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A RADIOFREQUENCY PULSE COMPRESSOR FOR SQUARE OUTPUT PULSES

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Abstract

"Energy doublers" based on the SLED principle which was developed at SLAC, have gained widespread interest and application for drivers of electron linacs [1, 2]. For application with "constant-gradient" structures, however, they suffer from the drawback that the highest part of the output pulse is at the low group-velocity end of the section when the electron beam passes, and hence is more compressed in geometrical length than the low-voltage parts. It seems therefore to be interesting to study whether there are pulse compressing schemes which yield a more favourable shape of the output pulse. In the paper, it is be shown that square shapes of the output pulse may be achieved by using sections of transmission line as energy stores. Characteristics, applications and limitations of such schemes are discussed.

1. Possible schemes can be subdivided into two classes: Transmissive and reflective ones. Transmissive schemes mean that time slices of a long input pulse are, after individually different delays, added by a power combining device in order to yield a shorter pulse of higher power. Reflective schemes make use of a storage line which is charged up to a certain energy during some time, and then discharged during a shorter time, possibly using some phase-switching mechanism as in SLED. For simplification, the principles of such a reflective scheme using lines will be treated in a graphic way as used for transients on high-voltage lines. This will amount, apart from some very simple formulae, to a drawing exercise rather than to mathematical theory. Afterwards, it will be discussed how to translate the results of this model into a practical radiofrequency set-up.

2. Fig. 1 depicts a snapshot of the square wave travelling along a line. The ratio of the voltage V to the current I on the line is given by its characteristic impedance, $V/I=Z$.

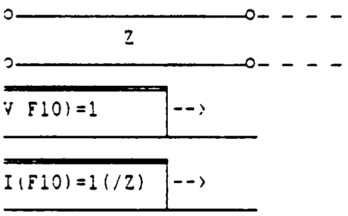

Fig. 1: Voltage and current of a travelling wave.

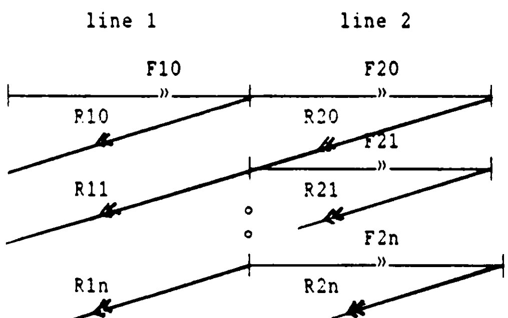

Fig. 2: Index tree

If this line is connected at its end to a second line having a different characteristic impedance, $Z_2 \neq Z_1$, a part of the wave will be reflected back on line 1, and another part will be refracted into line 2. The voltage and current amplitudes of the reflected wave are given by the reflection coefficient $r_{12}=(Z_2-Z_1)/(Z_2+Z_1)$; such that $V(R1)=r_{12}*V(F1)$ and $I(R1)=-r_{12}*I(F1)$. The refracted wave travelling on line 2 has the amplitudes $V(F2)=(1+r_{12})*V(F1)$ and $I(F2)=(1-r_{12})*I(F1)$.
In order to discuss the various forward- and backward-running waves, an indexing system is used which should be clear from the tree outlined in Fig. 2.The reflected and refracted waves, and the result of the superposition with the original, forward running wave are sketched in Figs. 3 and 4. For scaling the sketches, Z_2 was assumed to be $Z_1/2$; hence

$$r_{12}= (1/2 - 1)/(1/2 + 1) = -1/3, \text{ and } 1 - r_{12} = 4/3.$$

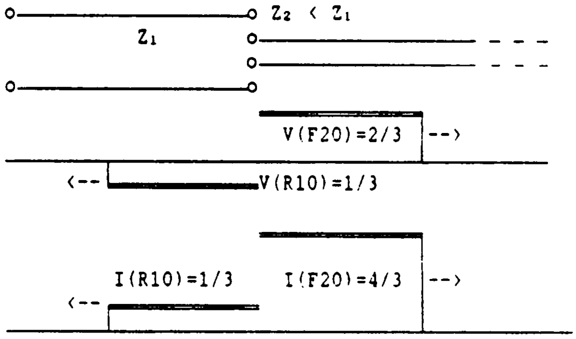

Fig. 3: Reflected and refracted waves

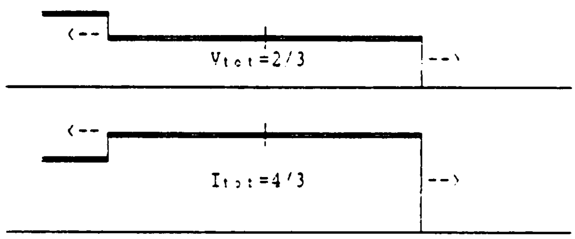

Fig. 4: Result of the superposition of the waves

3. If line 2 is terminated with an open-circuit or a short-circuit, the reflection coefficient at its end is $r_{20}=1$ or $r_{2s}=-1$, respectively. In both cases, the wave travelling on line 2 will be reflected, the amplitudes of the reflected wave being

$$V(R2) = r_2 * V(F2) \ ; \ I(R2) = -r_2 * I(F2)$$

This effect, and the result of the superposition of the waves existing so far on line 2 are sketched in Figs. 5 and 6, assuming an open-circuit at the end.

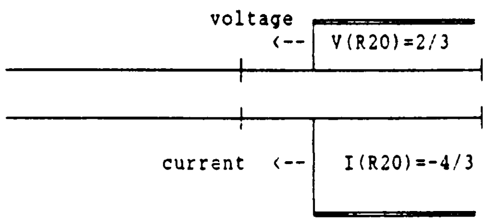

Fig. 5: Reflected wave from open-circuit on line 2

The total amplitudes on line 2 are given as

$$V_{tot2} = V(F2) + V(R2) = (1+r_2) * V(F2)$$
$$= (1+r_2) * (1+r_{12}) * V(F1), \text{ and}$$

$$I_{tot2} = I(F2) + I(R2) = (1-r_2) * I(F2) = (1-r_2) * (1-r_{12}) * I(F1)$$

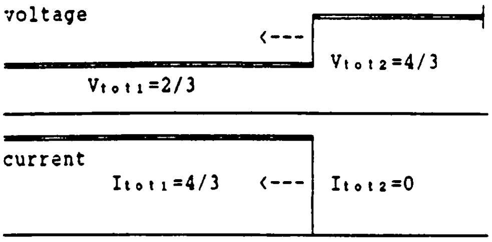

Fig. 6: Result of superposition of the waves

When the head of the reflected wave on line 2 reaches the junction to line 1, it is partially reflected back into line 2, and partially refracted into line 1. Considering that now the reflection coefficient of the junction has to be taken from the right, looking from line 2 into line 1, $r_{21}=-r_{12}$, the new reflected wave is given as

$$V(F21) = - r_{12}*r_2*V(F20) = -r_{12}*r_2*(1+r_{12})*V(F10)$$
$$I(F21) = - r_{12}*r_2*I(F20) = -r_{12}*r_2*(1-r_{12})*I(F10)$$

For the refracted wave on line 1, travelling backwards, we find

$$V(R11) = (1+r_{21}) * V(R20) = (1+r_{21}) * r_2 * V(F20) = (1-r_{12})*r_2*(1+r_{12})*V(F10) = r_2*(1-r_{12}^2)*V(F10)$$
$$I(R11) = (1-r_{21}) * I(R20) = (1-r_{21}) * (-r_2) * I(F20) = (1+r_{12})*(-r_2)*(1-r_{12})*I(F10) = - r_2*(1-r_{12}^2)*I(F10)$$

This is shown in fig. 7.

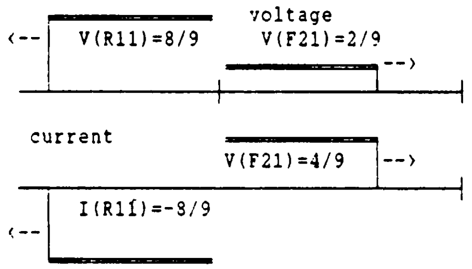

Fig. 7: Refracted on line 1, reflected on line 2

The resulting wave travelling backwards on line 1 is given by

$$V(R1,tot) = V(R10) + V(R11) = r_{12} * V(F10) + r_2 * (1-r_{12}^2) * V(F10) = r_{12} + r_2 * (1-r_{12}^2)) * V(F10)$$
$$I(R1,tot) = I(R10) + I(R11) = -r_{12} * I(F10) - r_2 * (1-r_{12}^2) * I(F10) = -(r_{12} + r_2 * (1-r_{12}^2)) * I(F10)$$

In order to get the total voltage and current for this moment on line 1, we still have to add the incident values:

$$V(1,tot) = V(F10) * (1 + r_{12} + r_2 * (1-r_{12}^2))$$
$$I(1,tot) = I(F10) * (1 - r_{12} - r_2 * (1-r_{12}^2))$$

It is easy to verify that the total voltage and current on on line 2 are the same, V(2,tot)=V(1,tot); I(2,tot)=I(1,tot).

After these rather explicit derivations, it is possible to be shorter for writing the formulae concerning multiple-reflected waves:

$$V(R2,n) = r_2 * V(F2,n)$$
$$V(R1,n+1) = (1+r_{21}) * V(R2,n) = (1-r_{12}) * V(R2,n)$$
$$V(F2,n+1) = r_{21} * V(R2,n) = -r_{12} * V(R2,n)$$

Hence,

$$V(R2,n)=-r_2*r_{12}*V(R2,n-1)=(-r_2*r_{12})^n*V(R20) =r_2*(-r_2*r_{12})^n*V(F20)=(1+r_{12})*r_2*(-r_2*r_{12})^n*V(F10)$$

and, for n#0,

$$V(R1,n) = (1 - r_{12}) * V(R2,n-1) = (1 - r_{12}^2) * r_2 * (-r_2 * r_{12})^{n-1} * V(F10)$$

which yields

$$\sum_{\mu=1}^{n} V(R1,\mu)=(1-r_{12}^2) * r_2 * V(F10) * \sum_{\mu=1}^{n} (-r_2 * r_{12})^{\mu-1}$$

and $V(R10) = r_{12} * V(F10)$.

If we evaluate the sum and add the term for V(R10), we get:

$$\frac{V(R1,tot)}{V(F10)} = (1-r_{12}^2) * r_2 * \frac{1 - (-r_2 * r_{12})^n}{1 + r_2 * r_{12}} + r_{12}$$

Eventually, (for n -> ∞), this expression approaches the value

$$(1 - r_{12}^2) * r_2 * \frac{1}{1 + r_2\, r_{12}} + r_{12} =$$

$$= \frac{r_2 - r_{12}^2 r_2 + r_{12} + r_{12}^2 r_2}{1 + r_2 r_{12}} = \frac{r_2 + r_{12}}{1 + r_2 r_{12}}$$

For $r_2 = -1$ (short-circuit at the end of line 2) this yields -1, and for $r_2 = 1$ (open-circuit), this yields 1, as it should be, independent of r_{12}.

4. If, after n "charging periods", the polarity of the input voltage is inversed, a forward-running wave with an amplitude of -2V is superimposed. At the junction of the two lines, it is reflected back into line 1 with the amplitude $-2r_{12}V$. During the time which the refracted part takes to run to the end of line 2 and back to the junction, the total backward running wave on line 1 is given by

$$\frac{V(R1,sw)}{V(F10)} = (1 - r_{12}^2) * r_2 * \frac{1 - (-r_2 r_{12})^n}{1 + r_2 r_{12}} - r_{12}$$

For $r_2 = -1$, this expression simplifies to

$$\frac{V(R1,sw)}{V(F10)} = (r_{12} + 1) * (r_{12}^n - 1) - r_{12}$$

The (modulus of the) ratio V(r1,sw)/V(F10) can become greater than 1. Therefore, it may be called a (voltage) enhancement factor. It is a function of n and r_{12}. It is shown in Fig. 8 as a function of r_{12}, for n=5. It can be seen that it has a maximum for a value of r_{12} between zero and one. For greater n, this maximum becomes bigger and moves closer to r_{12}=1.

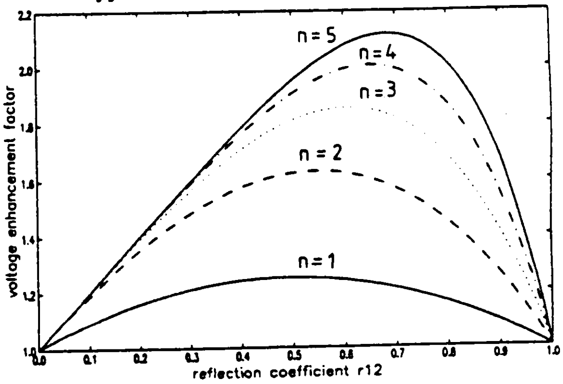

Fig. 8
Voltage enhancement factor as a function of r_{12}

Solving the equation for its maximum as a function of r_{12} yields the values in table 1. As it can be seen from this table, the behaviour of the arrangement is rather similar to that of a SLED device. The backward running wave, however, is given as a step function rather than by an exponential. This means that, assuming a lossless and nondispersive line 2, an input pulse can be compressed into a square output pulse of, for instance, one fifth of its length, and slightly above twice its amplitude (n=4).

n	v. e.	r_{12}	n	v. e.	r_{12}
1	-1.25	0.5	8	-2.32	0.75
2	-1.63	0.55	10	-2.41	0.78
3	-1.85	0.61	20	-2.63	0.86
4	-2.01	0.65	100	-2.89	0.95
5	-2.12	0.68	∞	-3	1
6	-2.20	0.71			

Table 1: Maximum voltage enhancement as a function of n

5. For an experimental study, a shortcircuited section of WR 284 waveguide was used. The mismatch between the two waveguides was realized by a diaphragm with a hole rather than by a step of the characteristic impedance as assumed until now. Thus, the theoretical considerations had to be different, as outlined here: Assuming the coupling network having the scattering matrix [S], the reflected wave towards the generator is given by S_{11}, and the refracted wave into the storage line by S_{21} times the incident wave amplitude A. The wave running on the storage line to the shortcircuit and back will undergo a phase lag and an attenuation, which can be expressed as a factor

$$D=r_2 e^{-2\Gamma l} \quad \text{with} \quad \Gamma=\alpha+j\beta=\alpha+j\omega/v_p,$$

the propagation constant of the line. After a time T corresponding to twice the group delay of the storage line, the signals at the coupling network are given by $A*S_{21}*D$ for the wave coming back on the storage line, $A*S_{21}*D*S_{22}$ for the wave reflected back into the storage line, and $A*S_{21}*D*S_{12}$ for the refracted wave towards the generator. One "period" T later, the partial signals are, in the same order,

$$A*S_{21}*D*S_{22}*D,\ A*S_{21}*D*S_{22}*D*S_{22},\ \text{and}$$
$$A*S_{21}*D*S_{22}*D*S_{12}.$$

It is easy to derive the expression valid after n cycles, and to write the result of superimposing all partial waves which run towards the generator:

$$SUM1=A*S_{11}+A*D*S_{21}*S_{12}*\sum_{\mu=1}^{n}(D*S_{22})^{\mu-1}$$

Superimposing a wave of the amplitude -2A (by phase-switching the driving signal) yields for the total backwards-running wave on line 1:

$$SUM2=-A*S_{11}+A*D*S_{21}*S_{12}*\sum_{\mu=1}^{n}(D*S_{22})^{\mu-1}$$

Replacing the reflection coefficients used before by the scattering matrix elements of a jump in characteristic impedance,

$$S_{11}=-S_{22}=r_{12},\ S_{21}=1-r_{12},\ S_{12}=1+r_{12},\ \text{and}\ D=r_2,$$

it can be seen that these equations are formally equivalent to the ones derived before.
In order to maximize this expression, all terms of the sum have to be in phase. As the elements of the matrix [S] and D need not a priori be real, this means that

$$\arg(D)=-\arg(S_{22}),\ \text{and}$$
$$\arg(S_{11})+\arg(S_{22})=\arg(S_{12})+\arg(S_{21})+n\pi.$$

The first equation is nothing but the resonance condition for the storage line, whereas the second one is fulfilled by any lossless coupling network; it follows from the unitarity of its scattering matrix. Fig. 9 shows the calculated shape of the reflected pulse for a SLED-type input pulse (phase-switching of the input pulse after 5 delay periods T), and Fig. 10 is a photograph of pulse shapes on the measuring setup. For the calculation of table 1, the attenuation of the storage line was assumed to be zero. As it will have a finite attenuation in practice, the voltage enhancement will be less favourable. If, in fact, an output pulse of about one microsecond is desired, it is clear that normal WR 284 waveguide, having an attenuation of 0.02 dB/m at 3 GHz, will be rather impractical, because the reflected waves have to travel between 300 m and 1500 m electrical length, and the voltage enhancement will be, for n=5, 1.22 instead of the 2.01 of table 1, and 1.23 instead of 2.12 for n=5. Still, an application for shorter (0.1 µs) pulses would be quite feasible, as is shown in the experimental setup.

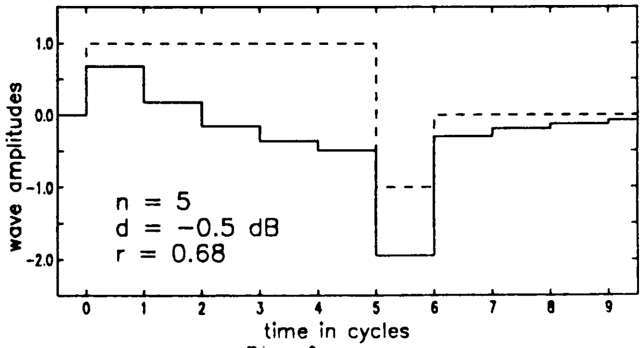

Fig. 9
Input pulse and reflected output pulse (calculated).

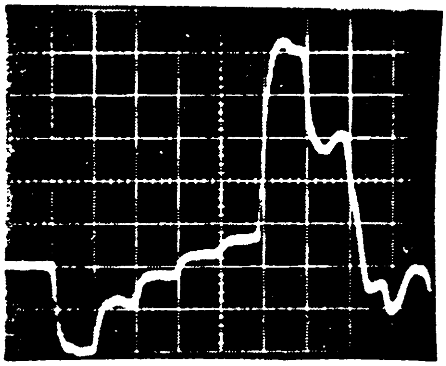

Fig. 10
Shape of the pulses on the experimental setup.

A possible solution for longer pulses could be using superconducting waveguides, or round waveguides with a H_{01} mode. In the latter case, and for a diameter of 20 cm, the attenuation would be 1.5 * 10^{-3} dB/m, yielding an enhancement factor of 1.9 for n=4 and 1.99 for n=5. Furthermore, it should be possible to replace the storage line by a chain of mutually coupled high-Q resonators. In all schemes, the separation of the forward- and backward-running waves would be done by a method similar to that applied for SLED.

References

[1] Z.D. Farkas, H.A. Hogg, G. A. Loew and P. B. Wilson, "SLED, a method for doubling SLAC's energy," presented at the IX. Conference on High Energy Accelerators, 1974.

[2] A. Fiebig, R. Hohbach, P. Marchand and J. O. Pearce, "Design Considerations, Construction and Performance of a SLED-Type Radiofrequency Pulse Compressor Using Very High Q Cylindrical Cavities," presented at the 1987 Particle Accelerator Conference, Washington.