EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/PS 91-27 (RF)

MIXED ION BEAMS NEAR TRANSITION ENERGY

S. Hancock

Paper presented at the
IEEE 1991 Particle Accelerator Conference
6-9th May, 1991, San Francisco, U.S.A.

Mixed Ion Beams Near Transition Energy

S. Hancock
CERN, CH-1211 Geneva 23

Abstract The standard derivations of the energy and phase of the synchronous particle in a proton accelerator assume, as if by definition, that said synchronous particle lies on the central orbit of the machine. This is manifestly unjustified in the particular case of the acceleration near transition of a mixture of ions, when a small difference in charge-to-mass ratio can produce a large radial separation of the different ion species. The development of a simple derivation of the parameters of the synchronous particle that involves no such *a priori* constraint has yielded some surprises; not least, a belated explanation for an apparent anomaly encountered in 1987, when a mixture of oxygen and sulphur ions was accelerated in the CERN Proton Synchrotron for the first time. These ideas are supported by measurements performed in 1990 during a second ion run.

1 Preamble

In the autumn of 1987, sulphur ions were accelerated at CERN to a world-record energy of 6.4 TeV. The $^{32}S^{16+}$ ions were supplied by an electron cyclotron resonance source which also produced $^{16}O^{8+}$ ions. Significantly, however, the oxygen ions constituted a ninety percent "contamination" of the total beam current. Since the relative charge-to-mass ratio difference of $^{16}O^{8+}$ with respect to $^{32}S^{16+}$ is only about -5.4×10^{-4}, the oxygen was essentially indistinguishable from sulphur throughout the acceleration process, with the exception of the region near transition. It was near transition in the PS that the oxygen was eliminated, albeit with some difficulty initially. It proved far more straightforward to remove the sulphur when the oxygen was required for setting-up.

The "asymmetry" of the selection process could not be satisfactorily explained. Despite the obvious fact of the beam control system being dominated by the oxygen, there seemed to be no convincing mechanism to cause an RF system which included a phase loop but no radial feedback to behave differently in the two cases.

2 The Synchronous Particle

According to the "standard creed", the momentum of the synchronous particle is given by the fundamental guide field relation

$$p_{\mathrm{nom}} = q_{\mathrm{ion}} B_z \rho_{\mathrm{nom}} \tag{1}$$

Here, q_{ion} is the particle's charge and ρ_{nom} is its radius of curvature in the dipole magnetic field, B_z, of the machine. For the suffix nom to denote parameters which pertain to a central orbit of mean radius R_{nom} presupposes the ideal RF (angular) frequency programme

$$\omega_{\mathrm{RF}}(B_z) = \frac{hc}{R_{\mathrm{nom}}} \left[1 + \left(\frac{E_{\mathrm{rest,ion}}}{q_{\mathrm{ion}} B_z \rho_{\mathrm{nom}} c} \right)^2 \right]^{-1/2} \tag{2}$$

where $E_{\mathrm{rest,ion}}$ is the rest-mass energy of the particle, c is the speed of light *in vacuo* and where the harmonic number, h, may be any positive integer. The corresponding synchronous phase (with respect to the zero-crossing of the RF), φ_{nom}, is given by

$$\sin \varphi_{\mathrm{nom}} = \frac{2\pi R_{\mathrm{nom}} \rho_{\mathrm{nom}}}{\hat{V}_{\mathrm{RF}}} \frac{dB_z}{dt} \tag{3}$$

where $\hat{V}_{\mathrm{RF}}$ is the total (peak) RF voltage and t is time.

A full derivation of the parameters of the synchronous particle for arbitrary ω_{RF} is given elsewhere [1]. The starting point is a more general form of (1):

$$p_0 = q_{\mathrm{ion}} B_z \rho_{\mathrm{nom}} \left(\frac{R_0}{R_{\mathrm{nom}}} \right)^{1/\alpha} \tag{4}$$

where the synchronous particle is now denoted by suffix zero and α is the momentum compaction factor. It follows that the synchronous energy, E_0, is obtained by solving

$$E_0 \left(E_0^2 - E_{\mathrm{rest,ion}}^2 \right)^{(\alpha-1)/2} = \frac{hc}{\omega_{\mathrm{RF}} R_{\mathrm{nom}}} \left(q_{\mathrm{ion}} B_z \rho_{\mathrm{nom}} c \right)^{\alpha} \tag{5}$$

While the synchronous phase is given by

$$\sin \varphi_0 = \frac{2\pi h}{q_{\mathrm{ion}} \hat{V}_{\mathrm{RF}} \omega_{\mathrm{RF}}} \frac{dE_0}{dt} - \frac{2\pi R_0}{\hat{V}_{\mathrm{RF}}} \mathcal{E}_\theta(R_0) \tag{6}$$

where $\mathcal{E}_\theta$ is the azimuthal component of the electric field which is produced in accordance with Faraday's Law by the time-variation of the magnetic field. The second, "betatron acceleration" term is hereafter neglected as small.

When the special case of an ideal frequency programme is considered by substituting (2) into (5), two roots are found. One solution corresponds to $R_0 = R_{\mathrm{nom}}$ and is thus consistent with (1) and (3). The other lies on the opposite side of transition and, except very near to transition, would fall well outside the physical aperture of a real machine.

3 Mixtures of Ions in the PS

An Open-loop Analysis

The acceleration of particles other than the usual protons requires the modification of the RF frequency programme to cater for different charge-to-mass ratios. In the

PS, this is achieved by the introduction, on a cycle-to-cycle basis, of a rate multiplication factor, b, which modifies the so-called B-train measurement of the dipole magnetic field. Thus, in the absence of feedback, the RF frequency would be generated as a slightly modified form of (2):

$$\omega_{\mathrm{RF}}(B_z;\, b) = \frac{hc}{R_{\mathrm{nom}}}\left[1+\left(\frac{E_{\mathrm{rest,p}}}{q_{\mathrm{p}} b B_z \rho_{\mathrm{nom}} c}\right)^2\right]^{-1/2} \tag{7}$$

with, ideally,

$$b = \frac{q_{\mathrm{ion}}/E_{\mathrm{rest,ion}}}{q_{\mathrm{p}}/E_{\mathrm{rest,p}}} \tag{8}$$

Consequently, (5) becomes

$$f_1(E_0;\ \mathrm{ion}) = f_2(B_z;\, b) \tag{9}$$

where

$$f_1(E_0;\ \mathrm{ion}) = q_{\mathrm{ion}}^{-\alpha} E_0 \left(E_0^2 - E_{\mathrm{rest,ion}}^2\right)^{(\alpha-1)/2} \tag{10}$$

is determined by ion species and

$$f_2(B_z;\, b) = \left(B_z \rho_{\mathrm{nom}} c\right)^{\alpha}\left[1+\left(\frac{E_{\mathrm{rest,p}}}{q_{\mathrm{p}} b B_z \rho_{\mathrm{nom}} c}\right)^2\right]^{1/2} \tag{11}$$

is a function only of the free variable B_z. Furthermore, assuming $\alpha = \gamma_{\mathrm{tr}}^{-2}$ to be invariant,

$$\sin\varphi_0(B_z;\ \mathrm{ion},b) = \frac{2\pi R_{\mathrm{nom}}}{q_{\mathrm{ion}} \hat{V}_{\mathrm{RF}} c}\left[1+\left(\frac{E_{\mathrm{rest,p}}}{q_{\mathrm{p}} b B_z \rho_{\mathrm{nom}} c}\right)^2\right]^{1/2} \times \frac{dB_z}{dt}\,\frac{f_2'(B_z;\, b)}{f_1'(E_0;\ \mathrm{ion})} \tag{12}$$

Equations (9) and (12) provide the synchronous energy and phase, respectively, for a given ion species under open-loop conditions. That is, the RF frequency of (7) is not considered to be influenced by feedback. However, it is a strength of the approach developed thus far that this frequency is not assumed to be ideal, so that b, although a fixed parameter, may be chosen freely.

Oxygen and Sulphur

The stage is now set for the (numerical) solution of (9) for $^{16}O^{8+}$ and $^{32}S^{16+}$ ions for the various permutations of ion species and b. The result is shown in fig.1, first in the case when b is calculated for oxygen and then in the case when the RF frequency programme for sulphur is chosen. Both cases exhibit a monotonically increasing solution for $\gamma_0 = E_0/E_{\mathrm{rest,ion}}$ as a function of B_z for the ion species which corresponds to the RF programme; this is the "standard creed". There is also a decelerating solution for that species. However, for the other species, the solutions in the two cases are very different. For sulphur ions on the oxygen programme, the synchronous energy solutions above and below transition never meet. While for oxygen ions on the sulphur programme, there is a period during which there is no solution for the synchronous energy.

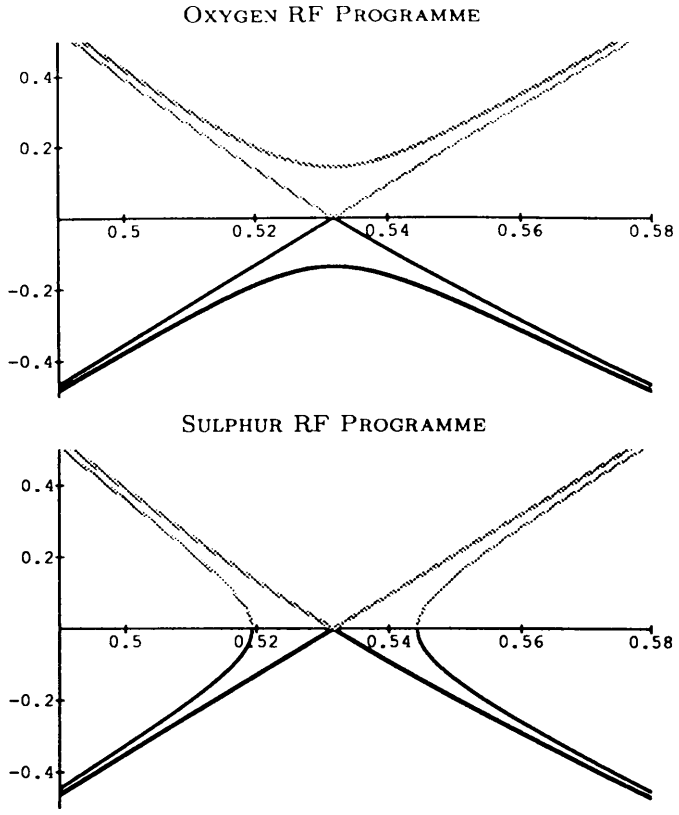

Figure 1: $(\gamma_0 - \gamma_{\mathrm{tr}})$ vs. $B_z/[\mathrm{T}]$ for $^{16}O^{8+}$ (thin lines) and $^{32}S^{16+}$ (thick lines) ions in the PS. (Dark/light lines correspond to synchronous particles which are below/above transition.)

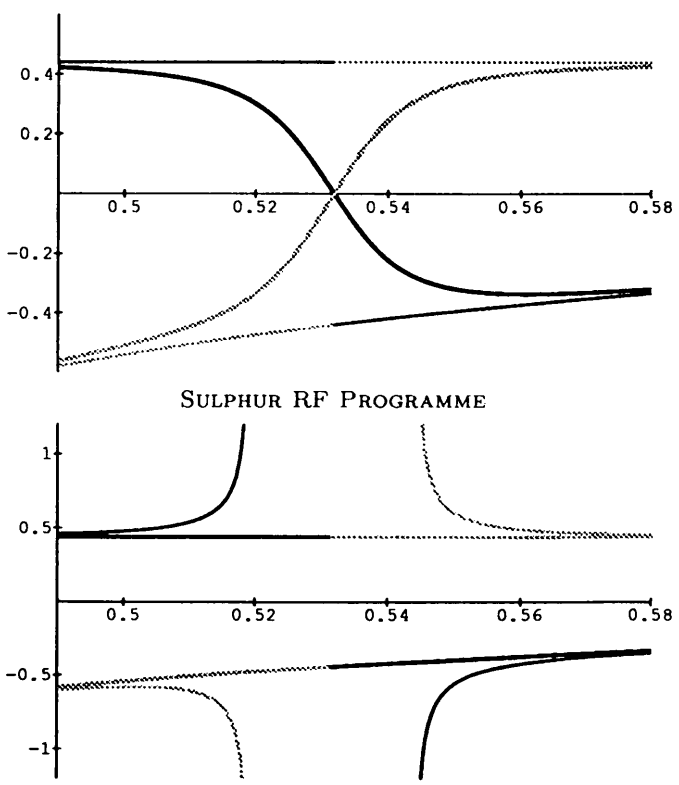

Figure 2: $\sin\varphi_0$ vs. $B_z/[\mathrm{T}]$ for $^{16}O^{8+}$ (thin lines) and $^{32}S^{16+}$ (thick lines) ions in the PS. (Dark/light lines correspond to synchronous particles which are below/above transition.)

The (scaled) synchronous energy solutions of fig.1 are readily converted to the corresponding mean orbital radii using (4). For sulphur ions on the oxygen programme, the synchronous particle orbit (approaching transition from below) heads towards the inside of the machine. However, at the point when oxygen reaches transition, this radial excursion for sulphur is no greater than that of the one in the opposite direction for oxygen on the sulphur programme. Thus, the "symmetry" of the radial separation persists.

The picture is completed by fig.2, which shows the solutions of (12) for the two RF programmes. On the oxygen programme, the sulphur buckets are stationary when oxygen crosses transition. While for oxygen ions on the sulphur programme, φ_0 becomes imaginary and the oxygen bucket collapses even before the solution for E_0 disappears.

The response of a phase loop to this longitudinal separation of the two species would depend upon their contributions to the total beam current. If the beam were dominated by its oxygen content, sulphur ions accelerated on the oxygen programme would head off towards 0° largely unnoticed. However, any migration of oxygen ions towards 90° on the sulphur programme would be opposed by the phase loop. Then, depending upon the gain and time-constant of the loop, the sulphur programme would tend to be made to resemble the oxygen one. Thus, a mechanism has been found which could account for the "asymmetry" observed when it was required to remove the oxygen ions.

Experimental Observations

The predicted longitudinal separation of $^{16}O^{8+}$ and $^{32}S^{16+}$ ions near transition is indeed observed. Fig.3 shows a series of longitudinal beam profile measurements that was obtained using a fast, resistive-wall pick-up when the RF system of the PS had been optimized for sulphur. These measurements were triggered by the RF at times successively closer to the transition phase jump, the last profile being triggered 8 ms later than the first (although, necessarily, on a different cycle). The inference from the previous section is that, here, the oxygen ions are prevented from heading towards 90° by the action of the phase

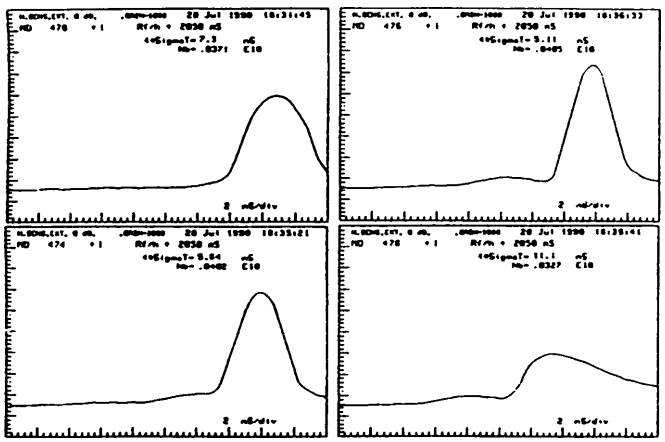

Figure 3: Longitudinal beam profiles taken on the RF programme for sulphur.

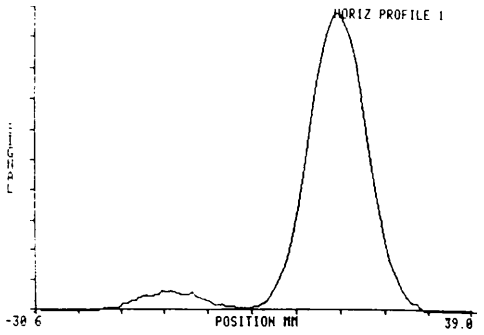

Figure 4: Horizontal beam profile taken on the RF programme for sulphur. The centre of the aperture corresponds to a measured position of zero.

loop. Thus, the net result of the expected longitudinal separation is the appearance of a second, much less intense bunch at earlier times with respect to the trigger as the sulphur is displaced to lower RF phase.

An additional feature of fig.3 is that the oxygen debunches. This could be indicative of the collapse of the oxygen bucket as the synchronous particle for oxygen reaches transition.

The radial separation of the two species is also observed. Fig.4 shows a horizontal beam profile measurement that was obtained using a flying-wire scanner when the RF system had been optimized for sulphur. The timing of this measurement corresponds approximately to that of the second frame of fig.3, but such a scan is made over many turns and is not a "snapshot" of a single bunch. The dominant oxygen profile appears at larger radial position. Significantly, the sulphur lies inside the central orbit, apparently having been displaced by the action of the phase loop. Operationally, a perturbation is added to the RF phase programme to counteract this effect.

Similar pictures to figs.3 and 4 are obtained when the RF system is optimized for oxygen, except that the oxygen does not debunch and remains closer to the centre of the aperture while the sulphur heads towards the inside of the machine. Operationally, no phase perturbation is required to eliminate the sulphur ions.

4 Conclusions

A simple derivation has been developed which yields the synchronous particle parameters without the tacit assumption of an idealized RF frequency.

Where they may be compared, the results obtained are entirely in agreement with those of more familiar derivations, except that a synchronous partner is revealed on the other side of transition. The predicted separation, both radially and longitudinally, of oxygen and sulphur ions as they approach transition are observed effects and it is the latter which explains why the dominant species is more difficult to eliminate from the mixture.

Reference [1] S. Hancock, "Transition Revisited", CERN PS/90-53 (1990).