CERN/PS/OP/77-12
March 1977

A SIMPLE MODEL FOR MULTITURN INJECTION INTO A.G. PROTON SYNCHROTRONS

P.D.V. van der Stok*)

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*) Paper presented at the 1977 Particle Accelerator Conference, Chicago, U.S.A.

FIGURE CAPTIONS

Fig. 1 Phase plane representation of slice movement.

Fig. 2 Partial acceptance evolution for $Q_H = 4.20$.

Fig. 3 Efficiency versus number of injected turns for various horizontal emittances.

Fig. 4 Optimum injection efficiency and setting (DI, DIP) as a function of Q_H.

Fig. 5 Computed current build-up and incoherent Q_H-domain for a high intensity beam.

Fig. 6 Horizontal amplitude distribution calculated and measured for four T_B-values.

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P.D.V. van der Stok
European Organization for Nuclear Research
1211 Geneva 23, Switzerland.

Summary

Analytical expressions are derived in order to compute the optimum injection parameters for betatron stacking into a chosen fraction of the total accelerator acceptance in the Q range $N + 0.1 \leq Q \leq N + 0.9$. The example of the CERN PS Booster (PSB) is worked out. The efficiency of the process is calculated using standard numerical integration algorithms with a Gaussian distribution of the incoming beam. Other types of distribution can also be dealt with. The results show that the installation of pulsed dipole and quadrupole supplies to obtain steering and focusing conditions which vary during the injection process will improve the injected current by only 3% in the case of the PSB. The measured performance of the PSB injection system is reported and compared with model prediction. Estimates of injected currents prove to be about 10% higher than the actual values. Some considerations on the influence of space-charge forces, which might explain this effect are presented.

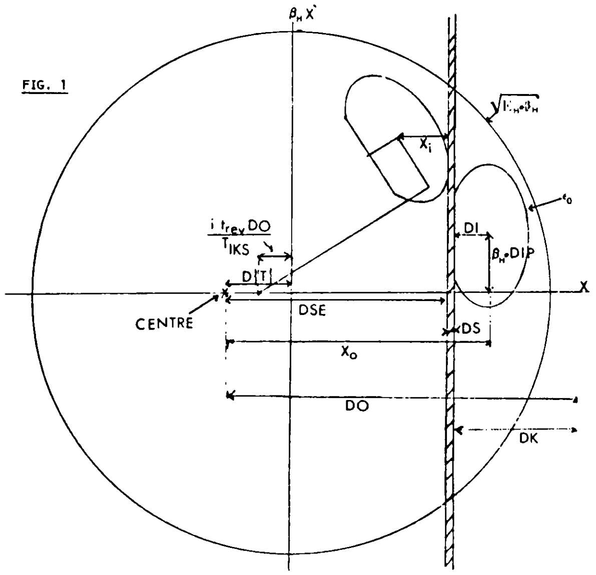
Dynamics of Beam Slice

The beams which are to be injected into the four PSB rings can be aligned and focused such that for each ring the injection conditions are optimal. Four kickers per ring provide a local closed orbit bump at the injection septum position, with a maximum amplitude DO, which falls linearly in a time T_{IKS}, that is: $D(t) = DO\ (1-t/T_{IKS})$. A slice of incoming beam enters the accelerator at the exit of the injection septum, after which the centre of the slice will perform betatron oscillations with respect to the instantaneous closed orbit, with an amplitude X_o at the injection point.

The centre of the slice was injected at a distance DI to the outer septum side and at an angle DIP with respect to the closed orbit. After each revolution i, the distance X_i of the slice centre to the inner septum side can be calculated (see Fig. 1) :

$$X_i = X_o\{1-\cos(2\pi Q_H i)\} - DI - DS + \frac{i.t_{rev}DO}{T_{IKS}} - \beta_H DIP \sin(2\pi Q_H i) \qquad (1)$$

FIG. 1



DS is the effective septum thickness (1.5-3 mm), Q_H is the number of horizontal betatron oscillations (4-5), β_H is the betatron function value at the injection point ($\sim$ 5.8 m), and t_{rev} is the revolution time ($\approx$ 1.66 μs at injection).

The symmetry of the Booster unit cells determines $\beta'_H = 0$ at the injection point. As it is useful to orientate the incoming beam equidensity ellipses in the same way as the Courant Snyder invariant ellipses at the injection point[1], the incoming beam is focused such that the equidensity ellipses in the horizontal PSB phase plane obey the relation :

$$(x-x_c)^2/\beta_L + (x'-x'_c)^2\beta_L = \varepsilon \qquad (2)$$

where (x_c, x'_c) are the coordinates of the incoming slice centre.

After each revolution a loss can be produced by the injection septum defining a loss boundary - cut assumed to be a straight line[2] - in the horizontal phase plane moving with the circulating slice. To correlate the distance X_i directly to the intensity, a normalisation function $F(iQ_H)$ is used such that :

$$X_i^N = X_i/F(iQ_H) \qquad (3a)$$

$$F(iQ_H) = \sqrt{\frac{\beta_H^2}{\beta_L^2}\sin^2(2\pi Q_H i) + \cos^2(2\pi Q_H i)} \qquad (3b)$$

Here X_i^N is a measure for the number of particles dependent on the particle distribution which will be lost on the injection septum if the losses of all other cuts are neglected.

The start of the injection process T_B, is chosen such that the centre of the horizontal phase plane is filled, yielding :

$$T_B = (-DSE-DS+DO)\ T_{IKS}/DO - t_{rev} \qquad (4a)$$

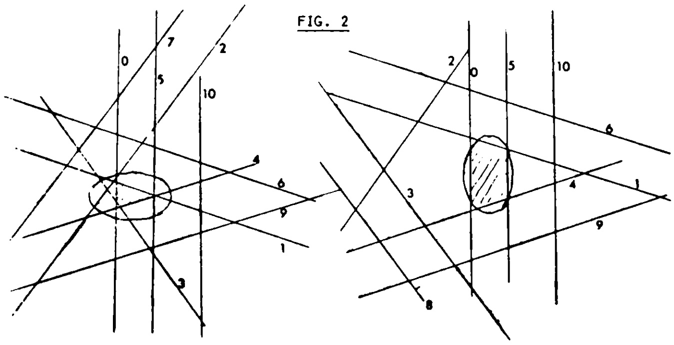
DSE is the distance from the inner septum side to the unperturbed closed orbit.

n_t is the number of injected turns, which determines the end of the injection process: $T_E = T_B + n_t\ t_{rev}$.

Partial Acceptance

For a slice of incoming beam, an area in the horizontal phase plane - the partial acceptance - is defined to be the area in which the particles have to be located to be accepted in the machine. All particles of

FIG. 2



the slice outside this area will be lost on the injection septum. A minimum of three cuts is required to describe such an area, but in some cases four cuts prove to be necessary. A higher number of cuts is not considered, as this only happens in the low efficiency part of the process, or only affects the low density tails of the particle distribution.

Fig. 2 shows an example of the partial acceptance evolution for the case $Q_H = 4.20$. At the beginning of the process the partial acceptance is determined by 3 cuts: 0 produced at the moment of injection and the cuts produced after 1 and 4 machine revolutions. Later on, cut 5 has to be considered as well, leading to a four angle partial acceptance.

The optimized parameter setting is determined by finding the largest ellipse which can be injected into the partial acceptance without losses. In the 3-angle case defined by cuts o, j and k, one poses:

$$X_j^N = X_k^N = DI \tag{5a}$$

and

$$\frac{d}{d\beta_L}\left(\frac{DI^2}{\beta_L}\right) = 0 \tag{5b}$$

As a result, DI and DIP can be written as a function of the moment of injection t and T_{IKS} for a given partial acceptance configuration :

$$DI(t,T_{IKS}) = \frac{A_{DI}t}{T_{IKS}} + \frac{B_{DI}}{T_{IKS}} + C_{DI} \tag{6}$$

$$DIP(t,T_{IKS}) = \frac{A_{DIP}t}{T_{IKS}} + \frac{B_{DIP}}{T_{IKS}} + C_{DIP} \tag{7}$$

Eqns. (6) and (7) are determined by eq. (5a) and β_L coming from eqn. (5b) is approximated by :

$$\beta_L = \frac{\sqrt{0.9}\,\beta_H\left|t_g\ (2\pi Q_H j)\right|}{\left\{0.738+\left[\frac{\cos(2\pi Q_H k)}{\cos(2\pi Q_H j)}\left|\frac{\sin(2\pi Q_H j)}{\sin(2\pi Q_H k)}\right| + 0.4\right]^2\right\}^{\frac{1}{2}}} \tag{8}$$

while j and k are chosen such that :

$$\cos(2\pi Q_H j) \geq \cos(2\pi Q_H k) \tag{9}$$

More detailed derivations and results are presented elsewhere[3].

For the four-angle case, the DIP is assumed to be completely decoupled from DI. The partial acceptance is determined by the cuts o, k, j and l, where j is almost parallel to cut o. For the calculation the requirements are :

$$DI = X_j^N \tag{10a}$$

$$\frac{\beta_H}{2\overline{DI}}\left\{\frac{X_k^N}{\left|\sin(2\pi Q_H k)\right|} + \frac{X_\ell^N}{\left|\sin(2\pi Q_H \ell)\right|}\right\} \stackrel{\sim}{\sim} \beta_L \tag{10b}$$

and $$\left\{\frac{X_k^N}{\sin(2\pi Q_H k)} + \frac{X_\ell^N}{\sin(2\pi Q_H \ell)}\right\} / 2\beta_H \stackrel{\sim}{\sim} DIP \tag{10c}$$

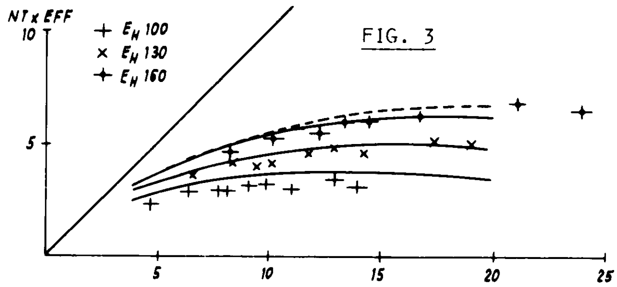


FIG. 3

which yields the same type of equation as eqn. (6) and (7), but β_L will be written as :

$$\beta_L(t) \stackrel{\sim}{\sim} \frac{\overline{DI}\beta_H}{A_{\beta_L}t+B_{\beta_L}}\ ,\ \overline{DI} = \text{mean}\{DI(t)\}_{T_B}^{T_E} \tag{11}$$

In the PSB it has proved sufficient to distinguish only two partial acceptance configurations during the whole injection process. The time T_o for switching from one configuration to the other is calculated by requiring that one specific parameter must be a continuous function with time. Of course this procedure can be refined, but such modifications necessitate considerable effort and do not significantly alter the results. Table 1 shows the three possible cases :

TABLE 1

Case	T_B-T_o	T_oT_E	Cont. Parameter
I	3-angle	4-angle	β_L
II	3-angle	3-angle	$DI/\sqrt{\beta_L}$
III	4-angle	3-angle	β_L

At last T_{IKS} can be calculated. In the PSB, the beam is injected until the horizontal PSB emittance containing 95% of the particles has the required size[4]. (Nominal $\varepsilon_H = 130\,\pi$ mm mrad at 50 MeV, while $A_H = 220\,\pi$ mm mrad). When DI(t), DIP(t) and $\beta_L(t)$ are known, it is possible to obtain a first approximation for T_{IKS} from geometrical considerations.

A linearization of the efficiency behaviour around T_E subtracting the particles lying outside ε_H will then finally yield the required T_{IKS}.

Efficiency Calculation

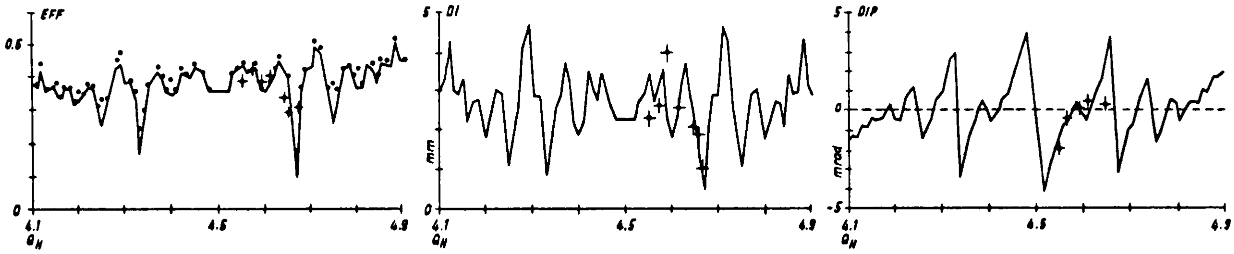
To calculate the efficiency of the process, use is made of a Gaussian distribution for the incoming beam, while the probability of finding a particle on an equidensity ellipse ε is given by:

$$P(\varepsilon) = \frac{1}{\varepsilon_o}\exp)-\ \varepsilon/\varepsilon_o) \tag{12}$$

$\varepsilon_o \stackrel{\sim}{\sim} 10\ \pi$ mm mrad for the incoming PSB beam.

For a given slice, the partial acceptance is split up into two sets of rectangles approximating the total partial acceptance area. One contains twice as many rectangles as the other. The summation over all rectangles yields the efficiency for a given slice of in-

FIG. 4



coming beam. A comparison of the results of the two sets shows the accuracy of the calculation. As efficiency is a smooth function of time, the total multi-turn process efficiency is calculated with Simpson's rule. To compare the injections with dynamic injection parameters and fixed ones, the fixed parameter $\overline{x}$ has to be calculated from the dynamic one x(t). A weight function w(t) is defined as :

$$w(t) = 1 - \exp\{-DI^2(t)/\varepsilon_o\beta_L(t)\} \qquad (13a)$$

and $$\overline{x} = \int_{T_B}^{T_E} w(t)\, x(t)\, dt \Big/ \int_{T_B}^{T_E} w(t)dt \qquad (13b)$$

Results obtained in the CERN PS Booster

Much information on the performance of the multi-turn injection process is contained in the graphs : number of injected turns (n_t) versus n_t times the efficiency. To diminish space charge effects, these curves have been measured in the PSB with a low intensity Linac Beam (17mA) for ε_H = 100, 130 and 160 π mm mrad (see Fig. 3). The graphcs can be fitted nicely with an effective septum thickness DS = 3mm. The dotted curve shows the prediction for DS = 1.5 mm (the calculated value). The probable cause for this high DS-value is the trajectory of the incoming beam in the injection septum.

For T_{IKS} = 60 µsec and β_L = 2m the efficiency plus optimized parameters are displayed in Fig. 4. Measurements in the Q_H value range 4.55 - 4.68 are also shown . The DIP discrepancy at Q_H = 4.67 is related to the influence of the horizontal acceptance, which is not considered in this model, owing to the particular emittance filling adopted in the PSB. For other accelerators it may however be useful to approximate the horizontal acceptance by a cut, thus introducing three partial acceptance configurations with time, instead of only two as in the case of the PSB. The dots in the efficiency plot in Fig. 4 represent the gain expected when injection is performed with dynamic injection parameters.

Space Charge Effects

At high intensities $3.8\mathrm{x}10^{12}$ppp are injected with the injection point $Q_H, Q_V \approx (4.23, 5.32)$. A horizontal blow-up has been observed coming from the line Q_H = 4.0 about 400 µsec after injection during the trapping process. One msec after injection the beam is stable. These conditions make it difficult to observe the injection process behaviour. Some deductions can, however, be made.

In Fig. 5 the current build-up and incoherent Q_H-domain during the injection process in the PSB is shown for two typical Q_H values. The Q_H spread and shift-induced partial-acceptance change has been verified by a comparison of the DI-values around Q_H = 4.67 and the DIP values around Q_H. 4.17 for two incoming beam intensities (17mA and 86mA). The measured values correspond to the predictions from Figs. 5 and 4.

For low-beam intensities it proved impossible to create horizontally hollow beams. Injection of four turns with different T_B values - T_{IKS}: 40µsec, I_{LINAC}: 17mA and $\Delta E_{LINAC} \leq \pm$ 35keV - should yield the horizontal amplitude distributions shown in Fig. 6. Different distributions have been measured with BEAMSCOPE[6] (Fig. 6), which were confirmed by target measurements. The hole fills up and simultaneously the number of high amplitude particles increases. The powering of zero-harmonic octupoles, which create a Q_H spread, diminishes this effect.

The phenomenon may be explained from the lumped beam structure. The space charge forces working on each particle are modulated with the betatron frequency. For a 17mA Linac beam maximum movements of $\pm$ 0.5 mm at the septum position have been calculated after five machine revolutions, assuming rigid slices and no Q_H spread at all. For different incoming beam intensities no significant influence of this movement on the injected intensity has been detected. This can be caused by the Q_H spread and the dependence of the sign of the particle movement on the initial conditions of slices and particles.

Acknowledgements

I am grateful to K. Schindl who has significantly contributed to this work. Many suggestions and remarks have been made by Prof. D. Harting, F. Sacherer and H. Schönauer. K.H. Reich and D. Dekkers supported this work. A. Faugier commented on the manuscript.

References

1) E.D. Courant, Fermi Lab. report NAL-FN-174.

2) C. Bovet, CERN SI/Note DL/69-18.

3) P. van der Stok, CERN/PS/OP/Int. 76-1.

4) K. Schindl, P. van der Stok, CERN/MPS/BR/Int.74-2.

5) K. Schindl, P. van der Stok, This conference.

6) Device developed by P. Krempl.

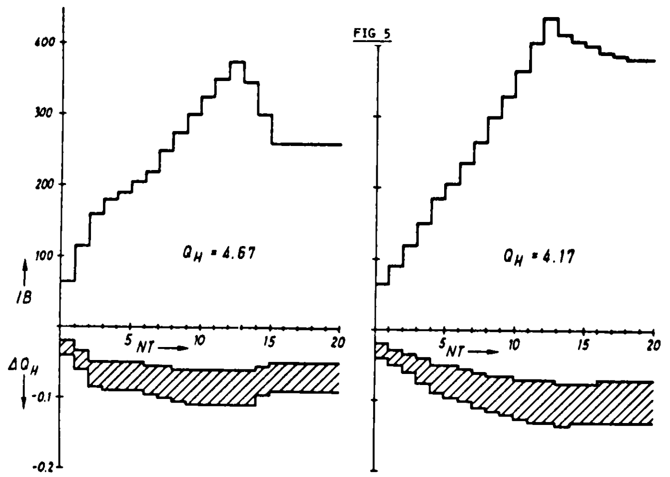


FIG. 6