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QUANTUM BREMSSTRAHLUNG: REPORT TO WORKSHOP ON LINEAR COLLIDERS

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A B S T R A C T

Recent developments in the quantum theory of bremsstrahlung, examining the adequacy of the "local-Sokolov-Ternov" approximation, are described.

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1. INTRODUCTION.

The quantum theory of bremsstrahlung [1-20] was seen to be relevant for the design of hypothetical future linear colliders by Himel and Siegrist[8]. When a charged particle describes an orbit with radius of curvature ρ the typical frequency radiated, in classical electrodynamics, is of order

$$\omega = \gamma^3/\rho \tag{1}$$

(with c=1). The corresponding photon energy is (with $h/2\pi=1$) the same. The ratio of typical photon energy to initial electron energy is then

$$\Upsilon = \gamma^3/\rho)/(m\gamma) = \gamma^2/\rho \tag{2}$$

Only for $\Upsilon \ll 1$ can classical theory be good, and this is not the case, in the linear colliders now considered, when the bunches collide.

When the Himel-Siegrist paper was written, detailed results of the quantum theory were available only for circular orbits, as presented in the book of Sokolov and Ternov[6]. In dealing with orbits with varying radii of curvature, Himel and Siegrist applied the Sokolov-Ternov formulae locally, with the local radius of curvature, and integrated. In the meantime the adequacy of this 'local-Sokolov-Ternov' (LST) approximation has been investigated by several authors[13,14,19,21]. These investigations will be reviewed briefly here.

2. LST

The mean fractional energy loss δ, of an electron traversing an electromagnetic field, is given [19], following Baier and Katkov, by an integral over radiated photon energy , $m\gamma(1-\xi)$, of a double integral over the classical particle orbit

$$\delta = {}_0\int^1 d\xi \int dt' \int dt'' \; .... \tag{3}$$

The integrand .... is given in Appendix A. With

$$t = (t'+t'')/2 \tag{4}$$

we have

$$\delta = {}_0\int^1 d\xi \ \int dt \ \int d_8 t'-t'') \qquad (5)$$

When the variation of ρ along the orbit is neglected, there is a factor in the integrand

$$\sin \Omega(\tau + \tau^3/12) \qquad (6)$$

where

$$\tau = (t'-t'')/(\rho/\gamma) \qquad (7)$$
$$\Omega = (1-\xi)/(2\xi\Upsilon) \qquad (8)$$

It is the oscillation of this factor which brings about the convergence of the integration over τ. So the important range of integration is that in which τ does not greatly exceed a characteristic value defined by

$$\Omega(\tau + \tau^3/12) = 1 \qquad (9)$$

This depends on ξ. For large Υ the spectrum of ξ is broad, and we may say that Ω is typically of order $1/\Upsilon$, and then (6) gives

$$\tau \text{ of order } \Upsilon^{1/3}, \text{ for } \Upsilon.>>1$$

For small Υ, $(1-\xi)$ is typically of order Υ over the important part of the spectrum, so that Ω is of order 1, and (6) gives

$$\tau \text{ of order } 1, \text{ for } \Upsilon<<1$$

Interplating between these two expressions, and taking advantage of the fact that the notion of characteristic time is well defined only as regards order of magnitude, we adopt as characteristic time,

$$t_k = \tau_k \rho/\gamma \ , \quad \tau_k = (.75 + \Upsilon^{2/3})^{-1/2} \qquad (10)$$

Of course there is much arbitrariness here; the convenience of this particular choice will appear in a moment. It seems reasonable to conjecture that when the radius of curvature ρ varies little over the section of orbit described in time τ_k, the formula for constant ρ, i.e. LST, is a good approximation.

In terms of t_k, the LST formula can be written

$$t_k \; d\delta/dt = .0027(1 + .75 \; \Upsilon^{-2/3})^{-3/2} \; f(\Upsilon) \tag{11}$$

where $f(\Upsilon)$ varies with Υ only within narrow limits near 1:

$$.9 < f < 1.1 \tag{12}$$

It is clear from (11) that to build up a δ of say 0.1 requires a transit time of some $30t_k$ (for Υ large) or more. Then unless the deflecting field has a strong fine structure, the requirement for the validity of LST will be met over the portion of the orbit giving the main contribution to the integral of LST. However where the field falls to zero at the beginning and end, ρ and t_k become indefinitely large, and LST becomes inapplicable. So a special investigation of end effects is required.

Note by the way that our tentative sufficient condition for the validity of LST for $d\delta/dt$ is certainly unnecessarily stringent in the case of very small Υ. For then classical radiation theory applies, and the total radiated power (but not the detailed spectrum) can be expressed locally in terms of local fields.

3. IMPROVED LST

It is natural to try to improve the LST approximation by not neglecting completely the variation of ρ over time t_k , but still treating it as small. This leads [19] to the correction factor, to multiply the LST formula (11),

$$1 - \; 156(\; t_k B^{-1} dB/dt)^2 + .117 \; t_k{}^2 B^{-1} d^2B/dt^2 \tag{13}$$

for Υ>>1. This exercise was started by P.Chen [13]. Chen and Yokoya have obtained a result equivalent to (13) (private communication; the corresponding formula (18) of [21] seems to contain misprints).

The formula corresponding to (13) for Υ not large has not been worked out, as far as we know. But Chen and Yokoya have obtained the corresponding integral over a Gaussian field [21]. According to them, the correction factor to multiply the integrated LST δ is

$$1 - 2.0\ (\rho_0/\sigma_z)^2\Upsilon_0, \qquad \Upsilon_0 \ll 1 \qquad (14)$$

$$1 - .33\ (\rho_0/\sigma_z)^2\Upsilon_0^{4/3}(\ln \Upsilon_0)^{1/2}, \qquad \Upsilon \gg 1 \qquad (15)$$

for a distribution

$$\Upsilon = \Upsilon_0 \exp(-2t^2/\sigma_z^2) \qquad (16)$$

With the parameters suggested for a 5+5 TeV collider by Himel and Siegrist, Chen and Yokoya find from (15) a correction of about 30%. But in our opinion this result, and these formulae, should be treated with some reserve, because of the increase of t_k at the entrance and exit, and the consequent gross violation of LST. Indeed our numerical integration (below) finds a much smaller correction in the Himel-Siegrist case.

4. END EFFECT

Another case which has been worked out, for large Υ, is that of a field constant over a limited interval and falling sharply to zero outside it:

$$\Upsilon = \Upsilon_0\ , \qquad |t| < l/2 \qquad (17)$$

$$\quad = 0\ , \qquad |t| > l/2 \qquad (18)$$

With

$$\delta = \delta_{LST} + \Delta\delta \qquad (19)$$

the end effect is [19], for $\Upsilon \gg 1$,

$$\Delta\delta = .0021\ (\ln\Upsilon_0 - 2.10) \qquad (20)$$

This increases slowly with Υ_0, but with Υ_0 = 10,000 is still only .015. The leading log in (20) was obtained by Jacob and Wu [14].

5. NUMERICAL INTEGRATION

In general, in a non-uniform field , we will have some combination of non-uniformity effects like those of section 3 above, and end effects like those of section 4. We have resorted then to numerical integration, and can reveal here some preliminary results for gaussian distributions. With

$$\Upsilon = \Upsilon_0(6/\pi)^{1/2} \exp(-2t^2/\sigma_z^2) \qquad (21)$$

We have taken a strength typical of the Himel-Siegrist 5+5 TeV collider:

$$\Upsilon_0 = 5000$$

and have adjusted the width to give values of δ_{LST}, from numerical integration of (11), around .07,.14, and .21. Numerical integration of the accurate equations, set out in Appendix A, gives slightly larger values of δ, and so a correction

$$\Delta\delta = \delta - \delta_{LST} \qquad (22)$$

The results are tabulated, with for comparison the end effect (20) for the sharply bounded uniform field with the same Υ_0. It is seen that the correction to δ_{LST} is somewhat smaller for the Gaussian case than for the other. And very much smaller than that suggested by the Chen-Yokoya perturbation formula (15), identified by (CY) in the table. (Note that in calculating this last we have to take account of the factor $(6/\pi)^{1/2}$ in (21) but not in (16).)

Table 1: Gaussian field distribution

δ_{LST}	.071	.141	.209
$\Delta\delta$	.008	.007	.006
$\Delta\delta(20)$	.012	.012	.012
$\Delta\delta(CY)$	.11	.056	.037

APPENDIX A

We start with the following formula [7] for the mean energy dE radiated into a frequency range $d\omega$ and solid angle do by a relativistic electron:

$$dE = (e^2\omega^2/(4\pi^2))\ d\omega\ do \int dt' \int dt''\ Ae^{i\phi}\ C \qquad (A1)$$

with

$$A = [(\varepsilon^2+\varepsilon'^2)/(2\varepsilon'^2)]\ (v(t').v(t'')-1) + (1/2)(\omega/\varepsilon')^2(m/\varepsilon)^2 \qquad (A2)$$

$$\phi = (\varepsilon/\varepsilon')\ \omega\ [(t'.-t'')-n.(r(t')-r(t''))] \qquad (A3)$$

where

$$\varepsilon = \text{initial electron energy} \qquad (A4)$$

$$\varepsilon' = \varepsilon - \omega \qquad (A5)$$

$$r(t) = \text{position on classical orbit unperturbed by radiation} \qquad (A6)$$

$$v(t) = (d/dt)r(t) \qquad (A7)$$

$$n = \text{unit vector in photon direction} \qquad (A8)$$

$$e^2 = 1/137 \qquad (A9)$$

$$h/(2\pi) = c = 1 \qquad (A10)$$

$$C = \exp(-\mu|t'|-\mu|t''|) \qquad (A11)$$

In the convergence factor C, μ is to be taken very small; we are interested in the limit in which μ approaches zero. For $\omega \ll \varepsilon$ (1) reduces to the (perhaps more familiar) classical formula.

With

$$\delta = E/\varepsilon, \quad \xi = \varepsilon'/\varepsilon, \quad (1-\xi) = \omega/\varepsilon \tag{A12}$$

the formulae can be rewritten

$$d\delta = d\xi\ do\ (e^2\omega^2/4\pi^2)\ \int dt' \int dt''\ Ae^{i\phi} \tag{A13}$$

$$A = [(1/2\xi^2)+(1/2)]\ (v(t').v(t'')-1)\ +\ (1/2)[(1-\xi)/\xi)]^2/\gamma^2 \tag{A14}$$

$$\phi = (\omega/\xi)(t'-t''-n.r) \tag{A15}$$

$$r = r(t') - r(t'') \tag{A16}$$

Performing the integral over solid angle ,i.e. over all directions of n, gives

$$d\delta/d\xi = (e^2/\pi)\ \omega x \int dt' \int dt''\ (A/r)\ \sin(\omega r/\xi)\ \cos(\omega(t'-t'')/\xi) \tag{A17}$$

with

$$r = |r|(t'-t'')/|t'-t''| \tag{A18}$$

In

$$\sin(\omega r/\xi)\cos(\omega(t'-t'')/\xi)=\{(\sin(\omega(r-t'+t'')/\xi)+\sin(\omega(r+t'-t'')/\xi)\ \}/2 \tag{A19}$$

the second term oscillates very quickly for a particle travelling with nearly the velocity of light. Neglecting the variation of v on this scale, and noting that r is well approximated by (t'-t'') and that

$$\int dt/t\ \sin\ (Wt) = \pi \tag{A20}$$

we have

$$d\delta/d\xi = (1/2)(e^2/\pi)\ \omega\xi \int dt' \int dt''\ (A/r)\{\pi\delta(t'-t'')-\sin(\theta)\} \tag{A21}$$

where

$$\theta = (\omega/\xi)(t'-t''-r) \tag{A22}$$

It is convenient to introduce normalized variables, denoted by capital letters, such that

$$t' = (\rho/\gamma)T', \quad t'' = (\rho/\gamma)T'' \qquad (A23)$$

$$v'_x = V'_x/\gamma \;, \quad v'_y = V'_y/\gamma \qquad (A24)$$

$$v''_x = V''_x/\gamma \;, \quad v''_y = V''_y/\gamma \qquad (A25)$$

$$(x',y') = (\rho/\gamma)((X',Y')/\gamma) \qquad (A26)$$

$$(x'',y'') = (\rho/\gamma)((X'',Y'')/\gamma) \qquad (A27)$$

where ρ is some typical radius of curvature,and where the z-axis of cartesian coordinates is taken in the direction of $v((t'+t'')/2)$. Then with

$$T = (T'+T'')/2, \quad \tau = (T'-T'') \qquad (A28)$$

and taking advantage of the symmetry of the integrand to restrict the integration to positive τ, we have

$$d\delta/d\xi = (e^2/\pi)\Omega \int dT \; S(\infty,T,\xi) \qquad A(29)$$

where

$$S(\tau,T,\xi) = \{{}_0\int^{\tau} d\tau (F/R)\sin(\theta) - (\pi/2)F_0\} \qquad (30)$$

where F, related to A, is given below. Assuming that the orbit turns through only small angles over the times required for the convergence of the τ integration, and expanding to second order in these small angles [21], we find that we have to integrate the following equations:

For $\tau = 0$,

$$X' = X'' = Y' = Y'' = 0 \qquad (A31)$$

$$V'_x = V''_x = V'_y = V''_y = 0 \qquad (A32)$$

$$\psi = 0 \qquad (A33)$$

$$S = -\pi\xi \qquad (A34)$$

And as τ varies

$$dX'/d\tau = V'_x/2 \quad , \quad dX''/d\tau = -V''_x/2 \qquad \text{(A35)}$$

$$dY'/d\tau = V'_y/2 \quad , \quad dY''/d\tau = -V''_y/2 \qquad \text{(A36)}$$

$$dV'_x/d\tau = -K_x(T')/2 \quad , \quad dV''_x/d\tau = +K_x(T'')/2 \qquad \text{(A37)}$$

$$dV'_y/d\tau = -K_y(T')/2 \quad , \quad dV''_y/d\tau = +K_y(T'')/2 \qquad \text{(A38)}$$

(where the quantity K, in general a function of T, is the deflecting force in units of that defining the typical radius of curvature ρ)

$$d\psi/d\tau = \Omega\{1 + (V'^2_x + V'^2_y + V''^2_x + V''^2_y)/2\} \qquad \text{(A39)}$$

$$\theta = \psi - \Omega\{(X'-X'')^2 + (Y'-Y'')^2\}/\tau \qquad \text{(A40)}$$

$$F = (1+\xi^2)\{1 + ((V'_x-V''_x)^2 + (V'_y-V''_y)^2)/2\} - (1-\xi)^2 \qquad \text{(A41)}$$

$$dS/d\tau = F/\tau\ C, \qquad C=\exp(-\mu|T'|-\mu|T''|) \qquad \text{(A42)}$$

Integration of this last to $\tau = \infty$ and substitution in (A29) gives δ.

$$\delta = (e^2/\pi) \int d\xi\ \Omega \int dT\ \ S(\infty, T, \xi) \qquad \text{(A43)}$$

1. REFERENCES.

1. A.A.Sokolov, N.P.Klepikov, and I.M.Ternov, ZETF 24 (1953) 249
2 J.Schwinger, Proc.Nat.Acad.Sci. 40 (1954) 132.
3. A.N.Matveev. ZETF 31(1956)479; Sov.Phys.JETP 4 (1957) 409.
4. A.I.Nikishov and V.I.Ritus, Sov.Phys.JETP 25 (1967) 1135.
5. V.N.Baier and V.M.Katkov, Sov.Phys.JETP 28 (1969) 807
   and Sov.Phys.JETP 26,(1968) 854
6. A.A.Sokolov and I.M.Ternov. Synchrotron Radiation (Pergamon 1971).
7. V.B.Berestetskii, E.M.Lifshitz, and L.P.Pitaevskii,
   Relativistic Quantum Theory (Pergamon 1971).
8. T.Himel and J.Siegrist, SLAC-PUB-3572, Feb. 1985, and in Laser

Acceleration of Particles (C.Joshi and T.Katsouleas,eds.) AIP Conf.Proc.No.130 (1985)

9. R.J.Noble, SLAC AAS-Note 3, Dec. 1985, SLAC-PUB 3871 (1986), Nuclear Instruments and Methods A256 (1987) 427
10. K.Yokoya, Nuclear Instruments A251 (1986) 1
11. R.Blankenbecler and S.D.Drell, Phys.Rev.B3 (1987) 277, And SLAC-PUB-4483 (November 1987)
12. M.Bell and J.S.Bell, CERN PS 87-53(DL) (June 1987); Particle Accelerators,to appear.
13. Pisin Chen, SLAC-PUB-4391 (Aug. 1987)
14. M.Jacob and T.T.Wu, Phys.Lett.197B (1987) 253
15. M.Jacob and T.T.Wu, CERN-TH.4848/87 (1987)
16. M.Jacob and T.T.Wu, CERN-TH.4907/87 (1987)
17. U.Amaldi, in Lecture Notes in Physics 296 (M.Month and S.Turner,eds., Springer-Verlag,Berlin, 1987) p.341
18. P.Chen, in Lecture Notes in Physics 296 (M.Month and S.Turner,eds., Springer-Verlag,Berlin, 1987) p.495
19. M. Bell and J.S.Bell, CERN-PS 88-13 (DL), CERN-TH 4936/87 (1987)
20. R.Blankenbecler and S.D.Drell, SLAC-PUB-4629 (May 1988)
21. M.Bell and J.S.Bell, CERN-TH.5056/88, CERN/PS 88-27(DL) (May 1988)
22. P.Chen and K.Yokoya, SLAC-PUB-4597 (April 1988)