Fig. 1

C_e is the self capacitance of the electrode and C_z is an additional loading capacitance, R is a resistance which together with C_z prevents the cathode-follower from overloading. We are interested in the voltage U_g for the following cases :

1. R = 0

In this case we may write :

$$U_g = \begin{cases} \beta t & 0 \leqslant t \leqslant \varphi \\ \beta(2\varphi - t) & \varphi \leqslant t \leqslant 2\varphi \\ 0 & 2\varphi \leqslant t \leqslant \tau \end{cases} \qquad (1)$$

where φ and τ are explained in fig. 2 which shows $U_g = f(t)$

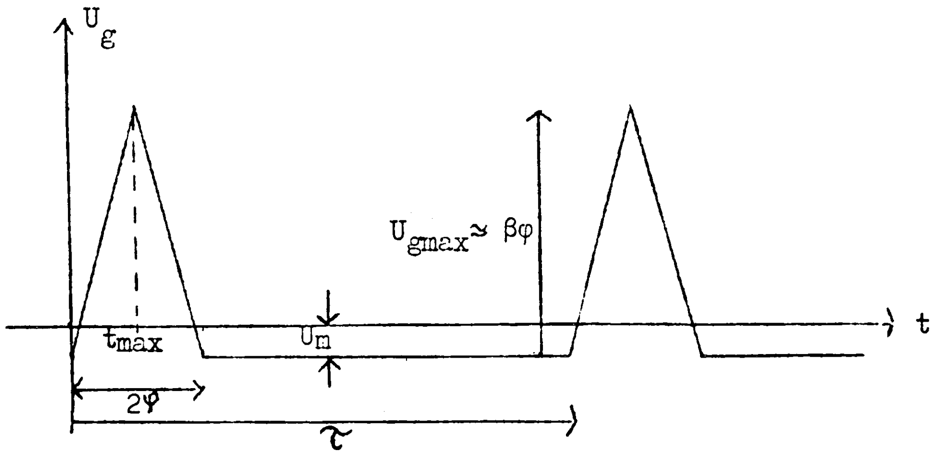


Fig. 2

In equation (1) the electrode load resistance R_g has been neglected. Since the overall time constant given by R_g and the electrode loading capacitances is large compared with the pulse width 2φ, the only effect of R_g is to make the mean electrode potential zero. The positive excursion of the electrode voltage is thus reduced by U_m where

$$U_m = \frac{\beta\varphi^2}{\tau}$$

Since $\varphi \ll \tau$, U_m is generally small compared with the peak positive excursion of electrode voltage and has been neglected in the following considerations.

We may therefore write :

$$U_{g\,max} \approx \beta\varphi \qquad (t_{max} = \varphi) \qquad (2)$$

The frequency analysis of equation (1) gives the peak amplitude of the fundamental and its harmonics as :

$$\hat{U}_n = \frac{\beta \tau}{\pi^2} \left(\frac{1 - \cos n \frac{2\pi}{\tau} \varphi}{n^2} \right) \tag{3}$$

where n is the harmonic number (n = 1, 2, 3, ..)

2. R ≠ 0

Where essentially the fundamental of the PU signal is of interest, a resistance R is included between the electrode and the capacitance C_z. As well as reducing the effective band-width of the PU station, the resistance R also reduces the signal amplitude reading the cathode-follower grid.

In this case we may write :

$$U_g = \begin{cases} \beta t + \frac{\beta}{\alpha} (e^{-\alpha t} - 1) & 0 \leqslant t \leqslant \varphi \\ 2\beta\varphi - \beta t + \frac{\beta}{\alpha} (e^{-\alpha t} - 2e^{-\alpha(t-\varphi)} + 1) & \varphi \leqslant t \leqslant 2\varphi \\ \frac{\beta}{\alpha} e^{-\alpha(t-2\varphi)} (e^{-2\alpha\varphi} - 2e^{-\alpha\varphi} + 1) & 2\varphi \leqslant t \leqslant \tau \end{cases} \tag{4}$$

The maximum amplitude of this voltage is :

$$U_{g\,max} = \beta \left[2\varphi - \frac{1}{\alpha} \ln (2e^{\alpha\varphi} - 1) \right]$$

$$\left(t_{max} = \frac{1}{\alpha} \ln (2e^{\alpha\varphi} - 1) \right) \tag{5}$$

The frequency analysis gives for the fundamental and its harmonics :

$$\hat{U}_n = \frac{\beta\tau}{\pi^2} \left(\frac{1 - \cos n \frac{2\pi}{\tau} \varphi}{n^2}\right) \frac{1}{\sqrt{1 + \left(n \frac{2\pi}{\alpha\tau}\right)^2}} \qquad (6)$$

$$\left(\operatorname{tg} \varphi_n = \frac{\sin n \frac{2\pi}{\tau} \varphi + n \frac{2\pi}{\alpha\tau} \cos n \frac{2\pi}{\tau} \varphi}{\cos n \frac{2\pi}{\tau} \varphi - n \frac{2\pi}{\alpha\tau} \sin n \frac{2\pi}{\tau} \varphi}\right)$$

φ_n gives the phase shift caused by the resistance R.

For the cases ($R = 0$, $R \neq 0$) so far considered

$\beta = \frac{q}{C_e + C_z}$, $\alpha = \frac{C_e + C_z}{C_e C_z R}$, $\tau = \frac{1}{f_a}$; f_a = accelerating frequency

q expresses the rate of change of electrode charge as a function of the number of accelerated protons (A_p) and the geometry of the electrode relative to the beam. If the electrode is cylindrical and encircles the beam, then the electrode geometry can be entirely expressed in terms of its effective length (ℓ_{eff}) .

We may write :

$$q = \frac{A_p \, e \, \tau}{L \, \varphi^2} \, \ell_{eff} \qquad (7)$$

= elementary charge = $1{,}6 \cdot 10^{-19}$ (coul.)

L = length of PS = 628 m

Most of the PU electrodes used in the PS have the same geometry, e.g. in the phase stations, the radial and the vertical position stations and the intensity stations. The effective lengths of these electrodes follows from ref. 1 from which we derive for the slope of the calibration curve :

$$\text{tg}\ \alpha = \frac{L}{\sqrt{2} \cdot e \cdot A_p} \cdot \frac{1}{\ell_{eff}} \qquad (8)$$

From the "coax method" from the same reference we find for the tg α_1 at 10^{11} protons :

$$\text{tg}\ \alpha_1 = 0.0784 \cdot 10^{12} \ (\frac{1}{F \cdot V_{eff}}) \qquad (9)$$

Similarly for the "charged rod" method :

$$\text{tg}\ \alpha_2 = 0.0844 \cdot 10^{12} \ (\frac{1}{F \cdot V_{eff}}) \qquad (1o)$$

Substitution of (9) or (10) respectively in (8) yield for the effective length :

$$\left.\begin{aligned} \ell_{eff\ 1} &= 0.355 \text{ m (coax method)} \\ \ell_{eff\ 2} &= 0.33 \text{ m (charged rod)} \end{aligned}\right\} \qquad (11)$$

The average of (11) gives :

$$\ell_{eff} \approx 0.3425 \text{ m}$$

which is in close agreement with ref. 2 where the effective length was found with the computer from the potential distribution as :

$$\ell_{eff} = 0.345 \text{ m}$$

The beam position electrodes are split into two halves. If the beam is central with respect to the two halves, then the influenced current q (eqn. 7) divides equally between them, and the current per electrode q_s is given by

$$q_s \approx 0.5\, q \tag{7}$$

If the beam is in its extreme position from the centre of the electrode pair, then the following relations are valid (ref. 4) :

$$\text{radial } q_{s\ max} \approx 0.7\, q \quad (\text{for } \Delta R_{rad} = 6 \text{ cm})$$

$$\text{vertical } q_{s\ max} \approx 0.62\, q \quad (\text{for } \Delta R_{vert} = 3.5 \text{ cm})$$

3. Electrode signal at transition

We know that in the neighbourhood of transition φ becomes very small while at transition itself φ becomes theoretically zero. As φ decreases the triangular form of U_g (R = 0) becomes increasingly rectangular because the effective length of the electrode becomes comparable with the physical length of a proton bunch. In the limiting case when $\varphi = 0$, the pulse length is given by :

$$\ell_p = \frac{\ell_{eff}}{V} \quad (\text{sec})$$

where V is the velocity of the proton beam which in the energy range above 4 GeV can be considered equal to the light velocity . With $v \approx c$

$$\ell_p \approx \frac{\ell_{eff}}{c} \quad (\text{sec}) .$$

In this case the maximum value of U_g with $R = 0$ is :

$$U_{g\,max,\;R=0} = \frac{A_p e}{20\,(C_e + C_z)} \tag{12}$$

The frequency analysis gives for the fundamental and its harmonics :

$$\hat{U}_n \approx \frac{2\,A_p e}{20\,(C_e + C_z)\,n\pi} \sin\left(n \frac{2\pi}{\tau} \frac{\ell_p}{2}\right) \tag{13}$$

And $R \neq 0$

$$U_{g\,max,\;R\neq 0} = \frac{A_p e}{20\,(C_e + C_z)} \left(1 - e^{-\alpha \ell_p}\right) \tag{14}$$

The frequency analysis gives :

$$\hat{U}_n \approx \frac{2\,A_p e}{20\,(C_e + C_z)\,n\pi} \sin\left(n \frac{2\pi}{\tau} \frac{\ell_p}{2}\right) \frac{1}{\sqrt{1 + \left(n \frac{2\pi}{\alpha\tau}\right)^2}} \tag{15}$$

4. Example

By way of example the analysis of the intensity PU station is given in Fig. 3. This analysis was made for $R = 0$ and $R = 330$ ohms with $C_e = 40$ pF, $C_z = 200$ pF, and $A_p = 10^{11}$ protons, $\ell_{eff} = 33$ cm.

R.K. Kaiser

Distribution : (open)

MPS Scientific and Technical Staff
RF Group

APPENDIX

In this appendix the frequency analysis is given for the case of an electrode loaded as shown in Fig. 4 :

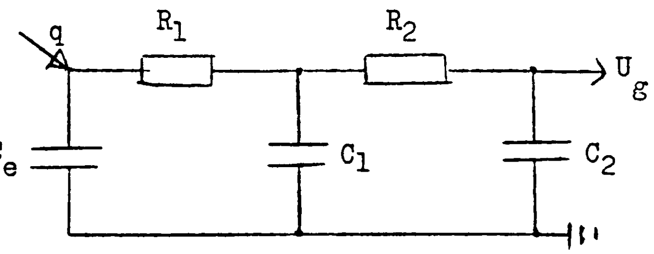


Fig. 4

$$\hat{U}_n = \frac{\beta^{*}\tau}{\pi^2}\left(\frac{1-\cos n\frac{2\pi}{\tau}\varphi}{n^2}\right)\frac{1}{\sqrt{1+\left(n\frac{2\pi}{\alpha^{*}\tau}\right)^2}} \qquad (16)$$

This equation has, of course, the same form as equation (6) but in the above case :

$$\beta^{*} = \frac{q}{C_e + C_1 + C_2} \quad , \qquad (17)$$

$$\frac{1}{\alpha^{*}} = \frac{1}{C_e+C_1+C_2}\sqrt{R_1^{\,2}C_e^{\,2}(C_1+C_2)^2 + R_2^{\,2}C_2^{\,2}(C_e+C_1)^2 + 2R_1R_2C_e^{\,2}C_2^{\,2}\left(1+\left(n\frac{2\pi}{\;}\right)^2 C_1^{\,2}R_1R_2\right)} \qquad (18)$$

To investigate the saturation possibilities of the cathode-follower with the computed input voltage which follows from this report, the reader is further referred to ref. 3.

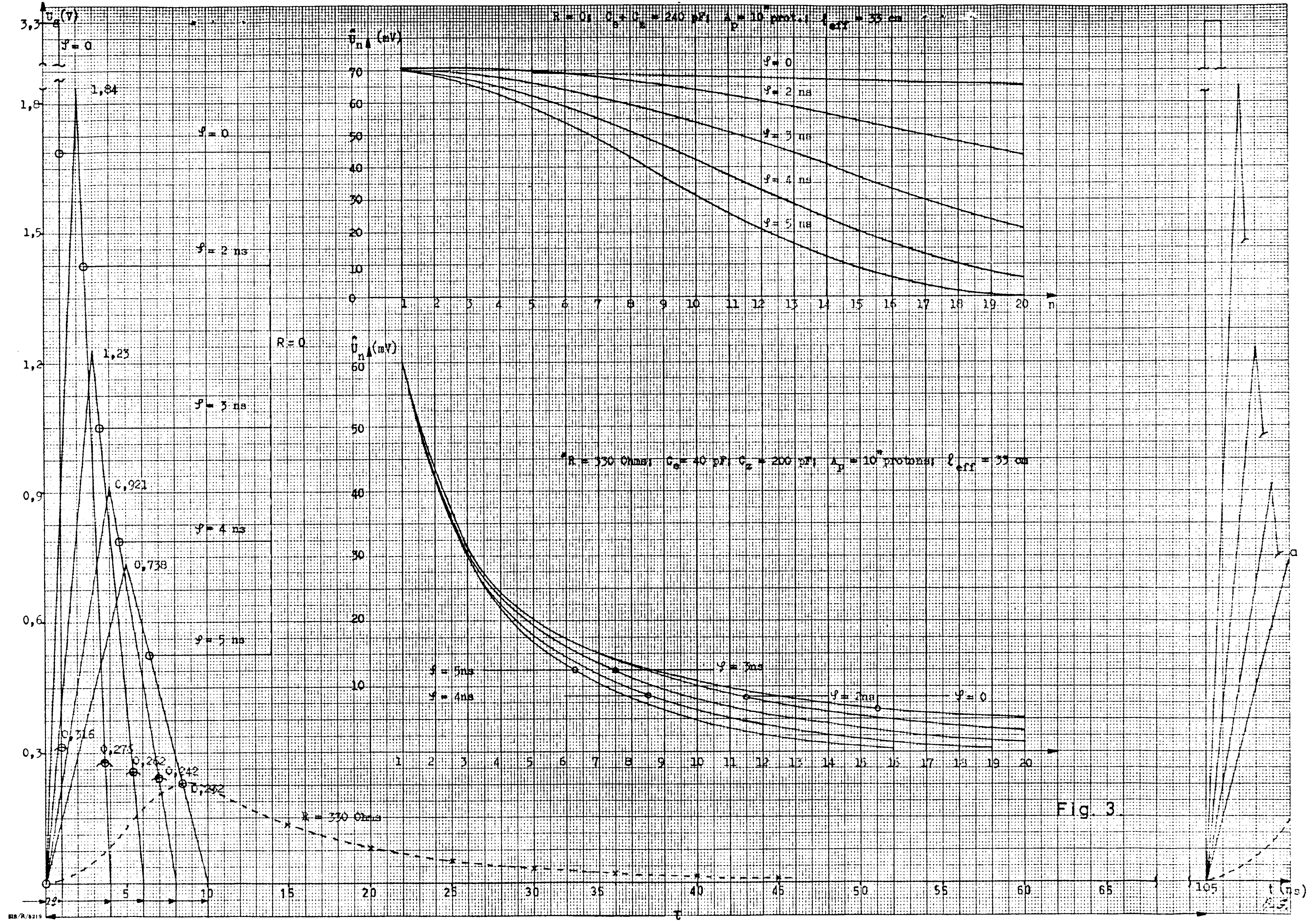
REFERENCES

1. R. Kaiser : On the Calibration of the intensity PU station, MPS/Int. RF 64 - 2

2. H.G. Hereward : Pick-up station effective length Memorandum of June 18th, 1962

3. K. Gase : Aspects of wide-band cathode-follower circuits MPS/Int. RF 63 - 7

4. K. Gase : Private communications



R = 0; $C_s + C_z$ = 240 pF; $A_p = 10^8$ prot.; ℓ_{eff} = 33 cm
$\hat{U}_n$ (mV)
φ = 0
φ = 2 ns
φ = 3 ns
φ = 4 ns
φ = 5 ns
n
R = 330 Ohms; C_e = 40 pF; C_z = 200 pF; $A_p = 10^8$ protons; ℓ_{eff} = 33 cm
U_s (V)
R = 0
1,84
1,23
0,921
0,738
0,316
0,275
0,262
0,242
0,232
R = 330 Ohms
t (ns)
τ

Fig. 3