

SUSPENDED VACUUM CHAMBERS:

STRESS AND DEFORMATION

H. Bargmann and H. Stucki

ABSTRACT

Equations are derived for stress and deformation of tubular vacuum chambers of quasi-elliptical cross-section, constrained at the minor axis by suspensions. Graphs are included to facilitate computations for race-track shaped tubes.

Geneva - 7 July 1977

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1. INTRODUCTION

There are several stress analyses available for elliptical or quasi-elliptical vacuum chambers, assuming a free deformation of the cross-section of the tube, without constraint due to any suspension¹⁻⁵⁾.

In order to obtain an increase in stability, together with a decrease in stress and deflection, eventually leading to a reduced wall thickness of such a tube, constraints of the cross-section by means of a suspension have been examined⁶⁻⁸⁾, Fig. 1.

In Ref. 6 the stress analysis for a thin-walled, quasi-elliptical, or elliptical, vacuum chamber was presented as the first part of a study of the effect of the constraint of the minor axis of the cross-section due to the suspension of the tube. The analysis of the deflection of such a tube was announced to follow as a second part. This is given in the present report. The basic equations are derived for a tube that has any cross-section with two orthogonal axes of symmetry, where one axis has an arbitrary elastic constraint. Hence, we allow for any constraint between a perfectly rigid (minor) axis and a completely free cross-section, Fig. 1. The equations are valid for variable bending stiffness along the ring, for example, for variable wall thickness. In addition, the equations provide estimates for tubes reinforced with ribs (see Section 5). The general results are specialized for the particular profile as given in Fig. 2. For the latter, numerical results for bending moment and deflection are computed in a non-dimensional form and presented as graphs.

2. BASIC EQUATIONS FOR SUSPENDED TUBES OF ARBITRARY DOUBLY-SYMMETRIC PROFILE, ELASTICALLY CONSTRAINED ALONG ONE OF THEIR PRINCIPAL AXES

2.1 Geometric relations

We restrict our attention to elastic tubes of any cross-section, but possessing two orthogonal axes of symmetry, for example, elliptical or quasi-elliptical tubes. It is then sufficient to consider one quadrant of the ring cut off the tube (of unit length), bounded by axes of symmetry, x, y , where (x, y, z) is a rectangular Cartesian coordinate frame, Fig. 1.

One axis, for example, the minor axis of the cross-section, is elastically constrained. This constraint on the ring may be represented by an elastic beam of length l_1 , cross-section area $2A_1$ (per unit length), and Young's modulus E_1 , Fig. 1,

The median line in the cross-section is represented in terms of polar coordinates by a relation $R = R(\phi)$, Fig. 1. The arc element ds and the angle β , between the tangent to the curve and a normal to the radius at a point (R, ϕ) of the curve, are given by⁶⁾

$$ds = (R^2 + R'^2)^{1/2} d\phi \quad (1)$$

$$\tan \beta = -(R'/R) . \quad (2)$$

The prime indicates differentiation with respect to ϕ .

2.2 Equations of equilibrium

If an external pressure p is applied and changed slowly enough so that inertia effects are avoided, then, at any time, equations of equilibrium relating normal and shear forces, P and S , respectively, and bending moment M (per unit length) in the ring to the external pressure and the shape of the cross-section may be established. At $\phi = \alpha$, a fictitious vertical force H has been introduced, which finally will be set equal to zero. This allows for a simple calculation of the vertical deflection v , at $\phi = \alpha$.

It follows from force equilibrium by inspection of Fig. 1, after simple calculations, and for $0 \leq \phi \leq \alpha$,

$$P = pR \cos \beta - (S_e - H) \cos (\beta + \phi) \quad (3a)$$

$$S = -pR \sin \beta + (S_e - H) \sin (\beta + \phi) , \quad (4a)$$

where the fact has already been used that, as a consequence of symmetry, shear forces must vanish at $\phi = 0, \pi/2$.

Moment equilibrium about point B, bearing in mind that $V = pR_0 - (S_e - H)$, with $R_0 \equiv R(0)$, yields

$$M = M_0 - \left(\frac{p}{2}\right)(R_0^2 - R^2) + (S_e - H)(R_0 - R \cos \phi) . \quad (5a)$$

Similarly, for $\alpha \leq \phi \leq \pi/2$,

$$P = pR \cos \beta - S_e \cos (\beta + \phi) \quad (3b)$$

$$S = -pR \sin \beta + S_e \sin (\beta + \phi) \quad (4b)$$

$$M = M_0 - \left(\frac{p}{2}\right)(R_0^2 - R^2) + S_e(R_0 - R \cos \phi) - H[R_0 - R(\alpha) \cos \alpha] . \quad (5b)$$

The statically indeterminate bending moment M_0 and the constraint force S_e are still to be determined. (Due to symmetry conditions, the original problem, statically indeterminate to the fourth degree, has already been reduced to a problem with 2 redundants, M_0, S_e , only.)

The normal circumferential stresses in the tube on the outer side, subscript o , and on the inner side, subscript i , may be written

$$\sigma_o = \frac{1}{h} \left(P - \frac{6M}{h} \right) , \quad \sigma_i = \frac{1}{h} \left(P + \frac{6M}{h} \right) , \quad (6)$$

where h is the local wall thickness (constant or slightly varying). The force P and the stresses σ_o, σ_i , are positive when they are compressive. The moment M is positive if it has the same sense as M_0 in Fig. 1.

3. SOLUTION FOR AN ELASTIC PROFILE WITH VARIABLE BENDING STIFFNESS

The solution of the statically indeterminate elastic problem is conveniently performed by employing the theorem of Castigliano, which states that the partial derivative of the strain energy U of the structure with respect to the redundant force S_e or moment M_0 is equal to zero,

$$\frac{\partial U}{\partial M_0} = 0, \quad \frac{\partial U}{\partial S_e} = 0. \quad (7)$$

This is a set of two simultaneous equations for the redundants M_0, S_e . Moreover, the deflection v under (and in the direction of) the given force H follows from

$$\frac{\partial U}{\partial H} = v. \quad (8)$$

The expression for the strain energy of the ring of unit length cut off the tube is

$$U = \frac{1}{2} \int_L \left(\frac{M^2}{EJ} + \frac{N^2}{EA} + \frac{\kappa Q^2}{GA} \right) ds, \quad (9)$$

where the terms on the right-hand side represent the strain energy due to bending moment, axial force, and shearing force, respectively. The elastic and cross-sectional constants have the usual meaning; s is the arc length along the median line of the cross-section, and the integration is performed, because of symmetry, along one quadrant of the profile, including the suspension.

In the present case, the influence of shear and of normal forces in the slender ring may be neglected. Thus, with EJ as the variable bending stiffness in circumferential direction of the ring, we have

$$U = \frac{S_e^2 \ell_1}{2E_1A_1} + \frac{1}{2} \int_0^{s(\pi/2)} \frac{M^2}{EJ} ds, \quad (10)$$

where $M \equiv M(M_0, S_e)$ from Eqs. (5a) and (5b) has to be introduced. According to Eqs. (7) we arrive at two equations:

$$\frac{\partial U}{\partial M_0} = \int_0^{s(\pi/2)} \frac{M}{EJ} \frac{\partial M}{\partial M_0} ds = 0 \quad (11)$$

$$\frac{\partial U}{\partial S_e} = \int_0^{s(\pi/2)} \frac{M}{EJ} \frac{\partial M}{\partial S_e} ds + \frac{S_e \ell_1}{E_1A_1} = 0, \quad (12)$$

from which, with $H \equiv 0$, M_0 and S_e can be determined. The vertical deflection v at $\phi = \alpha$ follows from Eq. (8):

$$\left. \frac{\partial M}{\partial H} \right|_{H=0} = \int_0^{s(\pi/2)} \frac{M}{EJ} \left. \frac{\partial M}{\partial H} \right|_{H=0} ds = v. \quad (13)$$

Upon substitution of Eqs. (5a,b) into Eqs. (11) to (13) we obtain the explicit expressions, with $EJ \equiv B(\phi) \equiv B$,

$$\frac{\partial U}{\partial M_0} = \int_0^{\pi/2} \left[M_0 - \frac{P}{2}(R_0^2 - R^2) + S_e(R_0 - R \cos \phi) \right] (R^2 + R'^2)^{1/2} \frac{d\phi}{B} = 0 \quad (11a)$$

$$\frac{\partial U}{\partial S_e} = \frac{S_e \ell_1}{E_1 A_1} + \int_0^{\pi/2} \left[M_0 - \frac{P}{2}(R_0^2 - R^2) + S_e(R_0 - R \cos \phi) \right] (R_0 - R \cos \phi) (R^2 + R'^2)^{1/2} \frac{d\phi}{B} = 0 \quad (12a)$$

$$\begin{aligned} \left. \frac{\partial U}{\partial H} \right|_{H=0} &= - \int_0^{\alpha} \left[M_0 - \frac{P}{2}(R_0^2 - R^2) + S_e(R_0 - R \cos \phi) \right] (R_0 - R \cos \phi) (R^2 + R'^2)^{1/2} \frac{d\phi}{B} \\ &\quad - \int_{\alpha}^{\pi/2} \left[M_0 - \frac{P}{2}(R_0^2 - R^2) + S_e(R_0 - R \cos \phi) \right] [R_0 - R(\alpha) \cos \alpha] (R^2 + R'^2)^{1/2} \frac{d\phi}{B} = \end{aligned} \quad (13a)$$

Note that the foregoing equations are valid for any cross-section that has two orthogonal axes of symmetry, and for variable bending stiffness $B \equiv B(\phi) \equiv EJ$. In particular, they allow numerical evaluation for any given profile.

4. THE RACE-TRACK TUBE

We now specialize for the particular quasi-elliptical (race-track) tube, the cross-section of which consists of two parallel straight sections and two semicircles at the major axis. (Such a cross-section is shown in Fig. 2 for the special case of a bending stiffness which is piecewise constant in four sections.) The median line of the cross-section is then given by the following equations.

For $0 \leq \phi \leq \phi_0 \equiv \arctan b/a$, we have

$$R = a \cos \phi + b\Delta, \quad \Delta \equiv \sqrt{1 - (a/b)^2 \sin^2 \phi} \quad (14a)$$

where $a + b \equiv R_0 \equiv R(0)$ and $b \equiv R(\pi/2)$ are (half the) major and minor axes of the cross-section, respectively. From Eq. (1) we then get for the element of arc:

$$ds = \left(\frac{a \cos \phi}{b\Delta} + 1 \right) b d\phi \quad (15a)$$

and for the angle β :

$$\tan \beta = \frac{a \sin \phi}{b \Delta} \quad (16a)$$

For $\phi_0 \leq \phi \leq \pi/2$, we have

$$R = \frac{b}{\sin \phi} \quad (14b)$$

for the element of arc:

$$ds = \frac{b}{\sin^2 \phi} d\phi \quad (15b)$$

and for the angle β , simply:

$$\beta = \pi/2 - \phi . \quad (16b)$$

Since we are interested in the deflections of the flat region of the tube, we assume $\alpha > \phi_0$.

Upon introduction of the normalized (non-dimensional) bending moment k_m , constraint force k_s , and deflection k_v , according to

$$M_0 \equiv k_m p R_0^2 , \quad S_e \equiv k_s p R_0 , \quad v \equiv k_v \frac{p R_0^4}{B_e} \quad (17)$$

together with the abbreviations

$$\xi \equiv \frac{a}{R_0} , \quad \eta \equiv \frac{b}{R_0} \equiv 1 - \xi , \quad \gamma \equiv \frac{B_e}{B(\phi)} , \quad (18)$$

where B_e denotes a reference bending stiffness, for example, Fig. 2, the basic equations (11a) to (13a) read, for the present profile,

$$\frac{B_2}{p R_0^3} \frac{\partial U}{\partial M_0} \equiv \int_0^{\phi_0} K_1 d\phi + \int_{\phi_0}^{\pi/2} K_2 d\phi = 0 \quad (11b)$$

$$\begin{aligned} \frac{B_2}{p R_0^4} \frac{\partial U}{\partial S_e} &\equiv k_s \frac{\rho_1 B_2}{E_1 A_1 R_0^3} + \int_0^{\phi_0} K_1 (1 - \xi \cos^2 \phi - \eta \Delta \cos \phi) d\phi \\ &+ \int_{\phi_0}^{\pi/2} K_2 (1 - \eta \cot \phi) d\phi = 0 . \end{aligned} \quad (12b)$$

$$\begin{aligned} k_v &= - \int_0^{\phi_0} K_1 (1 - \xi \cos^2 \phi - \eta \Delta \cos \phi) d\phi \\ &- \int_{\phi_0}^{\alpha} K_2 (1 - \eta \cot \phi) d\phi - \int_{\alpha}^{\pi/2} K_2 (1 - \eta \cot \alpha) d\phi , \end{aligned} \quad (13b)$$

where the abbreviations

$$\begin{aligned} K_1 &\equiv \left[k_m + k_s (1 - \xi \cos^2 \phi - \eta \Delta \cos \phi) - \frac{1}{2} (1 - \xi^2 \cos^2 \phi - \eta^2 \Delta^2 - 2\xi\eta\Delta \cos \phi) \right] \times \\ &\times \left(\xi \frac{\cos \phi}{\Delta} + \eta \right) \gamma \\ K_2 &\equiv \left[k_m + k_s (1 - \eta \cot \phi) - \frac{1}{2} \left(1 - \frac{\eta^2}{\sin^2 \phi} \right) \right] \frac{\eta}{\sin^2 \phi} \gamma \end{aligned} \quad (19)$$

have been employed. The normalized bending moment k_m and the normalized constraint force k_s follow from the first two, the normalized deflection k_v from the third of Eqs. (11b) to (13b) which may be written

$$a_{11}k_m + a_{12}k_s = b_1 \quad (11c)$$

$$a_{21}k_m + a_{22}k_s = b_2 \quad (12c)$$

$$k_v + a_{31}k_m + a_{32}k_s = b_3 \quad (13c)$$

Hence

$$k_m = D_1/D, \quad k_s = D_2/D \quad (20)$$

where

$$\begin{aligned} D &= a_{11}a_{22} - a_{12}a_{21} \\ D_1 &= b_1a_{22} - a_{12}b_2 \\ D_2 &= a_{11}b_2 - b_1a_{21} \end{aligned} \quad (21)$$

The coefficients a_{ik} and b_i are not written out explicitly here; they follow directly from Eqs. (11b) to (13b) and they contain integrals of the type

$$\int_0^{\phi_0} \frac{\cos \phi}{\Delta} \gamma d\phi, \text{ etc.} \quad (22)$$

For the special case of piecewise constant bending stiffness γ , these integrals reduce to pseudo-elliptical integrals which can be obtained in closed form⁹⁾.

Now the expression for the bending moment M , Eqs. (5a,b) with $H = 0$, may be written

$$M = k_\phi p R_0^2, \quad (23)$$

where the normalized bending moment k_ϕ is a function of $(\phi; b/R_0 \equiv \eta \equiv 1 - \xi; B\lambda_1/E_1A_1R_0^3)$ and is given by

$$\begin{aligned} k_\phi &= k_m + k_s [1 - (\xi \cos \phi + \eta \Delta) \cos \phi] \\ &\quad - \frac{1}{2} [1 - (\xi \cos \phi + \eta \Delta)^2]. \end{aligned} \quad (24)$$

Two particularly important cases are included:

- i) Free deformation of the cross-section. This implies $E_1 = 0$, and the normalized bending moment reduces to

$$k_\phi = k_m - \frac{1}{2} [1 - (\xi \cos \phi + \eta \Delta)^2], \quad (25)$$

where now $k_m = b_1/a_{11}$; k_ϕ is now a function of the position ϕ along the ring, and of b/R_0 .

ii) Rigid constraint of the minor axis. This implies $E_1 \rightarrow \infty$, and the normalized bending moment k_ϕ follows again from Eq. (24).

For comparison, for both the free tube and the rigidly supported tube k_ϕ is plotted as a function of ϕ for certain values of b/R_0 , giving the moment at any point along the profile, Figs. 3, 4. There, a uniform wall thickness has been assumed.

The expression for the normal force P , Eq. (3), can be evaluated in the same manner. In particular, we have at the major and minor axes

$$P(0) = pR_0 - S_e, \quad P(\pi/2) = pb, \quad (26)$$

respectively.

The stresses in the tube may finally be calculated from Eqs. (6). For a rough check of the stresses a good estimate is already obtained from k_ϕ .

A plot of the normalized deflection k_v is presented in Figs. 5 and 6, for both the free tube and the tube with rigid constraint; again, a uniform wall thickness has been assumed there.

5. RESULTS AND CONCLUSIONS

The program SUVAC (suspended vacuum chamber, see Appendix) has been established for the prediction of the deflection in the most interesting regions of the vacuum chamber, i.e. in the flat portion $\phi_0 < \alpha < \phi_1$, and at the point of suspension. Moreover, the program computes the values of the stresses at any point along the profile of the tube.

The program has been used for the prediction of stress and deflection of certain special vacuum chambers, as shown in Fig. 2, constructed for the West and North Experimental areas¹⁰⁾. There was rather good agreement between the results predicted for stress and deflection and actual measurements, see, for example, Fig. 7 (reproduced from Ref. 10). (In fact, the actual chambers had reinforcing ribs welded on to the tube in the region of the major axis of the cross-section; and the suspension straps were fixed at discrete points of the minor axis along the tube. Therefore, the actual chamber had to be replaced by a fictitious uniform tube where the discontinuities had been smeared out. The actual elasticity of the suspension had directly been taken into account.)

From the results for the chamber with profile according to Fig. 2 the following conclusions can be drawn:

- i) as in the case for corresponding elliptic profiles⁶⁾, for a free cross-section, i.e. without constraint, the maximum stresses occur always at the major axis ($\phi = 0$), and the stress values along the profile vary in a rather wide range. In the case of a rigid constraint the maximum stresses are much lower; they

occur at the minor axis ($\phi = \pi/2$), and there is not so much variation of the stress values along the profile. The wall thickness of the tube may then be considerably reduced.

- ii) For a given (major) diameter (say $2R_0$) of the chamber there exists a critical ratio of minor to major diameters b/R_0 [at about $(b/R_0)_{cr} \sim 0.35$ for the free tube, Fig. 5, and about $(b/R_0)_{cr} \sim 0.16$ for the tube with rigid constraint, Fig. 6] for which the deflection v in the flat portion of the tube is a maximum. Thus, starting with a circular tube^{*)}, $b/R_0 = 1$, the deflection increases for decreasing b/R_0 (i.e. increasing eccentricity), arriving at a maximum value for $(b/R_0)_{cr}$. For still lower values of b/R_0 (still higher eccentricity), the deflection decreases again.

Acknowledgement

We would like to thank Charles E. Rufer for his constant interest in the present work and for many valuable discussions.

*) We note that for a circular tube, even if its cross-section is free to deform, a zero deflection is predicted (in point $\phi = \pi/2$). This has to be expected, since bending cannot occur and since we have neglected from the outset any deformation due to circumferential stress. In fact, for practical elliptic or quasi-elliptic tubes the only significant contribution to the deflection is due to bending.

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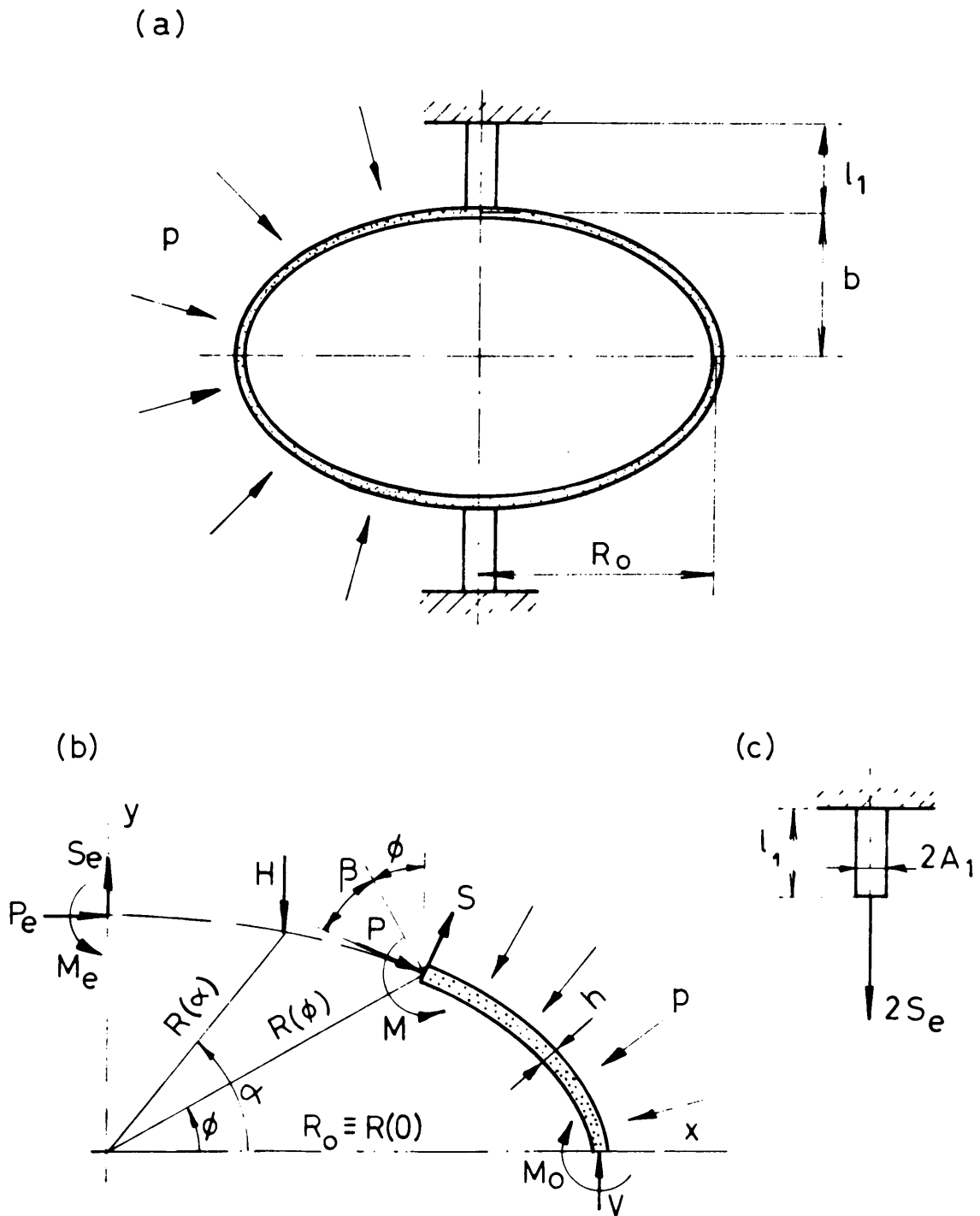


Fig. 1 a) Suspended tube under external pressure p ;
 b) shape of the median line of the cross-section, forces and moments acting on the ring;
 c) constraint force in suspension.

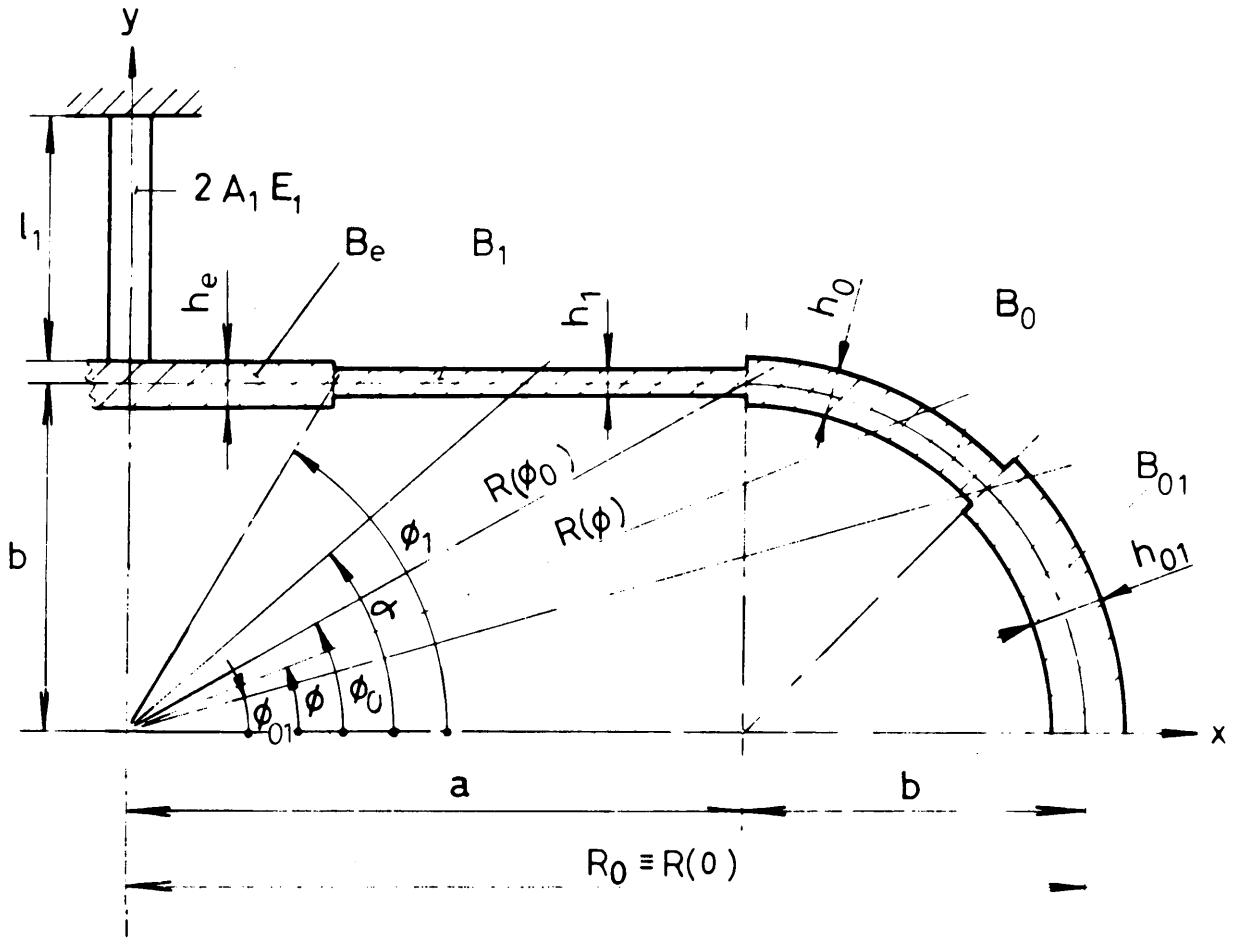


Fig. 2 Race-track-shaped tube with variable bending stiffness $EJ \equiv B(\phi)$.

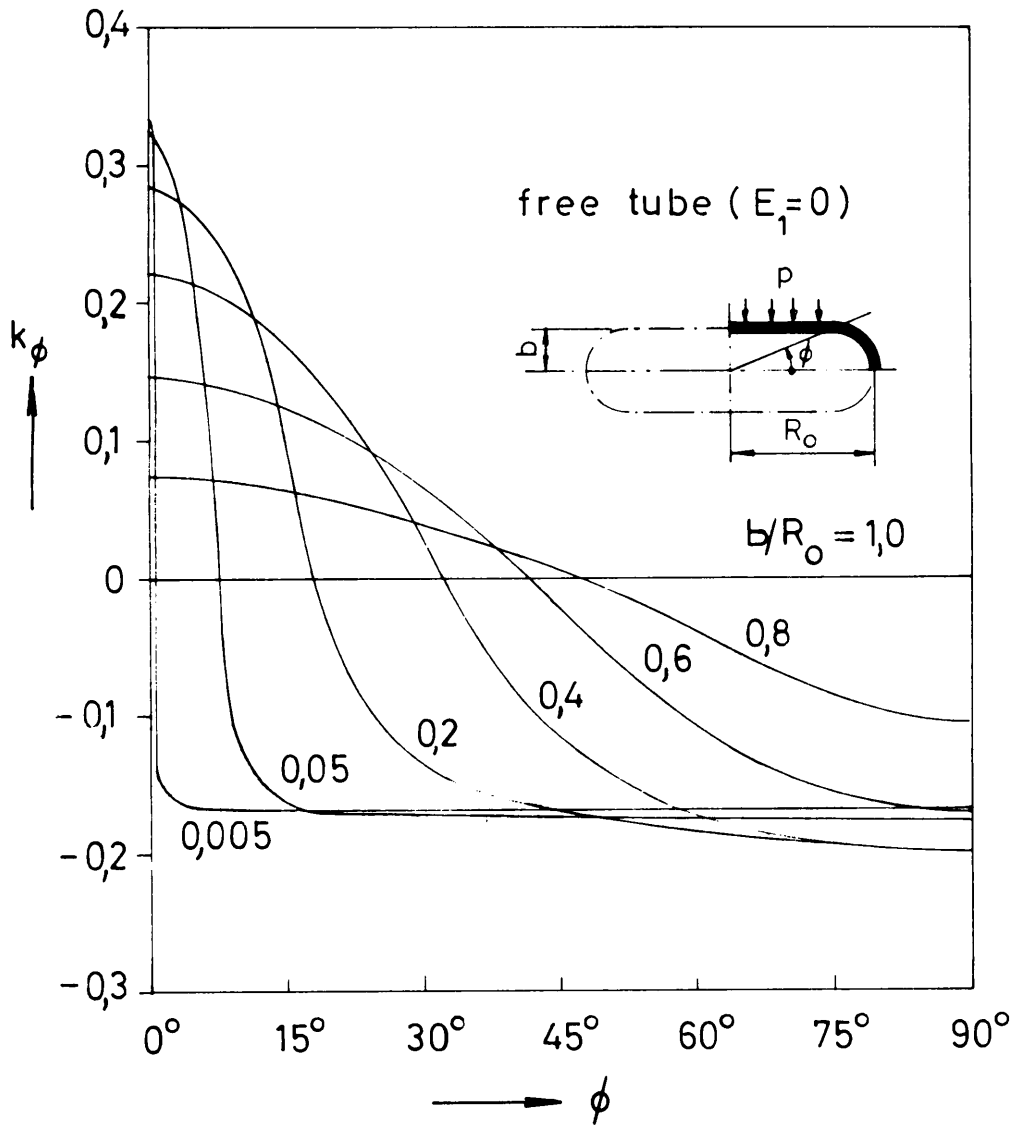


Fig. 3 Bending moment $k_\phi \equiv M_\phi / (pR_0^2)$ at intermediate points ϕ (free tube with uniform wall thickness).

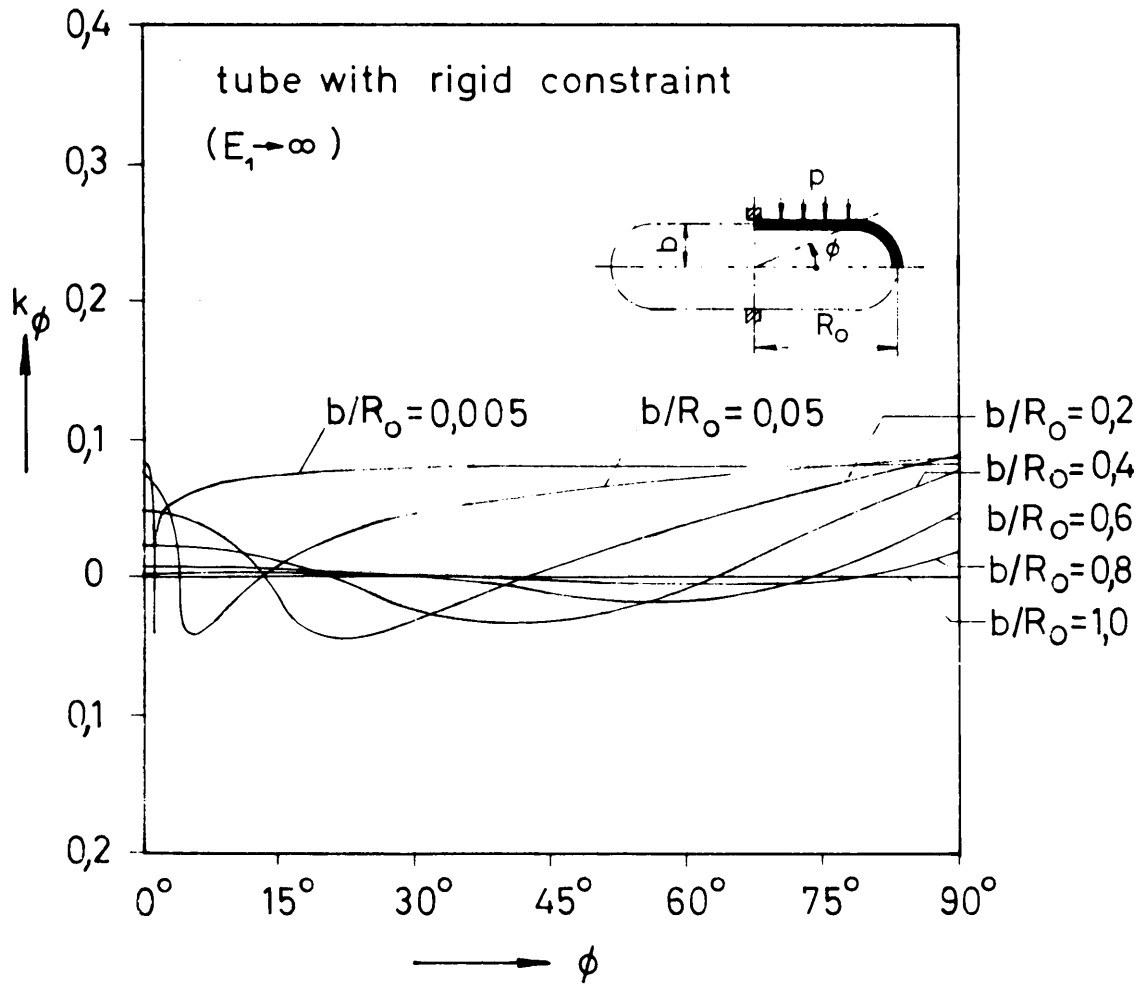


Fig. 4 Bending moment $k_\phi \equiv M_\phi / (pR_o^2)$ at intermediate points ϕ (rigidly constrained tube with uniform wall thickness).

free tube ($E_1=0$)

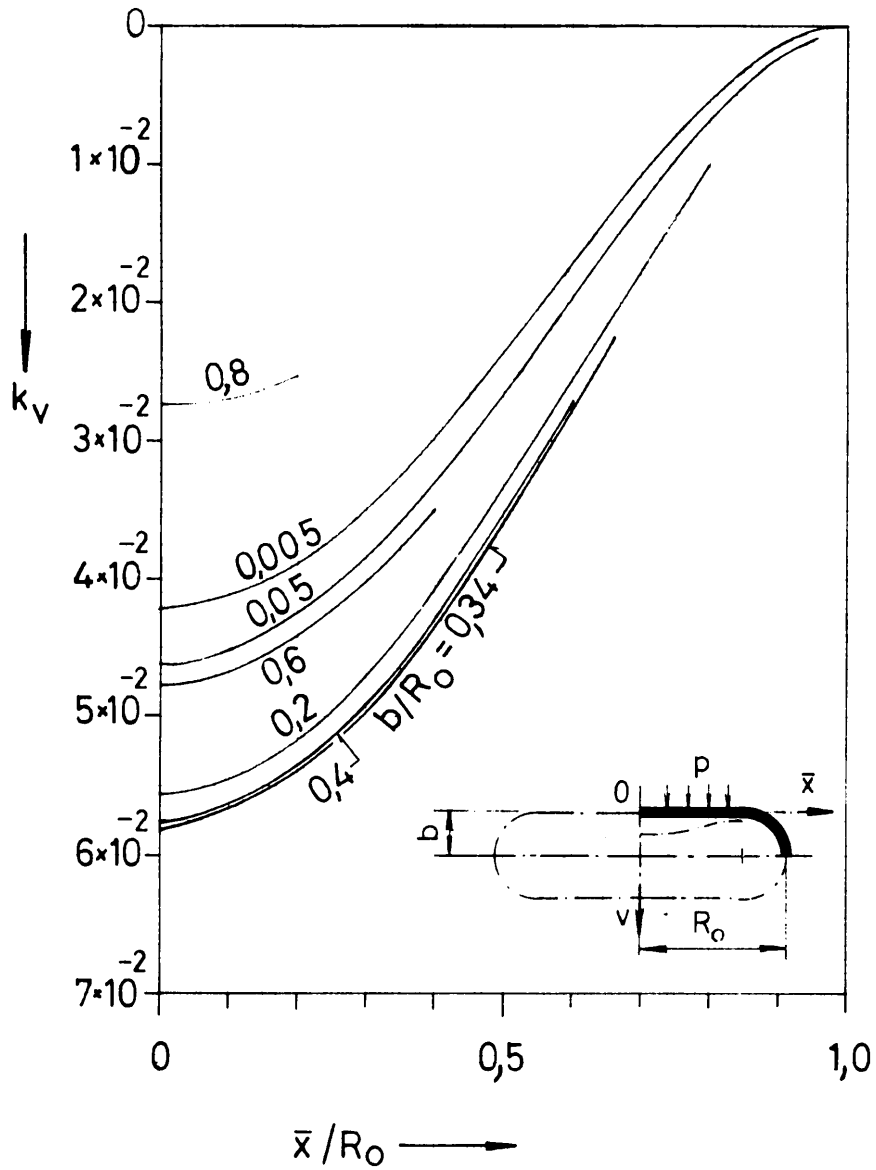


Fig. 5 Deflection $k_v \equiv vB_e/(pR_0^4)$ at intermediate points \bar{x}/R_0 (free tube with uniform wall thickness).

tube with rigid constraint ($E_1 \rightarrow \infty$)

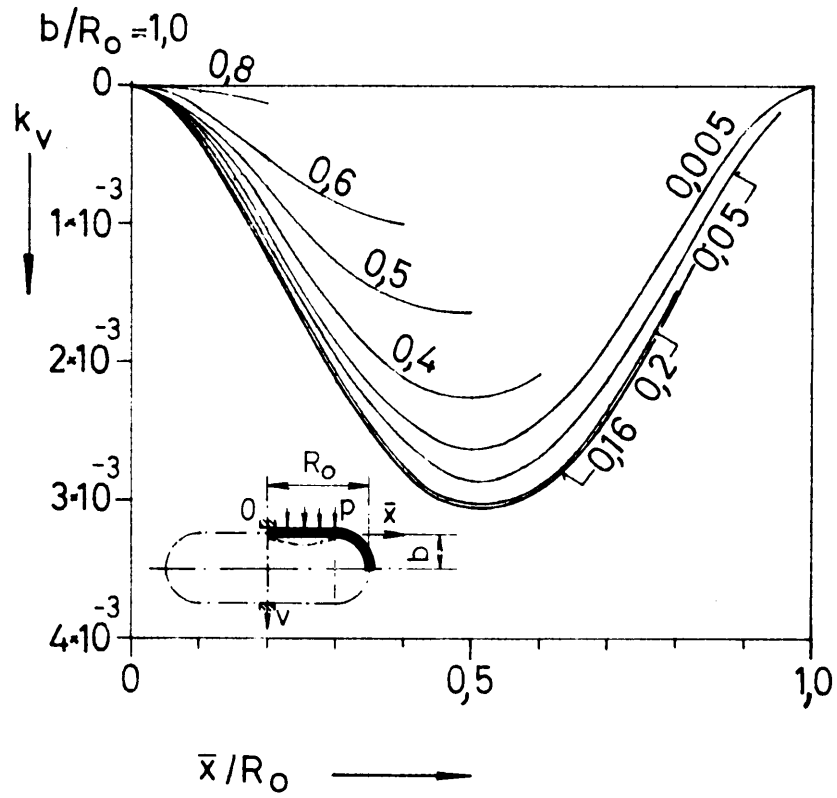


Fig. 6 Deflection $k_v \equiv vB_e/(pR_0^3)$ at intermediate points \bar{x}/R_0 (rigidly constrained tube with uniform wall thickness).

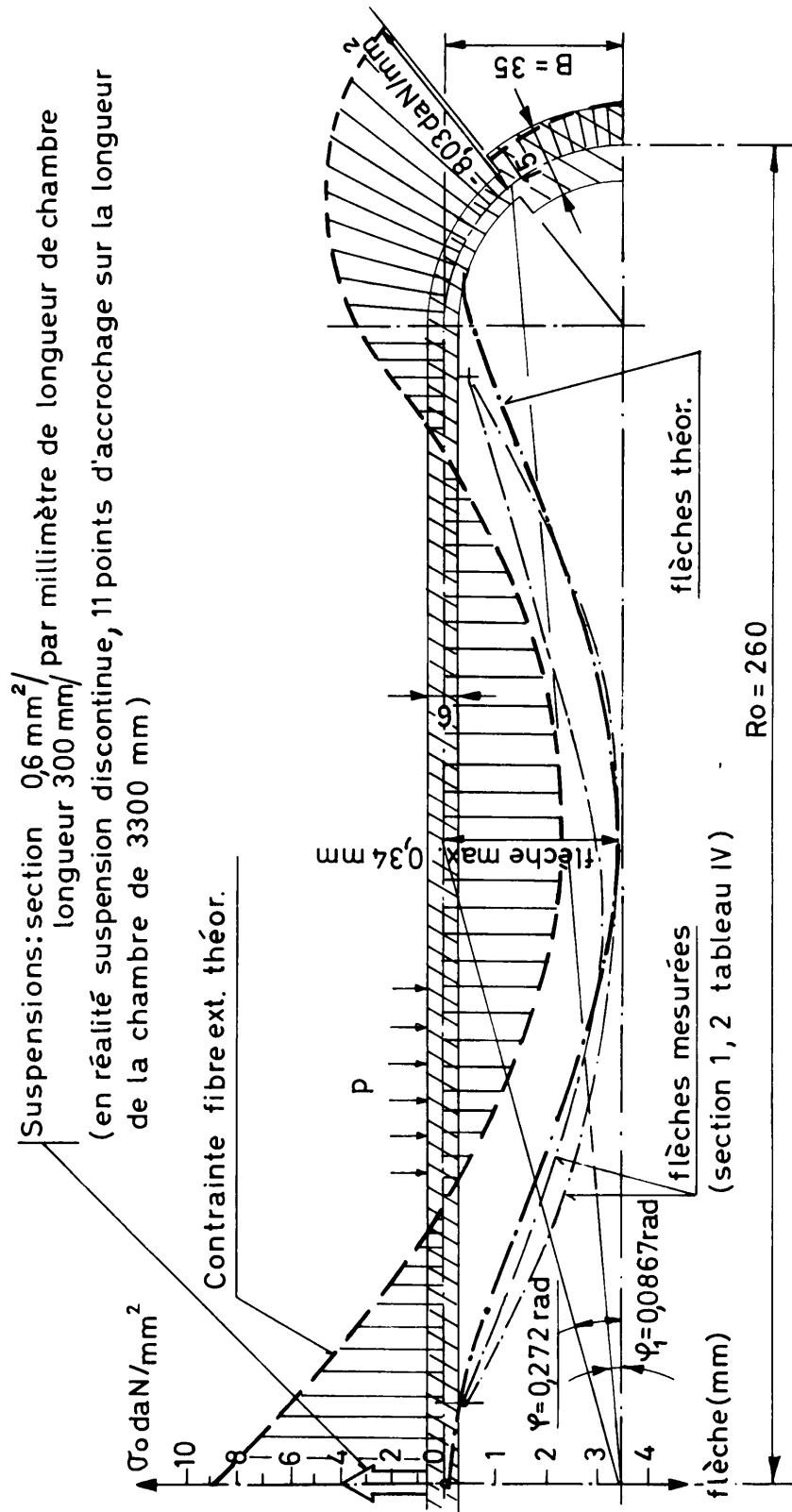


Fig. 7 Stress and deformation. Predicted deflection compared with actually measured deflection.

THE PROGRAM SUVAC

The program SUVAC (vacuum chamber suspended at a principal axis, stress and deflection analysis) is written for a vacuum chamber with piecewise constant wall thickness h_{01} , h_0 , h_1 , h_e , Fig. 2. Geometrical and material data (as well as pressure) must be given as shown on the two following pages for two typical cases. The output is self-explanatory.

PROGRAM SUVAC -VERSION MARCH 1977-

SUSPENDED VACUUM CHAMBER, STRESSES AND DEFLECTIONS

CASE NUMBER 2. (TUBE WITH RIGID CONSTRAINT)

SEMI MINOR AXIS B 50.00000 MM
 SEMI MAJOR AXIS RO 250.00000 MM
 ANGLE PH01 1.08673834 RAD
 ANGLE PH1 1.57079630 RAD
 WALL THICKNESS HO1 6.00 MM
 WALL THICKNESS HO 0.00 MM
 WALL THICKNESS H1 6.00 MM
 WALL THICKNESS HE 6.00 MM
 SECTION A1 100000.00 MM2
 LENGTH L1 0. MM
 ATM. PRESSURE P .01000 DAN/MM2
 SUSPENSION : MODULUS OF ELASTICITY E1 19000.00 DAN/MM2
 CHAMBER : MODULUS OF ELASTICITY E 19000.00 DAN/MM2

ANGLE PHI (RAD)	X (MM)	DEFLECTION (MM)
.2449786631	200.00000	.17096
.3775604268	126.07583	.34674
.5101421905	89.35825	.28327
.6427239542	66.77472	.20292
.775305179	51.01957	.13800
.9078874816	39.03882	.09073
1.0404692453	29.31782	.05578
1.1730510089	21.00690	.03074
1.3056327726	13.57791	.01365
1.4382145363	6.66821	.00348
1.5707963000	.00000	-.00000

ANGLE PHI	X (MM)	OUTSIDE STRESS	INSIDE STRESS	(DAN/MM2)
0.0000000		4.9464752	-5.3457842	
.0086738		4.9411141	-5.3402043	
.0173477		4.9250217	-5.3234550	
.0260215		4.8981697	-5.2955071	
.0346953		4.8605107	-5.2563109	
.0433692		4.8119767	-5.2057959	
.0520430		4.7524778	-5.1438686	
.0607168		4.6819004	-5.0704104	
.0693907		4.6001039	-4.9852753	
.0780645		4.5069181	-4.8882800	
.0867383		4.4021383	-4.7792295	

.0867383		4.4021383	-4.7792295	
.1025624		4.1802795	-4.5483152	
.1183864		3.9170066	-4.2743589	
.1342104		3.6098354	-3.9545877	
.1500345		3.2548288	-3.5850910	
.1658585		2.8465861	-3.1601853	
.1816825		2.3767971	-2.6712213	
.1975066		1.8319075	-2.1040913	
.2133306		1.1872595	-1.4331311	
.2291546		.3884862	-.6017549	
.2449787		-.7532620	.5865953	

.2449787	200.00000	-.7532620	.5865953	
.3775604	126.07583	-4.7982220	4.6315553	
.5101422	89.35825	-3.4219089	3.2552422	
.6427240	66.77472	-1.4593690	1.2927023	
.7753057	51.01957	.4131345	-.5798011	
.9078875	39.03882	2.1139634	-2.2806301	
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