ISBN: 978-3-95450-215-8 ISSN: 2226-0366 doi:10.18429/JACoW-LINAC2022-TUPORI13

BEAM LOADING SIMULATION FOR RELATIVISTIC AND ULTRARELATIVISTIC BEAMS IN THE TRACKING CODE RF-TRACK

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Abstract

Medical and industrial electron linacs can benefit from the X-band accelerating technology developed for the Compact Linear Collider (CLIC) at CERN. However, when high-intensity beams are injected in such high-gradient structures (>35 MV/m), the beam loading effect must be considered by design since this beam-cavity interaction can result in a considerable gradient reduction with respect to the unloaded case. Studying energy conservation, a partial differential equation (PDE) has been derived for injected beams in both the relativistic and ultrarelativistic limits. Making use of this, a specific simulation package within RF-Track has been developed, allowing realistic tracking of charged particle bunches under this effect regardless of their initial velocity. The performance of this tool has been assessed by reproducing previously obtained beam-loaded fields in CLIC main linac and CLIC Drive-Beam linac structures. In this paper, we present the analytic PDE derivation and the results of the tests.

INTRODUCTION

Accelerating cavities exhibit an ohmic response when electromagnetic waves travel through the structure, leading to energy losses. Furthermore, when a beam of charged particles enters the accelerating structure, it excites the cavity, and such excitation diminishes the stored energy and thus the accelerating gradient. This is called beam loading effect [1].

Previous attempts to model the beam loading effect rely on simplifications of the structure and beam properties. The most general analytic study is reference [2], which describes the beam-loading effect in arbitrary structures and shows that this interaction beam-cavity follows a transient model. However, [2] is limited to the ultrarelativistic case and focuses on the structure gradient rather than its implementation in a tracking routine.

So far, the way of performing tracking under this effect was using a beam-loaded precomputed field obtained from finite-difference solvers such as HFSS. This approach presents the inconvenience of being both computationally demanding and static, i.e. no transient behaviour can be retrieved.

To design a quick, flexible and non-ultrarelativistic tool to study the beam loading effect, the starting point is deriving a PDE describing the energy conservation in such a general scenario. Its numerical solution will be implemented in RF-Track, allowing tracking under the beam loading effect.

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PDE THEORETICAL DERIVATION

Energy Conservation in Accelerating Structures

Instantaneous energy conservation can be studied starting from the differential version of Poynting's theorem [3]:

$$-\frac{\partial u(\boldsymbol{r},t)}{\partial t} = \boldsymbol{\nabla}\cdot\boldsymbol{S}(\boldsymbol{r},t) + \boldsymbol{E}(\boldsymbol{r},t)\cdot\boldsymbol{J}(\boldsymbol{r},t), \quad (1)$$

where:

- u is the volumetric electromagnetic energy density stored in the accelerating cavity, defined in terms of the electric ($\boldsymbol{E}$) and magnetic ($\boldsymbol{H}$) fields as:

$$u(\boldsymbol{r},t) = \frac{1}{2}\varepsilon_0||\boldsymbol{E}(\boldsymbol{r},t)||^2_{\mathbb{R}^3} + \frac{1}{2}\mu_0||\boldsymbol{H}(\boldsymbol{r},t)||^2_{\mathbb{R}^3}\ [\mathrm{J/m^3}], \quad (2)$$

  with ε_0 the electric permittivity and μ_0 the magnetic permeability in vacuum (empty cavity).

- $\boldsymbol{S}$ is the Poynting vector defined as $\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H}$. Its flux takes into account power flow (P_{flow}) and dissipation (P_{diss}) along the structure as it has been reported in [3]. In an arbitrary volume bounded by a surface $\mathcal{S}$, its flux can be described as:

$$P_{\mathrm{total}}(z,t) = \oint_{\mathcal{S}} \boldsymbol{S}(z,t)\cdot \mathrm{d}\boldsymbol{\mathcal{S}} = P_{\mathrm{diss}}(z,t) + P_{\mathrm{flow}}(z,t). \quad (3)$$

- $\boldsymbol{J}$ is the current density flowing along the structure, defined in terms of the volumetric charge density λ_q and the particle's velocity $\boldsymbol{v}$ as:

$$\boldsymbol{J}(\boldsymbol{r},t) = \lambda_q(\boldsymbol{r},t)\boldsymbol{v}(\boldsymbol{r},t)\ [\mathrm{A/m^2}]. \quad (4)$$

Accelerating Gradient

The main quantity whose evolution is aimed to be studied is the effective accelerating gradient, defined as the average electric field that affects a particle. In a given cell of length L located at position z, it can be described as:

$$G_{\mathrm{eff}} = \frac{1}{L}\mathrm{Re}\left[\int_z^{z+L}\tilde{E}_z(\zeta,t)e^{j\omega t_q(\zeta,t_0,\beta(\zeta,t,t_0,\beta_0))}\,\mathrm{d}\zeta\right]\ [\mathrm{V/m}], \quad (5)$$

with ω the RF angular frequency, $j=\sqrt{-1}$ the imaginary unit, and t_q the time of flight of a particle with charge q entering the structure at a time t_0, calculated as [4]:

$$t_q(z,t_0,\beta(z,t,t_0,\beta_0)) = t_0 + \int_0^z \frac{d\zeta}{\beta(\zeta,t,t_0,\beta_0)c}\ [\mathrm{s}]. \quad (6)$$

Here, c refers to the speed of light and β to the ratio of v to c, which evolves with the particle's energy gain.

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Since the variation of the gradient with time is much slower than the RF field oscillation, one can consider the electric field phasor ($\tilde{E}_z$) dependency with time [5]. This is called the "quasi-static" assumption.

Figures of Merit

The description of the energy flow along the structure requires the presentation of three figures of merit:

- Normalized effective shunt impedance per unit length (ρ_{eff}): measures the efficiency of a structure in providing a high gradient field for a given RF power dissipation in the structure [6]. It is defined as:

$$\rho_{\text{eff}}(z,\beta(z,t,t_0,\beta_0)) = \frac{G_{\text{eff}}(z,t,\beta(z,t,t_0,\beta_0))^2}{Q(z)\langle p_{\text{diss}}(z,t)\rangle}\ [\Omega/\text{m}], \tag{7}$$

  where $p_{\text{diss}} = \frac{\partial P_{\text{diss}}}{\partial z}$ is the dissipated power per unit length and $\langle f\rangle = \frac{1}{T}\int_t^{t+T} f(t')\mathrm{d}t'$ denotes the time average over an RF-period ($T = \frac{2\pi}{\omega}$) of an arbitrary time-dependent function f.

- Quality factor (Q): the bigger its value is, the less power is dissipated at the cavity. It is defined as:

$$Q(z) = \frac{\omega\ \langle w(z,t)\rangle}{\langle p_{\text{diss}}(z,t)\rangle}, \tag{8}$$

  where w stands for the linear density of EM energy per unit length, defined as $w(z,t) = \int_S u(\boldsymbol{r},t)\mathrm{d}S$ [J/m].

- Group velocity (v_g): indicates the speed at which the energy flows along the structure. It is defined as:

$$v_g(z) = \frac{\langle P_{\text{flow}}(z,t)\rangle}{\langle w(z,t)\rangle}\ [\text{m/s}]. \tag{9}$$

For the sake of its analytical manipulation, the figures of merit have been presented as continuous functions of z as it is exposed in [2].

The General Relativistic Case

In order to obtain a PDE for G_{eff} in the variables (z,t), Eq. (1) must be time-averaged and integrated over the structure section ($\mathcal{S}$) so that the previous figures of merit can be substituted.

As a first approximation, the gradient is calculated for a witness particle characterized by the average properties of the considered bunch travelling on axis. Therefore, the paraxial approximation ($v_z \gg v_x, v_y$) is assumed as well as the previously mentioned quasi-static assumption. Doing so, one gets the following PDE:

$$-\frac{\partial G_{\text{eff}}}{\partial t} = \left(-\frac{1}{\rho}\frac{\partial \rho_{\text{eff}}}{\partial \beta}\frac{\partial \beta}{\partial t} - \frac{v_g}{\rho_{\text{eff}}}\frac{\partial \rho_{\text{eff}}}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z}\right)\frac{G_{\text{eff}}}{2} + v_g\frac{\partial G_{\text{eff}}}{\partial z} + \frac{\omega\rho_{\text{eff}}\tilde{I}}{2}, \tag{10}$$

with $\tilde{I}(z,t,\beta) = \langle I(z,t)e^{j\omega[t-t_q(z,t,t_0,\beta(z,t,t_0,\beta_0))]}\rangle$ [A] and $I(z,t) = \int_S \boldsymbol{J}(\boldsymbol{r},t)\cdot\hat{e}_z\ \mathrm{d}S$ [A].

The Ultrarelativistic Case for a Train of Bunches

Let us consider the case of a train of N ultrarelativistic bunches, each of them with charge q, time width σ and time-spacing given by an entry frequency f_b synchronous with ω.

In this case, $\beta \simeq 1$, $\frac{\partial\beta}{\partial t} \simeq 0$, $\rho_{\text{eff}} = \rho$ and Eq. (10) reads:

$$-\frac{\partial G_{\text{eff}}}{\partial t} = \left(-\frac{v_g}{\rho}\frac{\partial \rho}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z}\right)\frac{G_{\text{eff}}}{2} + v_g\frac{\partial G_{\text{eff}}}{\partial z} + \frac{\omega\rho\tilde{I}}{2}, \tag{11}$$

where $\tilde{I}$ is:

$$\tilde{I}(z,t,t_0) = \sum_{n=1}^{N}\chi(n,t-t_q(z,t_0))\frac{q}{T}e^{-\omega^2\sigma^2/2}, \tag{12}$$

with $\chi(n,t) = \begin{cases} 1 \text{ if the } n^{\text{th}} \text{ bunch is at } z \text{ at a time } [t,t+T] \\ 0 \text{ otherwise.}\end{cases}$

Equation (11) matches the expression found at [2]. In addition, it can be seen that Eq. (11) is a linear PDE and therefore the loaded case can be decoupled from the unloaded case [1].

RF-TRACK FOR BEAM LOADING SIMULATIONS

The ultrarelativistic case for travelling wave accelerators has been implemented into RF-Track [7], a code which tracks particles under given field maps including space charge effects and wakefields, among other phenomena. It has the advantage of tracking multiple species (regardless of their charge or mass) and is written in parallel C++, significantly reducing the computation time.

Pre-computing the Beam Loading Effect

The linearity of Eq. (11) allows the computation of the beam loading effect by calculating separately the beam induced field. This can be implemented in RF-Track [7] as a collective effect that can be superposed to an already existing field with the command `add.collective.effect()`.

The implementation is designed to solve Eq. (11) a priori. For that, knowledge of the beam (`q`, σ, t_0, `N`, `fb`) is required as a user-input for the calculation of the intensity. Information about the accelerating structure, namely v_g, Q and ρ_{eff} at some given positions, is also needed so that they can be cubic-interpolated along the structure.

However, combining Eqs. (7) and (8), one can find an alternative method for the computation of ρ_{eff} based on the field-map integration as well as $\langle w\rangle$ calculation [2]:

$$\rho_{\text{eff}} = \frac{G_{\text{eff}}^2}{\omega\langle w\rangle}. \tag{13}$$

Finally, the PDE is solved numerically by the finite-difference method and the solution for G_{eff} is obtained for a given space and time-mesh. Such space-mesh is fine enough so that the solution matches the limit as $L \to 0$ of Eq. (5). From there, the electric field phasor ($\tilde{E}_z$) is retrieved and the beam loading effect can be computed as an additional

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longitudinal kick (F_z) when a particle located at z_{part} at a time t_{part} is tracked. Such kick considers short-range effects by cubic-interpolating $\tilde{E}_z$ in the variables t and z as follows:

$$F_z = q\mathrm{Re}\left[\tilde{E}_z(z_{\text{part}}, t_{\text{part}})e^{j\omega t_{\text{part}}}\right]. \quad (14)$$

RESULTS AND DISCUSSION

The performance of the new RF-Track collective effect has been assessed by studying the beam-loading implementation in CLIC's main beam accelerating structure (CLIC AS) and CLIC's Power Extraction and Transfer Structures (PETS). The parameters used for both simulations are shown in Table 1.

Table 1: Structure and beam specifications [2, 8, 9]

Magnitude	Unit	CLIC AS	PETS
ρ_{average}	Ω/m	16178	2294.0
Q_{average}	-	5636	7200
$v_{g,\text{average}}$	%c	1.21	45.3
f_0	GHz	11.9936	11.9936
f_0/f_b	-	6	1
N_{bunches}	-	312.0	2930
σ	mm/c	0.3	1.0
$\langle I\rangle$	A	1.20	101
E_0	MeV	-	2400
t_0	ns	0	0
L_{total}	mm	213	211

CLIC Accelerating Structure

The beam-induced gradient for CLIC AS has been calculated and its superposition with an inital gradient of 120 MV/m is shown in Fig. 1.

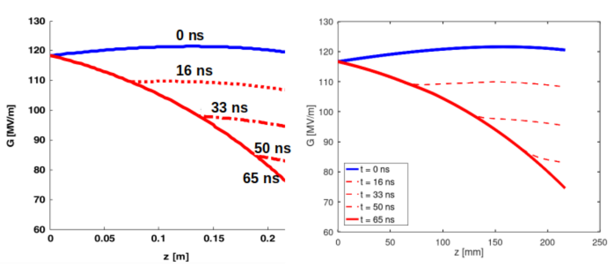

Figure 1: Time evolution of G_{eff} for CLIC AS. The left plot corresponds to the results exposed in [2]. The right plot shows beam-loaded field calculated with RF-Track.

It can be seen that, as the beam enters the structure, there is a transient response of the cavities, and a decelerating gradient is created which translates into a decrease of the available accelerating gradient. This stabilizes at a filling time t_f when the steady state is reached.

Such filling time corresponds to the time that it takes the excitation of the first cell to propagate through the structure:

$$t_f = \int_0^{L_{\text{total}}} \frac{\mathrm{d}z}{v_g(z)} = 62.2 \text{ ns} \quad (15)$$

Comparing with [2], where this case is studied analytically, excellent agreement can be found both for the filling time and the strength of the accelerating gradient.

CLIC Power Extraction and Transfer Structures

CLIC's novel acceleration scheme is based on a 2-beam model in which the power needed for its main beam acceleration is obtained from a drive beam deceleration [8]. The PETS are passive structures where CLIC's drive beam decelerates due to the pure beam loading effect.

The energy distribution resulting from the tracking of the drive beam is shown in Fig. 2. In this simulation, being the PETS passive structures, the beam loading effect has simply been added to an element of type "drift". It can be seen how a transient response leads to a non-homogeneous energy distribution between bunches.

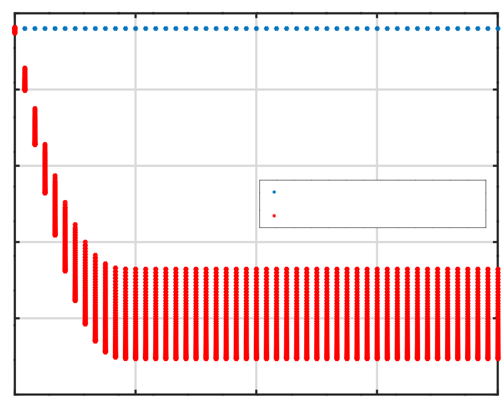

Figure 2: Energy distribution after the deceleration of a train of bunches at the PETS (see Table 1).

The minimum energy reached per bunch is 241.6 MeV, which differs a 0.67% with respect to reference [9]. Moreover, stabilization occurs at the 11$^{\text{th}}$ bunch in both studies, which confirms the good agreement between the filling times.

CONCLUSIONS & OUTLOOK

Starting from basic principles, the response of accelerating cavities to beam loading has been modelled in a self-consistent way, and a generic PDE has been proposed. For the ultrarelativistic case, its linearity shows that this effect can be superposed to the already existing fields. A module has been developed in RF-Track with fast computation times and it has shown excellent performance in two different cases. This opens new simulation scenarios for high-intensity machines, including Energy-Recovery Linacs and others.

The present RF-Track feature will be upgraded to solve Eq. (10) for relativistic bunches so that electron photo-injectors can be studied, among other accelerating structures.

ACKNOWLEDGEMENTS

The authors would like to thank Alexej Grudiev and Hermann Pommerenke for the material provided as well as the useful discussions on the beam loading effect.

ISBN: 978-3-95450-215-8 ISSN: 2226-0366 doi:10.18429/JACoW-LINAC2022-TUPORI13

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