~~G. GIESCH~~

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# AIR FLOW BEHAVIOUR IN A SMALL ORIFICE

## Introduction

In the pressurised spark gaps which are at present under development for the fast ejection system, the air will be exchanged at a rate of 30 lit/min of free air, at a pressure of 8 kg/ $cm^2$ abs. per gap.

The air pressure in the gap will be set according to the voltage at which the gap is to work. The figure of 8 $kg/cm^2$ corresponds to the highest operational voltage level of 120 kV. The minimum pressure in the gap will be of the order of 2 $kg/cm^2$ abs.

The volume of each gap will roughly be 6 litres which means that, at a purge rate of 30 lit/min of free air, the volume will be exchanged at a rate of 1 volume per 12 seconds. With a repetition rate of 1 "long" shot per 3 secs in the ejection system of which the the spark gaps form an integral part, the exchange rate of 1 volume per 4 shots seems adequate.

It is thought that the bleed orifice could simply be one littel hole, drilled in an appropriate position through the spark gap housing. The dimensions of this orifice will be such that, at the maximum working pressure, the masse flow will be the 30 lit/min. As a consequence, the mass flow will be lower at lower pressures.

However, the flow pattern in the orifice is not necessarily constant over the pressure range. This pattern is governed by the Reynolds number which can be written as

$$Re = \frac{V_{av} \cdot 2R}{\vartheta}$$

where $V_{av}$ = average air speed

$2R$ = Diameter of orifice

$\vartheta$ = kinematic viscosity

For a Reynolds value of about 2300 or smaller the flow is viscous or laminar and the air speed and hence the mass flow through the orifice increases with the pressure in a linear manner.

For a Reynolds value greater than 2300 the air flow is turbulent. Under turbulent flow conditions the air speed, and the mass flow increase by the 4/7th power of the pressure increase.

The transition point, where the Reynolds value for a given orifice becomes critical, should possibly be outside the working pressure range. The reason for this condition is mainly this : should the pressure in the gap be such that the resulting air speed would bring the Reynolds value very close to the critical value, then the instability of the control system might bring it temporarily beyond this point. The sudden change in mass flow might cause the control system to become unstable, an effect which should clearly be avoided.

In our particular case it is interesting to bring the critical Reynolds value to the bottom of the working pressure range. As the mass flow under turbulent flow conditions changes with the 4/7th power of the pressure change, the air exchange rate maintained when the pressure is decreased is higher than under laminar flow conditions.

This investigation into the flow conditions was undertaken as a result of the fact that during the ordering period for the air control system two representatives of firms contacted proposed adjustable orifices or even two orifices per sprrk gap in order to adjust the flow conditions.

In the following it will however be shown that with a hole of 6 mm length and 0.6 mm diameter the flow is turbulent over the entire pressure range and that the mass flow under maximum pressure conditions meets the specification.

Thus the author does not see a necessity for a more complicated orifice control system. One single cylindircal hole will suffice.

Orifice dimensions

The mass flow at maximum pressure is 30 lit/min $= 5 \cdot 10^{-4}$ $m^3$/sec. The maximum differential pressure is $7 \cdot 10^4$ kg/$m^2$. The spark gap housing has a wall thickness of 6 mm, and applying this value as well as the one quoted in Appendix 3 for the dynamic viscosity, we get from equation (8), Appendix 1, for the dimension of R :

$$R = 0.12 \text{ mm}$$

Substitution of this value in equation (9) gives an air speed of

$$V_{av} \approx 10^4/\text{sec}$$

And the Reynolds value for these values of R and $V_{av}$ becomds

$$Re = 2 \cdot 10^5$$

At the lowest pressure ($10^4$kg/$m^2$ rel) the air speed is :

$$V_{av} \approx 25 \cdot 10^3$$

This signifies that, with a hole of these dimensions, the air flow over the whole pressure range is turbulent.

However, under turbulent flow conditions the mass flow will not come up to the required value of $5 \cdot 10^{-4}$ $m^3$/sec.

Calculated on the basis of the Blasius formula (Appendix 2) the hole radius becomes :

$$R \approx 0.3 \text{ mm}$$

After computation of the air speed from the same formula it is found that the Reynolds value for both the maximum and the minimum value of the pressure range is over-critical and hence the air flow is turbulent over the entire pressure range. With a hole of this diameter, the Reynolds value becomes critical at a differential pressure of 26.4 kg/m$^2$ which is much lower than the lowest value of our pressure range.

It may be of interest to calculate the hole diameter when the mass flow has to be increased with the transition point remaining under the working range. Thus, if the Reynolds value becomes critical at a differential pressure of 1 kg/cm$^2$, from the definition of the Reynolds value (0) we may write :

$$V_{av} \cdot R = 1650 \cdot 10^{-5} \qquad (20)$$

From equation (9) we get for $V_{av}$ at the minimum differential pressure :

$$V_{av} = 125 \cdot 10^{9} \cdot R \qquad (21)$$

From (20 and (21) it follows that the maximum radius of the orifice may be of the order of 2.5 mm. The flow will still be turbulent then, but the mass flow will of course be many times higher than the 30 lit/min.

K. Gase

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## Appendix I

## Derivation of equations for laminar flow conditions

Consider a cylinder of 2R inner diameter and a length $\ell$. Due to the viscosity, the air speed at the wall is zero and maximum in the centre of the cylinder.

Hence $\frac{dV}{dR}$ is negative.

Friction force on cylinder walls equals

$$\eta \frac{dV}{dr}$$

where $\eta$ is the dynamic viscosity.

For the total friction force on the inner surface of the cylinder one may write:

$$K_1 = - 2 \pi r . \ell . \eta \frac{dV}{dr} \qquad (1)$$

The pressure component, at a pressure difference of $p_1 - p_2$ between both ends of the cylinder, which pushes the air column through our cylinder is

$$K_2 = \frac{\pi r^2}{2} (p_1 - p_2) \qquad (2)$$

For the condition that $K_1 = K_2$ we obtain for the velocity component

$$dV = \frac{-(p_1 - p_2) . r \, dr}{4 \eta \ell} \qquad (3)$$

Integrated

$$V = \frac{-(p_1 - p_2) r^2}{4 \eta \ell} + K_3 \qquad (4)$$

The integration constant $K_3$ follows from the condition that for $r = R$, $V = 0$.

Hence

$$K = \frac{(p_1 - p_2)R^2}{4 \eta \ell} \tag{5}$$

and

$$V = \frac{(p_1 - p_2)(R^2 - r^2)}{4 \eta \ell} \tag{6}$$

The air speed is at the maximum for $r = o$ so

$$V_{max} = \frac{(p_1 - p_2) R^2}{4 \eta \ell} \tag{7}$$

The mass flow follows from

$$Q = \int_o^R 2 \pi r \, dr.V = \frac{(p_1 - p_2)\pi.R^4}{8\eta \ell} \tag{8}$$

and the average air speed follows also from equation 8

$$Q = \pi.R^2.V_{av} \qquad \text{hence} \quad V_{av} = \frac{(p_1 - p_2)R^2}{8 \eta \ell} \tag{9}$$

To arrive to these equations the Bernouilli laws could not be applied while the viscosity is not taken into consideration in the Bernouilli statements.

## Appendix II

## Equations for turbulent flow conditions.

A simple derivation of an equation for flow characteristics under turbulent conditions cannot be given.

However, "Blasius" experimental formula can be applied with fair accuracy.

Written in the same dimensions as used in Appendix 1 this formula is

$$(p_1 - p_2) = \frac{0{,}158\ \rho . V_{av} . \ell}{\sqrt[4]{Re}\ 2\ R} \qquad (10)$$

where $\rho$ = density

Re = Reynolds number

Substituting Re by $\frac{V_{av} 2\ R}{\vartheta}$ , and taking into account that

$\rho_{air}$ at $20^{o}C = 1{,}27\ kg/m^3$

and $\ell = 6.10^{-3}$ m ,

equation(10) can be written

$$V_{av} = \frac{(p_1 - p_2)\ R^{5/4}}{5.10^{-4} . \vartheta^{1/4}} \qquad (11)$$

The dimensions of R can be calculated from equation(11) and the statement of equation 9 :

$$Q = \pi . R^2\ V_{av}$$

## Appendix III

For the calculations in this report, the following values for density and viscosity have been used:

| | |
|---|---|
| Dynamic viscosity | $\eta = 18.10^{-7}$ kg.sec/$m^2$ |
| Kinematic viscosity | $\vartheta = 144.10^{-7} m^2/sec$ |
| Density | $\rho = 1.27$ kg/$m^4$sec. |

These have been taken from tables and valid for air at a remperature of $20^{o}C$.