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METHODS TO COMPARE AND TO OBTAIN REPRESENTATIVE EMITTANCE VALUES FROM FUNDAMENTALLY DIFFERENT

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Methods to Compare and to Obtain Representative Emittance Values from Fundamentally Different Measurement Devices

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Amongst the four different types of emittance measurement devices used in the CERN PS Complex, two measure the betatron amplitude distribution (Beamscope, flip targets) while the other two (SEM-grids, fast wire scanner) are recording the projected beam density. This paper briefly describes the measurement systems and concentrates on the mathematical methods applied to eliminate noise and measurement errors, to transform between the phase plane and its projection and to unfold the contribution from an assumed momentum spread in order to obtain the net betatron component. As the emittances are often quoted as derived from one standard width of the projected beam density, the goal is to provide comparable figures of this definition regardless of the measurement instrument and method.

I. Introduction

The four emittance measurement devices used in the CERN PS Complex belong to two fundamentally different classes

i. Devices of destructive type recording betatron amplitude distributions (Beamscope, flip targets).

ii. Devices measuring the projected density (SEM-grids, fast wire scanner). They are (nearly) non-destructive and sometimes referred to as "profile detectors".

Both classes have their "natural" definition of emittance resulting from the measured entities : Class (i) easily yields an emittance confining a given fraction (e.g. 95%) of all particles having betatron amplitudes less or equal to that limit; class (ii) suggests straightforward evaluation of the variance of the profile and an emittance expressed by this quantity.

Flip targets being almost extinct today, most instruments belong to class (ii), in particular in the client machines of the PS. There remains the Beamscope [1, 2] device of the PS Booster, the machine where the initial emittances of the hadron beams are "made" according to the physics involved in the multi-turn injection process. To enable the PS to monitor emittance conservation and identify possible blow-ups during transfers and acceleration, a reliable conversion between the two emittance definitions has to be provided - for practical reasons even "on-line".

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Theoretically the rotationally symmetric distributions of phase space (betatron) amplitude and its projected density are related by the well-known Abel transform [3], and computer codes for its numerical evaluation exist for many years [4]. However, its straightforward use in the horizontal dimension is difficult and unreliable for two reasons: (i) Beamscope encounters instrumental problems due to reactions of the rf system (cf. below) and (ii) the Abel transform is strictly speaking no longer meaningful in presence of momentum spread and finite lattice dispersion. For these reasons a conversion scheme has been chosen based on fitting the widely used "binomial" distribution [5, 6] to the measured data with subsequent analytical computation of all information wanted from the fitted function.

SEM-grids are the most widely used instruments for transverse emittance measurements in the PS. A new standard method using SEM-grids to measure emittances at the entrance of the PS and in the transport lines of the PS Complex has been put into operation in the framework of the rejuvenation of the PS control system [7]. The method involves a wide range of statistical tools to analyze the SEM-grid measurements and reliably derive the transverse beam dimensions. The numerical treatment of SEM-grid measurements first eliminates faulty wires. Then, it smooths the beam profile using a choice of curve fitting functions to reduce the noise effect on wire signals, and carries out a treatment of the beam tails to establish the base line. In order that a general beam quality diagnostic may be undertaken, none of the profile measurements will be automatically discarded. It is left to the user to keep or discard the proposed results. In the presence of lattice dispersion the momentum width of the beam is folded with the betatron width, and the emittance obtained from the measured profile may no longer be meaningful. It may then be needed to unfold the momentum component from that of the beam profile in order to restore the betatron beam width.

II. The measurement systems

A. *Beamscope*

Beamscope (BEtatron AMplitude Scraping by Closed Orbit PErturbation) is a device developed more than a decade ago at the CERN PS Booster [1, 2] and consists basically of a three simultaneously pulsed dipoles exciting a fast-rising local orbit bump which drives the beam into a precision scraper where it is lost within a few ms; the dipole shunt signals, the beam current and its derivative are recorded by transient digitizers. After a fair amount of processing using resident databases (e.g. for the magnetization curves of 24 dipoles) and a lattice code, applying corrections for eddy currents in the vacuum pipe etc., the bump amplitude at the scraper location is computed. The beam radius is simply the difference between the bump amplitudes at 95% circulating and at zero current. The latter is best found from the betatron amplitude distribution, which is computed from the recorded beam current derivative. This works very well in the vertical plane but turned out to be unreliable in the radial one [8] because of (i) the dependence of the beam centre (or closed orbit) on particle momentum and (ii) due to the reaction of the rf beam control system to the change in circumference due to the orbit bump, entailing the effect that the beam radii and amplitude distributions are not exactly the same for inward and outward directed bump. While for simple evaluation of the 95% emittance this problem can be circumvented by taking the average of the two results (in which case one need not know the exact beam centre), the Abel transform of these data sometimes generates erroneous results.

B. Flip-Targets

Beside their limited accuracy (individual position errors up to 1 mm have been reported) their operation in a supercycle environment is considered to be too tedious and the eight flip targets of the Booster have been withdrawn from the rings. Those of the PS are no longer used for emittance measurements for the same reason.

C. SEM-grids

A SEM-grid (Secondary Emission Monitor) consists of an array of ribbons or wires placed in the beam path. Profile shapes are obtained by measuring the secondary electron current from each wire due to the impact of the particles. Beam profiles are then sampled by the array of evenly separated ribbons or wires. SEM-grids can only be used to measure single passage beams (in transport lines or in rings provided the beam crosses the grids only once). The measurement is practically non-destructive. The resolution is quite satisfactory because SEM-grids feature wire step sizes from 3.5 mm down to 0.35 mm (even less by rotating the whole array of wires), adequate for minimum transverse beam size of, say 3 mm, for the LHC.

D. Fast wire scanner

The fast wire scanners of the PS consist of a single wire moving at a speed of up to 20 m/s across the beam. A scintillator and a photomultiplier detect the secondary particles produced in the interaction of the beam with the wire. The signal is sampled against the wire position and yields the beam profile, at intervals more dense than those obtained by SEM-grid (say 100 samples against a maximum of 40 for the SEM-grids) [9, 10]. Fast wire scanners are used to measure circulating beams only. The measurement is nearly non-destructive, the effect of multiple Coulomb scattering on the beam being small. The wire itself is fragile and may breaks if overheated accidentally.

E. Emittance measurement definition

When measured with SEM-grids or fast wire scanners the transverse emittance may be quoted in terms of the beam half-dimension w_x obtained from the projected transverse phase space density distribution onto the plane of the profile. A useful measure of the beam half-dimension is twice the r.m.s. value σ_x of the profile distribution. This definition appears as the most appropriate to characterize the density of the bulk of the beam. In the presence of dispersion the betatron beam half-width w_{x_β} taken as twice the betatron r.m.s. beam width σ_{x_β} is to be used instead of the overall beam half-size w_x to derive the "2σ-emittance"

$$\mathcal{E}_x = \frac{w_{x_\beta}^2}{\beta_x} \tag{1}$$

where β_x is the beta-function at the monitor location. In transfer lines, the emittance of the beam may be obtai from beam profiles measured at three different SEM-g with known transfer matrices between the detectors [7].

The betatron r.m.s. beam width may be obtained by subtracting the contribution δ of the momentum dispersion[1] from the beam profile using the quadratic formula

$$\sigma_{x_\beta}^2 = \sigma_x^2 - D_x^2\,\sigma_\delta^2 \tag{2}$$

in which D_x is the dispersion function in the plane of the profile and σ_δ is the r.m.s. momentum dispersion.

III. SIGNAL PROCESSING AND METHOD APPLIED

A. Beamscope

Abel transform The projected density $p(x)$ of a rotationally symmetric phase space density distribution $P(r)$ onto the coordinate axis x, with $r = \sqrt{x^2+\beta_x^2 x'^2}$, can be found simply to be

$$p(x) = 2\int_x^{x_L} \frac{P(r)\,r}{\sqrt{r^2-x^2}}\,dr$$

where x_L is the upper limit of the amplitude. Eq. 3 is a case of the Abel transform

$$g(x) = \int_x^{x_L} \frac{f(t)}{\sqrt{t^2-x^2}}\,dt \tag{4}$$

in which the function $g(x)$ is given and $f(t)$ is to be determined. Its unique inverse transform

$$f(x) = -\frac{2}{\pi}\frac{d}{dx}\int_x^{x_L} \frac{t\,g(t)}{\sqrt{t^2-x^2}}\,dt \tag{5}$$

exists if $g(x)$ is differentiable on $(0, x_L)$. As a consequence, the phase space distribution $P(r)$ can be computed formally from the projected density, with $p'(x) = dp/dx$

$$P(r) = -\frac{1}{\pi}\int_r^{x_L} \frac{p'(x)}{\sqrt{x^2-r^2}}\,dx \tag{6}$$

[1] $\delta \overset{def}{=} (p-p_0)/p_0$ is the relative momentum deviation of a particle with momentum p from the reference value p_0.

As its numerical evaluation has to deal with a derivative, the usual complications associated with numerical differentiation arise. This is not the case for the forward transform, and P. Krempl's code for the numerical Abel transform [4] has been included into the Beamscope processing software without major modifications or problems, to obtained the projected density $p(x)$.

Ivanov fit This method [11, 12] is based on the fitting of a binomial distribution

$$P(a) = \frac{m}{\pi}(1-a^2)^{m-1} \tag{7}$$

of phase space amplitudes $a = \sqrt{u^2 + v^2}$, where u and v are the coordinates x and x' normalized to limiting amplitude x_L, $(u = x/x_L,\ v = \beta_x x'/x_L)$. Its integral I_a describes the normalized beam current as a function of the normalized position u of a scraping target

$$\begin{aligned} I_a(u) &= 2\pi \int_0^u aP(a)\, da \\ &= 2m \int_0^u a(1-a^2)^{m-1} da = 1-(1-u^2)^m \end{aligned} \tag{8}$$

To fit this expression to the n_d measured beam current samples $I_d(x)$, it is evaluated at a number n_a of equidistant arguments $x(i)$, with $n_a \simeq 3 \rightarrow 10n_d$. Measured data $I_d(i)$ are also evaluated at these points by three-point Lagrange interpolation. Differences $I_d(i) - I_a(i)$ are summed separately to sums $S_+(k,j)$ and $S_-(k,j)$ according to the sign of the difference; $j = 2m$ and k are iteration parameters, where k represents the variation of the limiting amplitude $x_L(k) = x_L^{\text{in}} - (k-1)dx$, dx denoting the step between arguments $x(i)$ and x_L^{in} the initial amplitude limit equal to the data maximum. The procedure starts with $j = k = 1$ and consists in finding the value j_{fix} of j for which the sums over positive and negative differences are equal: $S_+(k, j_{\text{fix}}) = |S_-(k, j_{\text{fix}})|$. Then these sums $S_+(k, j_{\text{fix}})$ are computed for increasing k's, until a minimum is found at k_{fix}. Both j_{fix} and k_{fix} are interpolated values and in general not integers. The final fitted function is then given by Eq. 7 with $m = j_{\text{fix}}/2$ and $x_L = x_L^{\text{in}} - (k_{\text{fix}} - 1)dx$ and the r.m.s. width of the projected density is found by the simple expression $\sigma = x_L/\sqrt{2(m+1)}$.

The principal attraction of this technique is - beside the obvious desirability of an analytical description by a handy class of functions - that it does not require precise knowledge of the beam centre. In this respect it matches well the problematic of Beamscope measurements in the radial plane. In the practical implementation the measured data set $I_d(x)$ is already the average over two measurements, one with inward and one with outward bump amplitude.

B. *SEM-grids*

The accuracy of the emittance figures depends on how well the transverse beam profile, from which the beam dimensions have to be derived, is known.

SEM-grid wire output signals v_i are approximations of the unknown beam profile distribution. Erroneous wire output signal due to faulty wire or to bad gain of the amplifier signal would cause gross errors in the evaluation of the beam size. Hence, it is advisable to discard as "outliers" any unrealistic data which are too far away from the rest of the sample. To this end, it is assumed that the second derivative v_i'' of any erroneous wire signal v_{i_0}'' significantly differs from the other quantities v_i'' so that the "doubtful" signal v_{i_0} may be eliminated performing statistical tests on the data v_i''. This method is iteratively applied until no more data points are eliminated.

Moreover, random errors of the wire signals could strongly influence the value of the beam size derived by direct analysis of the measured beam profile. When no hypothesis is made on the shape that the data are to fit, flexible approximation of beam profile data by means of spline functions may be successfully used. Assume that approximate values v_i of the beam profile distribution $v(x)$ are known at the wire locations x_i. A cubic spline approximation of $v(s)$ consists of polynomials of degree three pieced together so that their values and those of their first two derivatives match at the points x_i

$$v(x) = \sum_{j=0}^{3} a_i^{(j)}(x_i - x)^j \tag{9}$$

The expressions for the coefficients $a_i^{(j)}$ of the spline function may be found elsewhere [7]. Spline approximations yield in some sense the best fitting of the beam profiles because they provide smooth approximating curves which have minimal oscillatory behaviour.

Often physical considerations suggest that the beam profile should be of Gaussian shape. A more readily acceptable model may therefore be obtained by fitting a Gaussian to the measured beam profile rather than a spline function

$$v(x) = a_0 \exp\left[-\left(\frac{x-a_2}{a_1}\right)^2\right] \tag{10}$$

where the parameters a_0, a_1 and a_2 are to be determined.

The general approach for deriving least squares estimates for Gaussian curves is to minimize the sum of squared residuals by successive improvements to initial guesses a_{0_0}, a_{1_0} and a_{2_0}.

In addition to the random wire signal errors, possible systematic instrumental errors could shift the base line, which will render the beam size calculations meaningless. One procedure for the search of the base line that has proven efficient is described in [7]. No automatic elimination of the base line will be performed. It is left to the user to decide, on observation of the displayed beam profile, whether the elimination has to be carried out or not.

C. *Fast wire scanners*

The data processing consist first of calculating the real wire position data (from laboratory calibrations), then of evaluating the r.m.s. beam size. To this end, it is assumed that the transverse particle density is a near-Gaussian distribution with possible bias in the tails due to instrumental errors. Thus, the excessive influence of profile tails may be discarded by subtracting systematically a 7.4% offset from the profile data. This procedure is justified in [13] and sketched hereinafter.

Given a Gaussian distribution with r.m.s. value σ and a bias expressed as a fraction ν of the peak distribution value, a new r.m.s. value σ_ν may be calculated from the distribution obtained by ignoring the part of the Gaussian below the bias-line. Computations yield

$$\frac{\sigma_\nu^2}{\sigma^2} = 1 - \frac{4\nu(-\ln\nu)^{3/2}}{3\left(\sqrt{\pi}\,\mathrm{erf}(\sqrt{-\ln\nu}) - 2\nu\sqrt{-\ln\nu}\right)} \qquad (11)$$

For $\nu = 0.074$ one computes the ratio $\sigma_\nu/\sigma = 0.85$, which gives a r.m.s. value 15% smaller than the true r.m.s. value for negligible bias. Thus, the beam dimensions used for emittance calculations must be corrected by a factor $\sigma/\sigma_\nu = 1.18$.

IV. Unfolding of the Momentum Component in the Profile

When measuring transverse emittances, the momentum width of the beam is folded with the betatron width so that the emittance derived from the measured profile may be meaningless when momentum spread and dispersion are large. Eq. 2 is always correct although not always meaningful, in particular when the momentum distribution is known to be far from a bell-shape distribution. A treatment must then be performed to unfold the momentum width from the measured profile so as to extract the betatron part of the beam profile [14]. At present this is only implemented in the processing of the SEM-grids. The sampled data f_i of the beam profile may be written

$$f_i = \sum_{j=0}^{n} h_{i-j} g_j \qquad f = Hg \qquad (12)$$

where g_i and h_i are the betatron and momentum width distributions, and H is a band matrix whose element h_i in every diagonal are identical.

Difficulties may occur when solving Eq. 12 because unfolding is an "ill-posed" problem, i.e. even the smallest noise in the measured signals may cause very large errors in the unfolded data g. Hunt's regularization method has been retained to unfold the beam profile. It is based on the idea that the discrepancy of the calculated values Hg from the given noisy data f may be stabilized by means of a "smoothing" function C chosen here as being the tri-diagonal matrix of the second derivative operator. The solution takes the form

$$g = \left(H^t H + \alpha C^t C\right)^{-1} H^t f \qquad (13)$$

which is dependent on the positive constant α. The choice of α implies iterative applications of Eq. 13 until the residual error $(f-Hg)^t(f-Hg)$ is of the same order as the noise $(\Delta f)^t \Delta f$ on the raw data.

The relative momentum distribution of the beam is supposed to be a known unperturbed function. A good model for the longitudinal phase-space density is the binomial distribution which, once integrated over the phase excursion yields the momentum width distribution

$$h(u) = \frac{\Gamma(m+1)}{\sqrt{\pi}\Gamma(m+\frac{1}{2})}\left(1-u^2\right)^{m-\frac{1}{2}} \qquad (14)$$

where $u = D_x\delta/x_L$ and $x_L = \sqrt{2(m+1)}D_x\sigma_\delta$ is a limiting amplitude.

The unfolding process has been applied to radial beam profiles measured by a SEM-grid device placed in the PS injection region (the grid is made of 32 wires spaced by 2 mm).

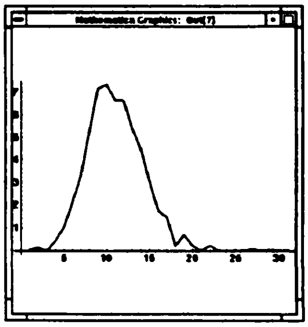

Figure 1: Measured beam profile: $\sigma_x = 6.52$ mm

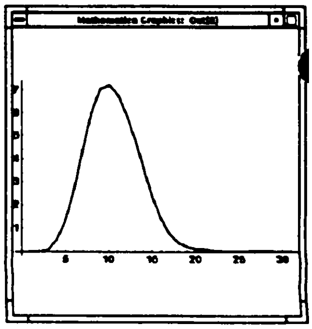

Figure 2: Unfolded beam profile: $\sigma_{x_\beta} = 6.25$ mm

Figures 1-2 show the acquired and the unfolded beam profiles considering a parabolic momentum density (binomial distribution with $m = 3/2$). The r.m.s. momentum spread of the beam was $\sigma_\delta = 0.75 \times 10^{-3}$. The best value $\alpha = 0.69$ has been determined assuming a noise on wire data of 3% of the peak signal (30 mV on a 1 V peak signal).

The r.m.s. values of the measured and unfolded profiles have been evaluated to be $\sigma_x = 6.52$ mm and $\sigma_{x_\beta} = 6.25$ mm, respectively. Eq. 2 yields with $D_x = 2.31$ m

$$\sigma_{x_\beta} = \sqrt{6.52^2 - (2.31 \times 0.75)^2} = 6.29 \text{ mm}$$

in satisfactory agreement with the result of 6.25 mm from unfolding. Eq. 2 is thus sufficiently accurate when either the signal-to-noise ratio of the measured data is high or the momentum width of the beam is small compared to the betatron width. However, when the signal-to-noise ratio is low and/or the momentum width is not small compared to the betatron width, the unfolding method, described in this paper, needs to be applied.

V. Results and Conclusion

For comparison of the instruments, numerous transverse emittance measurements have recently been carried out at different beam intensities, with the PSB Beamscope; the SEM-grids at the PS entrance, and with the PS Fast wire scanner respectively [15]. Tables 1-2 show some emittance measurement results for these three devices.

SFT[2]	Normalized[3] 2σ-emittances $[\pi\mu m]$		
Beam intensity	Beamscope (PSB)	SEM-grids (PS)	Wire scanner (PS)
5.20×10^{12}	10.3	12.1	11.4
1.08×10^{13}	16.3	21.0	23.8
1.33×10^{13}	24.9	25.3	28.5
1.60×10^{13}	21.0	29.5	31.1
1.88×10^{13}	26.9	36.7	36.1

Table 1: Comparison of horizontal emittances measurements between Beamscope, SEM-grids and Fast wire scanner at 1 GeV kinetic energy.

SFT	Normalized 2σ-emittances $[\pi\mu m]$		
Beam intensity	Beamscope (PSB)	SEM-grids (PS)	Wire scanner (PS)
5.20×10^{12}	10.7	7.8	7.0
1.08×10^{13}	17.2	14.3	14.5
1.33×10^{13}	22.4	17.5	18.4
1.60×10^{13}	25.8	19.9	21.9
1.88×10^{13}	28.2	22.6	26.4

Table 2: Comparison of vertical emittances measurements between Beamscope, SEM-grids and Fast wire scanner at 1 GeV kinetic energy.

Horizontal emittance figures given by the Beamscope are systematically smaller than those delivered by the SEM-grids and the Fast wire scanner, while vertical emittance given by the Beamscope are always larger than those measured by the SEM-grids and the wire scanner [8]. On the other hand, emittance figures given by the SEM-grids and the Fast wire scanner almost agree.

The systematically smaller horizontal Beamscope emittance results are believed to be due to an instrument fault which is not entirely explained by the reaction of the rf system to the orbit change [8] in the presence of beam loss. The vertical Beamscope measurements, however, appear flawless and the discrepancies, if due to this measurement, should rather be ascribed to errors in the numerical Abel transform. Although the latter was tested with analytically generated (binomial) distributions, its sensitivity to perturbing noise remains to be verified.

VI. Acknowledgments

We would like to express our gratitude to M. Arruat for help in designing the SEM-grid processing software.

[2]SFT is the proton beam delivered to the SPS by "continuous transfer" for fixed-target physics.

[3]The normalized emittance is $\mathcal{E}_x^* = \beta\gamma\mathcal{E}_x$.

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