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NEW GENERAL BEAM DYNAMICS FORMULATION FOR THE PROGRAM DYNAC

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ABSTRACT

Until recently beam dynamics programs for electrons and ions have been fundamentally different for two reasons: longitudinally the energy can change much more quickly with respect to the rest mass for electrons than for ions; transversally the integral approach used for ions seemed not to apply to electrons. At the previous linac conference in Albuquerque it was shown that using a reduced variable a single, more accurate equation of transverse motion could be used for any type of particle. A program, DYNAC, was proposed with the aim to treat long accelerating elements as currently used in superconducting systems. To obtain a high accuracy, keeping a relatively simple formalism, DYNAC is now using a new concept of equivalent accelerating fields. Computing bunch dynamics in an accelerating element (e.g. a multi gap element of more than 1000 deg effective length) is done in the following way : first, calculations are done on the central particle, followed by a fast but accurate calculation of the deviation of each individual particle with respect to the central one. Many examples have been treated (different fields and particles) and results will be presented including the comparison with an elaborate step by step integration method with a realistic electromagnetic field.

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Abstract

Until recently beam dynamics programs for electrons and ions have been fundamentally different because longitudinally the energy can change much more quickly with respect to the rest mass for electrons than for ions. A program, DYNAC, was proposed with the aim to treat long accelerating elements as currently used in superconducting systems for any type of particle. To obtain high accuracy, keeping a relatively simple formalism, DYNAC is now using a new concept of equivalent accelerating fields. Many examples have been treated (different fields and particles) and results will be presented including the comparison with an elaborate step by step integration method with a realistic electromagnetic field.

Introduction

In the second part of the 1960's PARMILA and MAPRO codes, developed to compute beam dynamics in drift tube linacs, made use of a set of quasi Liouvillian equations derived from the so called Panofsky equations [1],[2]; these equations are of the form:

$$\Delta W = qVT(k)I_0(k_r r)\cos\varphi + qV\frac{\delta}{\delta k}[T(k)k_r I_1(k_r r)]r'\sin\varphi \qquad (1)$$

$$\Delta\varphi = \frac{qV}{2W}k\frac{\delta}{\delta k}[T(k)I_0(k_r r)]\sin\varphi - \frac{qV}{2W}k\frac{\delta^2}{\delta k^2}[T(k)k_r I_1(k_r r)]r'\cos\varphi$$

where ΔW and $\Delta\varphi$ are the changes in energy and phase. Similar expressions can be written for the change in radial position Δr and slope $\Delta r'$. In Eq.(1) a "thin lens" approach is used in the middle of the gap, across which a voltage V is applied, q is the particle's charge, I_0 and I_1 are Bessel functions, T(k) is the transit time factor corresponding to the particle velocity v with $k=\omega/v$ and $k_r^2=k^2-\omega^2/c^2$ i.e. the amplitude of the wave k in a Fourier analysis of the axial field distribution E_z . Velocity v, phase φ and radial coordinates r and r' are the real coordinates of the particle in the middle of the gap, all of which require another set of equations to be computed by an iterative method. The above mentioned types of codes are still in use nowadays. Several limitations exist, however.

For heavy ions, complex structures such as multigap or helical shaped ones have been developed. These structures are not correctly treated with the first order perturbation method [3] used in Eq. (1). Second order terms have been computed in 1986 [4] and 1987 [5], giving extremely good results, at the cost, however, of computing time. Treating the transverse motion in the same way remained unsuccesful.

The motion of electrons can not be treated by equations like Eq.(1); therefore a step-by-step integration method is used in PARMELA.

In 1990 a new code, DYNAC, was presented [6], able to treat both the transverse and the longitudinal motion in complex accelerating structures and applicable to electrons, protons and ions. This was obtained by replacing the non-canonical coordinates r and r' by canonical "reduced coordinates" ,

$$R = r\sqrt{\beta\gamma} \qquad R' = dR/dz \qquad (2)$$

where β and γ are the classical relativistic coefficients and z the axial coordinate. In the paraxial approximation, the correct transverse equation of motion is :

$$\frac{d^2R}{dz^2} = \frac{1}{(\beta\gamma)^3}\frac{q}{2m_0c^2}\frac{\delta E_z}{\delta t}R - \frac{\gamma^2+2}{(\beta\gamma)^4}\frac{q^2}{(2m_0c^2)^2}E_z^2R \qquad (3)$$

In MAPRO and PARMILA only the first of these two terms is considered. The second term is generally prominent for the case of low energy electrons. The use of "reduced coordinates" avoids the presence of correcting terms for non-canonical variables as the equations applied in MAPRO and PARMILA and provides a much smoother evolution of the transverse extension, justifying a treatment over longer distances.

Another change in DYNAC is the use of coordinates at the input and output, rather than at the middle of a gap (the latter is still available if necessary).

A last change was the use of an "averaging method" instead of second order corrections, improving the accuracy.

Equivalent Accelerating Field

The above mentioned "averaging method" has now been improved by means of a new concept with an "equivalent accelerating field ", providing a better estimate of the k, r and r' terms in Eq.(1) along the accelerating structure.

In the original version of DYNAC an average value was taken for k in T(k) and a linear law was assumed for the energy evolution (or γ). As can be seen from fig.1A, the evolution of γ can be very different from linear, for instance when accelerating particles in long structures with a large energy or phase offset relative to the synchronous particle. In fact, the evolution of γ appears as a "rapidly" changing curve around a smooth one. The most important error resulting from an approximation in the evolution of γ comes

from the smooth part. This can be checked with a very accurate step by step integration method.

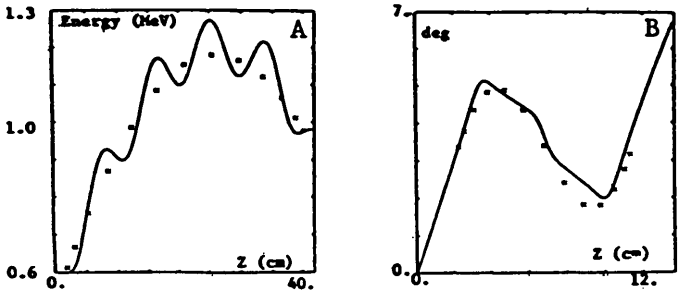

Fig. 1 Energy evolution of a 0.6 MeV electron injected at -90 deg from the crest into a 5 cell structure at 1.5 GHz adjusted to β=0.84 with a 7MV/m gradient (A) and phase slip for 0.15 MeV/nucleon Pb^{25+} ions injected at 90 deg from the crest into a Quasi-Alvarez cell at 200 MHz (B).Solid line represents real energy or phase, crosses represent equivalent field value.

It can be shown by Fourier analysis that an accelerating field generally consists of a superposition of waves of which only one is prominent. In the case of a small accelerating field and very little change in energy, it is sufficient to take the main wave only, with correct values for T(k), dT(k)/dk and $d^2T(k)/dk^2$ in Eq.(1) (see Appendix: Equivalent Wave Properties). With this "equivalent field" concept, however, even for large field amplitudes a good description of the beam dynamics is obtained.

Computational Methods and Results

An average k is computed which gives, at the field entrance and exit, the same r.f. phase as the particle considered . From the field entrance to the exit , a phase shift δ_s occurs between the particle considered and the "equivalent field" corresponding to the average k, from which the value of γ can be deduced at any position (see fig. 1A). The exact phase evolution with respect to the longitudinal coordinate z can be approximated by a 5th order curve in z (see fig. 1B). A similar approach, also with a 5th order law in z, is used for the transverse motion (the calculation of the quadratic term in Eq.(3) is explained in the Appendix: Calculation of the Quadratic Transverse Term). With this, all the particle coordinates are known at any position along the accelerating field, allowing an accurate integration of the beam dynamics terms like those in Eq.(1). An accelerating field with asymmetries can be treated; large asymmetries may, however, reduce the accuracy.

For multi-particle calculations, the coordinates are computed through expansions around the central particle.

Several accelerating structures have been tested with different kinds of particles such as low beta heavy ions and low energy electrons passing through long accelerating elements with high field amplitudes (see figs. 2, 3, and 4) [7],[8]. A slow but accurate step by step integration routine of Hamiltonian form allows a check of the direct expressions, both for transverse and for longitudinal motion.

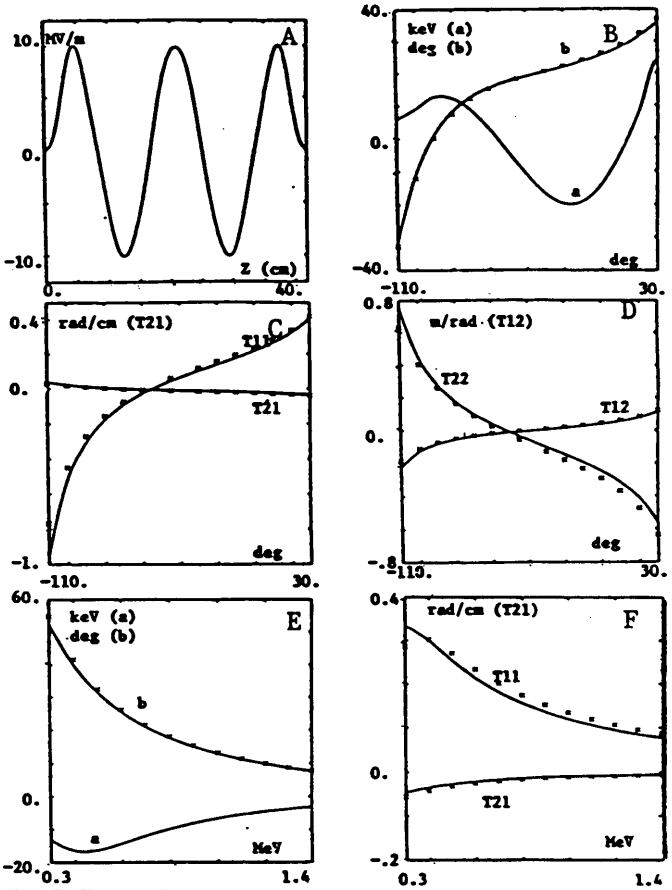

Fig. 2 Comparison between a numerical computation (solid line) and the equivalent accelerating field method (crosses) for a 5 cell structure as in fig 1A with the axial electric field (A), residual error in energy gain and phase jump with a peak energy gain of 1.7 MeV at 0 deg phase (B), matrix coefficients in a thin lense formalism with reduced variables (C, D) and E,F as B,C but for phase with peak gain and variable input energy.

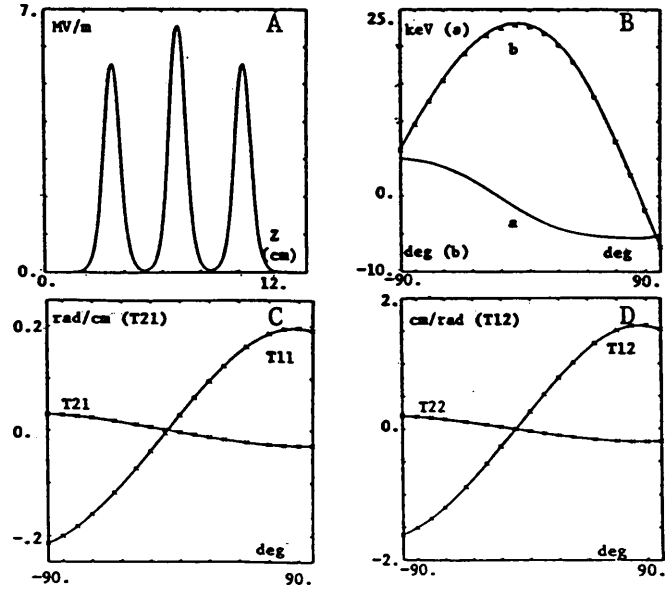

Fig. 3 As fig. 2 but for 0.24 MeV/nucleon Pb^{25+} ions injected into a Quasi-Alvarez cell at 200 MHz (peak energy gain of 3.5 MeV)

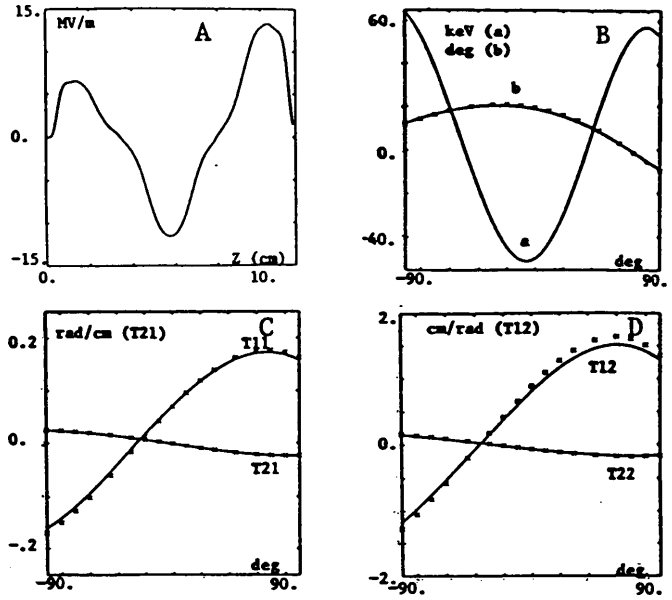

Fig.4 As fig. 3 but for acceleration through 3 gaps at the entrance of an interdigital H structure at 101 MHz (peak energy gain 6 MeV)[8]. Note the asymmetry in the electric field.

Conclusion

The new version of DYNAC can treat long and complex structures for electrons, protons or heavy ions used in a large energy range. Its limit of validity is a compromise between the length of the accelerating element (total phase shift of up to 10π) and the field strength (peak energy gain corresponding to 10% velocity change).

It is planned to complete the code with a space charge routine.

Appendix: Equivalent Wave Properties

Fourier analysis of the on-axis E_z field of an accelerating device can be made with the z origin in any position. Taking the median plane, the field expansion can be written according to the parity of the symmetry :

$$E_z = \frac{1}{2\pi}\int T_0(k)\cos(kz)\,dk \qquad \text{or} \qquad \frac{1}{2\pi}\int S_0(k)\sin(kz)\,dk$$

If the origin is displaced, one can show that the field expression, which now includes both T(k) and S(k) terms is such that, for any k :

$$T^2(k)+S^2(k)=T_0^2(k) \quad \text{or} \quad S_0^2(k)$$

Consider a particle moving at constant velocity throughout a wave of constant amplitude; its total phase shift is δ_s. The energy gain is reduced with respect to a synchronous particle by the classical transit time factor $\sin(\delta_s/2)/(\delta_s/2)$.

In order to have the same T_0 and T_0' for the real field and the equivalent wave over the length L_e, one has to solve :

$$\cot g\frac{\delta_s}{2}-\frac{2}{\delta_s}=\frac{2}{L_e}\frac{T(k)dT/dk+S(k)dS/dk}{T^2(k)+S^2(k)}$$

The length L_e may be chosen to have the same d^2T_0/dk^2 for the two fields [9].

A particle with a velocity corresponding to the average k and with the average phase φ (phase in the middle of the system with respect to the equivalent wave) will be subject to the field :

$$\tilde{E}_z(z)=\frac{1}{4L_e}[T(k)\cos(\varphi+\delta_s(z-\frac{L_e}{2}))-S(k)\sin(\varphi+\delta_s(z-\frac{L_e}{2}))]\frac{\delta_s}{\sin(\delta_s/2)}$$

from which the energy as function of z is computed.

Appendix: Calculation of the Quadratic Transverse Term

The real longitudinal field E_z can be considered as the sum of a smooth part E_{sm} and an oscillatory part E_{osc} which can be neglected for the linear term of motion. This is not the case for the quadratic term. One has:

$$E_z^2=E_{sm}^2+2E_{sm}E_{osc}+E_{osc}^2$$

Replacing the third term by a pure sinusoid one obtains:

$$\int E_{osc}^2\,dz \approx \frac{T^2+S^2}{32L_e}\frac{\delta_s^2}{\sin^2(\delta_s/2)}$$

Such an estimate gives good results.

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