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## PROJECT FOR A VARIABLE CURRENT ELECTRON GUN FOR THE LEAR ELECTRON COOLER

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### ABSTRACT

A new variable current electron gun has been developed in order to improve the performance of the LEAR Electron Cooler. The present gun is of the resonant type and therefore provides no operational flexibility although necessitating relatively low magnetic fields. The new gun will be of the adiabatic type. Consequently, it works at a high magnetic field and it will allow an on-line control of the electron beam intensity while ensuring low transverse and longitudinal temperatures. The present paper is a theoretical and numerical investigation of the planned adiabatic gun for which the design, the construction and the initial tests have been entrusted to the Centre of Applied Physics and Technology (CAPT), a Department of the Institute for Nuclear Physics (INP) at Novosibirsk.

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#### 1. INTRODUCTION

Electron cooling has been described in many papers like those mentioned in Refs. [1, 3]. Amongst all the constitutive components of this type of cooler the gun plays an important role. From its properties, and essentially the way it can provide energetic high-intensity electron beams with small rms velocity spreads, depend the final properties of the cooled ion beam [4].

Many investigations and studies have been made on this subject. The one presently installed on the LEAR electron cooler is of resonant type [3, 6]. It provides rather small electron transverse velocities at low magnetic fields to the prejudice of flexibility when operating in the frame of a variable energy accelerator.

In order to obtain the required flexibility, the use of adiabatic guns is being more and more considered by the users [5]. This type of gun, however, makes use of relatively high-magnetic fields (of the order of 0.1 to 0.2 T), which is a drawback in the case of cooling low-momentum (<100 MeV/c) ions [3].

The gun that will be described in this paper is of the adiabatic type but using magnetic fields a factor of two smaller than what is usually used for this type of gun. This lowering of the operational magnetic field was made possible due to the appropriate optimization of the form of the anodes in order to reach the same low transverse velocities obtained with the other guns.

The present paper is a theoretical and numerical investigation on an adiabatic gun requiring relative low-magnetic fields. Sections 2 and 3 may be skipped by readers already familiar with high-intensity guns. In Section 4 we give a more physical than mathematical approach of the resonant and adiabatic guns. The next sections are devoted to the results obtained by numerical simulations and to some technical problems.

## 2. AIM OF THE NEW GUN

With a view to improve the performance of LEAR electron cooler (ECOOL) a variable intensity gun is foreseen. The design, construction and test of this device has been entrusted to the Centre of Applied Physics and Technology (CAPT, Lipetsk) which is a branch of INP at Novosibirsk.

According to the experience gained with the present cooler [7] it would be desirable to be able to:

- switch the electron beam ON and OFF without disturbing the ion beam itself and the vacuum pressure,
- adjust the electron beam intensity, in a smooth manner, in order to obtain the shortest cooling time,
- adjust the cooling force so that to stay within the limit of stability since, when reaching a threshold of density, the ion beam tends to become unstable.

2

The latter two requirements cannot be fulfilled with the present gun which operates at well defined intensities and cooling forces.

## 2.1 The Cooling Force

A rough expression of the cooling force is given by:

$$\vec{F} = -Const \times n_e^* \int \frac{\vec{v}_i^* - \vec{v}_e^*}{\left|\vec{v}_i^* - \vec{v}_e^*\right|^3} f(v_e^*) dv_e^{*3}$$
(1)

where :

 $n_e^*$  is the electron beam density in m<sup>-3</sup>.

 $\vec{v}_i^*$  and  $\vec{v}_e^*$  are the ion and electron velocities in a frame  $R_0$  moving at the nominal velocity  $\vec{v}_0 = c\vec{\beta}_0$  and  $\vec{v}_0 = \langle \vec{v}_i \rangle$ .

 $f(v_e^*)$  is the electron velocity distribution.

The symbol (Figs. 1a and 1b)

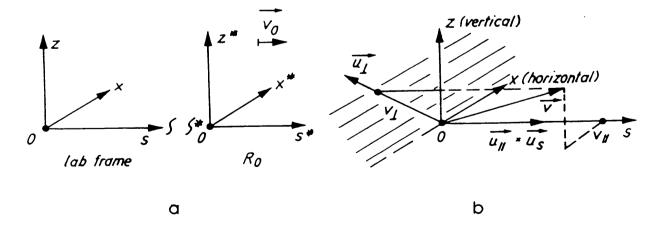
\* is related to the moving frame  $R_0$  (if omitted, one operates in the laboratory frame),

is related to the components in the parallel plane (0,s,x),

is related to the components in the transverse plane (0,x,z).

so that one may write :

$$\vec{v} = v_{||} \vec{u}_s + v_{\perp} \vec{u}_{\perp}$$
$$\vec{v}^* = v_{||}^* \vec{u}_{||} + v_{\perp}^* \vec{u}_{\perp}$$
$$\vec{v}_0 = v_0 \vec{u}_s = v_0 \vec{u}_{||}$$





Usually  $f(v_e^*)$ , the electron velocity distribution, can be expressed in a Gaussian form:

$$f(v_e^*) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\left(\Delta_{e_\perp}^*\right)^2 \Delta_{e^{\parallel}}} \exp\left[-\frac{1}{2} \left(\frac{v_{e^{\parallel}}^{*2}}{\left(\Delta_{e^{\parallel}}^*\right)^2} + \frac{v_{e_\perp}^{*2}}{\left(\Delta_{e_\perp}^*\right)^2}\right)\right]$$
$$1 = \int f(v_e^*) dv_e^3, \quad \Delta_{e_{\parallel}}^* << \Delta_{e_\perp}^*$$

Considering Eq. (1) one sees that the cooling force is enhanced, and therefore the cooling time reduced when :

- the electron-beam density  $n_e^*$  is increased,
- the electron-beam rms velocities  $\Delta_{e_{\parallel}}^{*}$  and  $\Delta_{e_{\perp}}^{*}$  are small with respect to the ion velocity components; the ideal case being:

$$f(v_e^*) = \delta(v_{e_{\parallel}}^*, v_{e_{\perp}}^*)$$

Therefore, an adequate control of  $n_e$  and the achievement of a very cold electron beam  $(\Delta v_{e\parallel} \text{ and } \Delta v_{e\perp} \text{ very small})$  are some of the necessary ingredients for a good electron-cooling process.

This, apart from the gun reliability problems that could arise due to a complicated cooling circuit and delicate high-voltage feedthroughs, justifies the need for a new electron gun which is the subject of the next sections.

#### 3. SOME FUNDAMENTAL PROPERTIES OF THE ELECTRON BEAM AND GUN

#### 3.1 Principle of the Gun

The principle of the present gun is shown in Fig. 2.

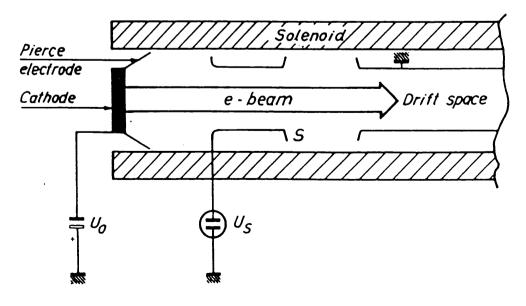


Fig. 2 - Principle of the electron gun

The electrons are created by a cathode of radius a = 2.5 cm, heated at temperature  $T_0$ . The cathode, at potential  $-U_0$ , with respect to the ground, is surrounded by a focusing electrode, the Pierce electrode.

The electrons on their way towards the drift space (where they mix with the ions to be cooled), at ground potential, passes through a steering electrode "S" at potential  $U_s$ . The ensemble has a symmetry of revolution. At the end of the gun, which is the entrance of the drift space, the electrons have a velocity  $v_0$ , such as:

$$eU_0 = (\gamma_0 - 1)mc^2 \approx \frac{1}{2}mv_0^2, \quad \vec{v}_0 = v_0\vec{u}_s = \vec{\beta}_0c, \quad \gamma_0 = (1 - \beta_0^2)^{-1/2}$$

where e and m are the electron charge and mass, respectively.

The overall (gun + drift space) is embedded in a solenoid giving a longitudinal magnetic field  $\vec{B}_0 = B_0 \vec{u}_s$ . The magnetic field will induce, on electrons having a transverse velocity  $v_{\perp}$ , a spiralling trajectory of angular velocity



with cyclotron radius

$$\rho_{\perp} = \frac{v_{\perp}}{\omega_H} = \frac{p_{\perp}}{eB_0}$$

where  $p_{\perp}$  is the electron transverse momentum. We can also define an -improperly- called Larmor radius  $\rho_l$  (Fig. 3)

Fig. 3

271 91

$$\lambda = 2\pi\rho_l = v_0 \frac{2\pi}{\omega_H}$$

The current density emitted by the cathode at temperature  $T_c$  is given by the Richardson-Dushman equation :

$$J = \operatorname{Const} T_c^2 \exp\left(-\frac{W_{\varphi}}{kT_c}\right)$$

where:

 $W_{\varphi}$  the extraction work,

k the Boltzmann constant;  $k = 1.38 \times 10^{-23}$ ,  $J \cdot K^{-1} = 8.618 \times 10^{-5} \text{ eV} \cdot \text{K}^{-1}$ .

The longitudinal and transverse electron rms energies are each about  $kT_c$  so that, by analogy with thermodynamics, we can state that, at the cathode output :

$$kT_c = m \left\langle v_{c_{\parallel}}^2 \right\rangle = m \left\langle v_{c_{\perp}}^2 \right\rangle$$

Example: if  $T_c = 1\ 000$  K,  $kT_c = 8.6 \times 10^{-2}$  eV.

#### 3.2 Influence of the Steering Electrode

The gun operates in space-charge regime, which means that the electron-beam current is related to the voltages by:

$$I_0 = p_g (|U_0 - U_s|)^{3/2} = p_g (U_t)^{3/2}, \quad [A]$$
$$U_t = |U_0 - U_s|$$

where  $p_g \approx 0.5 \times 10^{-6} \text{ AV}^{-3/2}$  is the perveance determined by geometrical considerations and the voltages  $U_0$  and  $U_s$  are referred to the ground.  $U_t$  is therefore the voltage difference between the cathode and the steering electrode.

For the theoretical model consisting of infinite plane electrodes, distant from each other by d, the current density is:

$$J = \frac{4\sqrt{2}}{9} \frac{\varepsilon_0}{d^2} \sqrt{\frac{l}{m}} U_t^{3/2} = \frac{2.33 \times 10^{-6}}{d^2 (cm^2)} U_t^{3/2}$$

In the case of the new gun with an e-beam having a radius a = 2.5 cm, one gets d = 10 cm when  $p_g = 0.5 \times 10^{-6}$ . This is quite in agreement with the mechanical layout given by Fig. 6.1a.

At this level, it is useful to introduce a new parameter  $p_b$ , also expressed in A·V<sup>-3/2</sup>, such that:

$$I_0 = p_b U_0^{3/2}$$

 $p_b$  which can take its values in the range of  $[0.125 \times 10^{-6}, 5 \times 10^{-6}]$  is a function of  $U_s$  and must not be confused with  $p_g$  which is constant.

For a fixed electron kinetic energy  $E_{c0}$  determined more or less by  $U_0$ , since  $E_{c0} \equiv eU_0$ , the steering-electrode voltage will fix the current  $I_0$ . For practical reasons we have limited  $I_0$  to 3 A and  $U_0$  to 30 kV.

Having a radial symmetry this electrode will induce a radial perturbing electrical field  $\vec{E_{el}}(r,s)$ , dependent on r and s, and therefore a radial force acting on the electrons:

$$\vec{f}_{el} = -e\vec{E}_{el}$$

(See Fig. 4 for symbols).

### 3.3 Influence of the Space Charge

We consider the cylindrical coordinate system of Fig. 4, where the position is given by:

$$O\vec{M} = r\vec{u}_r + s\vec{u}_s$$

and the velocity by:

$$\vec{v} = \frac{dOM}{dt} = v_r \, \vec{u}_r + v_\varphi \, \vec{u}_\varphi + v_s \, \vec{u}_s = v_\perp \, \vec{u}_\perp + v_{\rm H} \, \vec{u}_{\rm H}; \quad (v_s = v_{\rm H}).$$

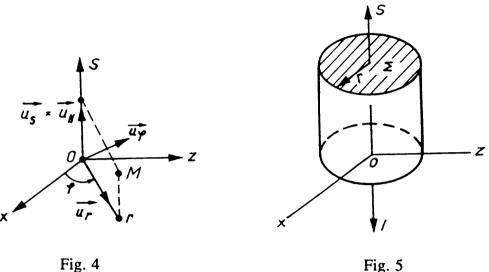


Fig. 5

Inside the electron beam, at a point where the actual velocity is v ( $0 < v < v_0$ ), the current density is :

$$\vec{J} = -n_e e \vec{v}$$
.  
Note that  $v_{\perp} \ll v_s$ , so we can take  $v \equiv v_s = v_{\parallel}$  and  $\beta = v_s / c \equiv v / c$ .

The intensity passing through the surface  $\Sigma$  is: (Fig. 5)

$$I = \vec{J} \cdot \vec{\Sigma} = n_e e v_s \, \pi r^2$$

Since for r = a, the electron-beam radius,  $I = I_0$ , the electron-beam density will be:

$$n_e = \frac{I_0}{e v \pi a^2} = \frac{p_b U_0^{3/2}}{e \beta c \pi a^2}, \, \mathrm{m}^{-3}$$
(2)

#### 3.3.1 Radial electrical field

Using Gauss Theorem the radial electrical field is:

$$\vec{E} = -\frac{en}{2\varepsilon_0} r \vec{u}_r \tag{3}$$

where  $n = n_e - n_+$ ,  $n_+$  being the density of the motionless positive charges inside the electron beam mainly due to the ionization of the residual molecules by the electrons.

7

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The radial force applied to an electron at radius r is therefore:

$$\vec{f}_1 = \frac{e^2 n}{2\varepsilon_0} r \vec{u}_r$$

For a neutralized beam, n = 0, this force cancels while it is maximum when  $n = n_e$ .

## 3.3.2 Axial magnetic field

Using Ampere Theorem the magnetic field due to the electron current itself is:

$$\vec{B} = -\frac{\mu_0 I}{2\pi r} \vec{u}_{\varphi} = -\frac{e\mu_0 n_e \beta cr}{2} \vec{u}_{\varphi}$$

Thus, the force acting on one electron is:

$$\vec{f}_2 = -e(\vec{v} \times \vec{B}) = e(v_s B_{\varphi} \vec{u}_r - v_r B_{\varphi} \vec{u}_s)$$

## 3.3.3 Total space-charge force

The total force due to the space charge  $\vec{f_{sp}} = \vec{f_1} + \vec{f_2}$ 

$$\vec{f}_{sp} = \frac{e^2 n_e}{2\varepsilon_0} r \left(\frac{n}{n_e} - \beta^2\right) \vec{u}_r + \frac{e^2 \mu_0 n_e v_r \beta c}{2} r \vec{u}_s \quad [\text{m} \cdot \text{kg} \cdot \text{s}^2]$$

## Remarks

- If the electron beam is neutralized, n = 0, then the radial force is reduced.
- When  $n = n_e$  the magnetic force is  $\beta^2$  times smaller than the electrical force. In our case  $\beta^2 < 0.1$ .
- The space-charge force is proportional to r.
- A longitudinal component exists which will not be considered in this report.

## 3.4 Influence of the Longitudinal Magnetic Field

The force due to the ideal solenoidal field  $\vec{B}_0 = B_0 \vec{u}_s$  is:

$$\vec{f}_m = -e(\vec{v} \times \vec{B}_0) = -ev_{\varphi}B_0\vec{u}_r + ev_rB_0\vec{u}_{\varphi}$$

#### 4. **REVIEW OF THE DISTURBANCES**

As mentioned in section 3.1, the rms velocity spread components at the cathode output are each of the order of  $kT_c$ .

At the gun output the electron velocity can be written in the following form:

$$\vec{v} = \left(v_s + dv_{e_{\parallel}}\right)\vec{u}_s + dv_{e_{\perp}}\vec{u}_{\perp}$$

where  $v_s = v_0$ , the longitudinal spread  $dv_{e\parallel} \ll v_0$  and the transverse spread  $dv_{e\perp} \ll v_0$ .

The aim of a good gun is to keep these spreads as small as possible.

#### 4.1 Longitudinal Spread

Concerning the longitudinal spread it is easily shown that its rms value  $\Delta_{e\parallel}^2 = \langle dv_{e\parallel}^2 \rangle$  is given by:

$$\Delta_{e_{\parallel}} = \frac{kT_c}{m\nu_0} \tag{4}$$

such that the equivalent electron longitudinal temperature is:

$$kT_{e_{||}} = m \left(\Delta_{e_{||}}^{*}\right)^{2} = \frac{\left(kT_{c}\right)^{2}}{2(eU_{0})}$$
(5)

*Example*: If  $kT_c = 0.1$  eV and  $U_0 = 10$  kV, then  $kT_{e\parallel} = 0.125 \times 10^{-6}$  eV, showing that the dynamic contraction results in a very small longitudinal spread.

#### 4.2 Transverse Spread

Transversely the rms velocity spread  $\Delta v_{e\perp}^2 = \langle dv_{e\perp}^2 \rangle$  would remain equal to that at the cathode output in the absence of any disturbance and will thus be much larger than the longitudinal rms spread. Unfortunately,

- the space-charge force  $\vec{f_{sp}}$ ,
- the radial electrical field due to the accelerating electrodes  $\vec{f}_{el}$ ,
- the magnetic field force  $\vec{f_m}$

will introduce a disturbance on the transverse electron movement such as the electron will have its transverse energy increased when entering the drift space:

$$kT_{e_{\perp}} = m\left(\Delta_{e_{\perp}}^{2}\right) = \left[m\left(\left\langle dv_{e_{\perp}}^{2}\right\rangle\right)\right] > kT_{c}$$

The aim of a good gun design is to keep  $kT_{e\perp}$  small, if possible less than 1 eV, for any foreseen accelerating voltage  $U_0$  and intensity.

## 5. TRANSVERSE VELOCITY, AN ANALYTICAL APPROACH

The basic equation of motion can be written as follows:

$$m\gamma \frac{d\vec{v}}{dt} = \vec{f}_{el} + \vec{f}_{sp} + \vec{f}_m;$$

$$m\gamma \frac{dv_r}{dt} = F_r, \quad m\gamma \frac{dv_{\varphi}}{dt} = F_{\varphi}, \quad m\gamma \frac{dv_s}{dt} = F_s$$

so that

$$\frac{dv_r}{dt} = \frac{e}{\gamma m} [(E_r - v_s B_{\varphi}) - v_{\varphi} B_0] \quad a)$$

$$\frac{dv_{\varphi}}{dt} = \frac{1}{\gamma m} [ev_r B_0] \quad b) \quad (6)$$

$$\frac{dv_s}{dt} = \frac{1}{\gamma m} (-eE_s)$$
 c)

We set:

$$\omega_H = \frac{eB_0}{\gamma m}$$
, [rad  $\cdot$  s<sup>-1</sup>], and  $F_r = \frac{e}{\gamma m} (E_r - v_s B_{\varphi})$ , [m  $\cdot$  s<sup>-2</sup>].

 $F_r$  = radial force/ $\gamma m$  includes, all together, the influence of the space charge and of the electrodes.

We then get:

$$\frac{dv_r}{dt} + \omega_H v_{\varphi} = F_r$$

$$\frac{dv_{\varphi}}{dt} - \omega_H v_r = 0$$
(8)

Multiplying the second equation by the complex *i* and adding it to the first one gives:

$$\frac{dv_{\perp}}{dt} - i\omega_H v_{\perp} = F_r \tag{9}$$

÷.

where  $v_{\perp} = v_r + i v_{\varphi}$ .

Without second member,  $F_r$ , the solution is:

$$v_{\perp} = a \exp(i\omega_H t)$$
;  $a \in \mathbb{C}$ , (C is the complex space)

## 5.1 Solution in the drift tube

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In this case the gun electrodes have no influence;  $\vec{f}_{el} = 0$ . We can then use the forces induced by the space charge only. They have been defined in 3.3.3. such that:

$$F_r = \frac{1}{\gamma m} \cdot \frac{e^2 n_e}{2\varepsilon_0} r \left( \frac{n}{n_e} - \beta^2 \right) = Constant$$
(10)

By the method of variations of the constant, we come to :

$$v_{\perp} = be^{i\omega_{H}t} + iV_{d}; \ b \in \mathbb{C} \quad \text{and} \ V_{d} = \frac{F_{r}}{\omega_{H}}, [\mathbf{m} \cdot \mathbf{s}^{-1}]$$
(11)

If, for t = 0:  $v_{\perp} = V_{\perp 0}$ , the radial velocity at the cathode output, we come to:

$$v_r = V_d \sin \omega_H t + V_{\perp 0} \cos \omega_H t$$

$$v_{\varphi} = V_d (1 - \cos \omega_H t) + V_{\perp 0} \sin \omega_H t$$
(12)

$$|v_{\perp}| = \sqrt{v_r^2 + v_{\varphi}^2} \ge V_{\perp 0} \tag{13}$$

This result shows that without any other disturbance than that introduced by the space charge the transverse energy will be increased.

The only way to reduce the transverse velocity  $v_{\perp}$  is to reduce  $V_d$  and consequently  $F_r$ . This occurs when n = 0, i.e. when the e-beam is neutralized.

It is worthwhile to put some emphasis on the drift velocity  $V_d$ . When using Eq. (10), with  $n = n_e$  and Eq. (2), we come to:

$$V_d = \frac{F_r}{\omega_H} = \frac{re^2 n_e}{\gamma^3 m 2\varepsilon_0 \omega_H} = \frac{e^{1/2} p_b r U_0}{m^{1/2} 2^{3/2} \pi \varepsilon_0 \gamma^3 a^2 \omega_H}$$

Since  $mv_0^2 = 2eU_0$ 

$$\theta_d = \frac{V_d}{v_0} = \left[\frac{p_b}{4\pi\varepsilon_0\gamma^3} \left(\frac{r}{a^2}\right) \frac{1}{\omega_H}\right] U_0^{1/2} \propto \sqrt{U_0}$$
(14)

Looking now at the transverse temperature  $T_d$  one obtains:

$$kT_d = mV_d^2 = 2e \left[\frac{p_b}{4\pi\varepsilon_0\gamma^3} \left(\frac{r}{a^2}\right) \frac{1}{\omega_H}\right]^2 U_0^2 \propto U_0^2$$

*Example*:  $p_b = 0.5 \,\mu \text{A·V}$ ,  $r = a = 2.5 \times 10^{-2} \text{ m}$ ,  $B_0 = 636$  gauss gives:

$$\omega_H = 1.12 \times 10^{10} \text{ rad} \cdot \text{s}^{-1}, \left[\frac{p_b}{4\pi\varepsilon_0} \cdot \frac{r}{a^2} \cdot \frac{1}{\omega_H}\right] = 1.6 \times 10^{-5}$$
$$\theta_d = 1.6 \times 10^{-2} \sqrt{U_0 \text{ (volt)}} \text{ mrad},$$
$$kT_d = 2e \left[1.6 \times 10^{-5}\right]^2 U_0^2 = 5.12 \times 10^{-10} U_0^2 \text{ (volt)}, \text{ eV}$$

Figure 5.1 gives a plot of the theoretical  $\theta_d$  and  $kT_d$  as a function of the acceleration voltage, with the data taken from the example.

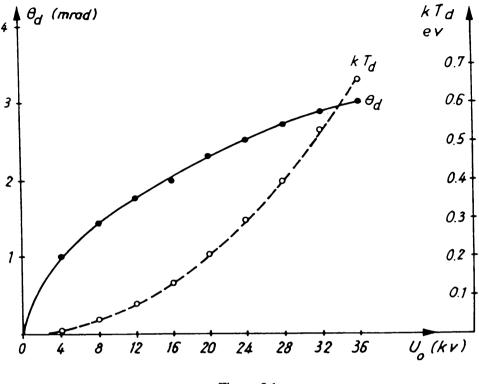


Figure 5.1

#### 5.2 General Solution

In that case,  $F_r$  can not be considered as constant; this is mainly true when the electron beam passes through the gun when  $\vec{f}_{el}$  is not negligible and also  $v_s$  not constant.

The general solution is then:

$$v_{\perp}(s) = v_{\perp 0} + e^{i\omega_{H}t} \int_{0}^{t} F_{r}(t_{1}) e^{-i\omega_{H}t_{1}} dt_{1}$$
(15)

with  $v_{\perp 0} = v_{\perp} (t = 0)$ 

If, instead of time, we use s, the longitudinal length, as independent variable we come to:

$$v_{\perp}(s) = v_{\perp 0} + e^{i\omega_{H}t(s)} \int_{0}^{s} \frac{F_{r}(z)}{v(z)} e^{-i\omega_{H}\int_{0}^{z} dz/v_{s}(z)} dz$$

In order to simplify we can average the longitudinal velocity over s, such that :

$$\int_0^z \frac{dz}{v_s(z)} = \frac{z}{v(\xi)} = \frac{z}{\omega_H \rho_l} \qquad 0 < \xi \le s$$

where  $\rho_l$  is the Larmor radius defined in Section 3.1.

After integration by part, the integral term:

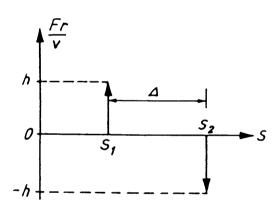
$$A(s) = e^{i\omega_H t} \int_0^s \frac{F_r(z)}{v(z)} e^{-iz/\rho_I} dz = i \frac{F_r(s)}{\omega_H} - i\rho_I e^{i\omega_H t} \int_0^s f(z) e^{-iz/\rho_I} dz$$
(16)  
with  $f(z) = \frac{d}{dz} \left[ \frac{F_r(z)}{v(z)} \right].$ 

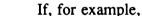
Remarks on Eq. (16):

- The first term,  $i[F_r(s)/\omega_H]$  is quite equivalent to  $iV_d$ , as analyzed in (5.1), and this is definitive when it is taken outside the electrodes, i.e. in a region where the space-charge forces only apply.
- The function f includes now all the perturbations and, of course, those due to the steering electrode.

At this level computer programs must be used.

In order to get a physical feeling of the cathode operation two approximations can be made. They are the "Resonant gun" and the "Adiabatic gun" principles.





5.2.1 Resonant gun

If, for example, one uses two thin electrodes, instead of the single steering electrode, such that at each passage under the electrode:

$$\frac{F_r(z)}{v_s(z)} = \pm h\delta(z), \ z = s - s_{1,2},$$

where the sign + or - is shown in Fig. 5.2, the integral term of Eq. (16) then becomes:



$$\int_{0}^{s} \frac{F_{r}(z)}{v(z)} e^{-iz/\rho_{l}} dz = h \Big[ e^{-is_{1}/\rho_{l}} - e^{-is_{2}/\rho_{l}} \Big] = h e^{-is_{1}/\rho_{l}} \Big[ 1 - e^{-i\Delta/\rho_{l}} \Big]$$

which cancels if  $\Delta/\rho_l = 2\pi n$  (*n* an integer).

This justifies the denomination of resonant optic [6] since it is based on a good relation, or resonance, between  $v_s$  and  $\rho_l = v_s/\omega_H = v_s(\gamma m/eB_0)$  dependent on  $v_s$  and  $B_0$ .

One must remember that this is an approximation since  $v_s$  may depend on s and so  $\rho_l$ .

A similar result can be obtained when considering a "rectangular" function ( $F_r$  = Const,  $s_1 \le s \le s_2$ ). In this case the resonance takes place when  $\Delta/\rho_i = \pi$  [6].

#### 5.2.2 Adiabatic gun

Let us for example consider an electrode configuration which induces a Lorentzian radial electrical field.

$$E_r(z) = \frac{E_0}{1 + \left[ \left( (z - \Delta) / k\Delta \right) \right]^2}$$

with  $k \leq 0.3$ ,  $\Delta$  in meter.

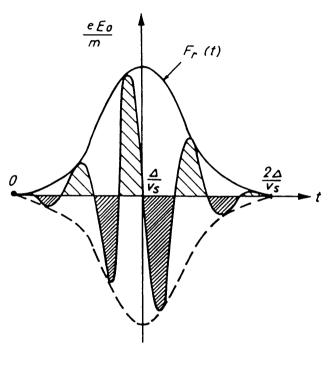


Fig. 5.3

The corresponding expression of  $F_r(t)$ 

$$F_r(t) = \frac{eE_0}{m} \frac{1}{1 + \left[ (vt - \Delta)/k\Delta \right]^2}$$

is represented in Fig. 5.3 together with its product by  $\cos \omega_H t = \Re_e (e^{-i\omega_H t})$  in order to describe the integrant of Eq. (15). One sees that if :

$$\rho_l = \frac{v_s}{\omega_H} = \frac{v}{\omega_H} << \Delta, \text{ or } \omega_H >> \frac{v_s}{\Delta}$$

$$(v_s = v \text{ considered constant for simplicity}).$$

the sum (or integral) of positive and negative terms will tend to cancel. More precisely the result of Eq. (15) becomes (see Appendix 1):

$$v_{\perp}(t) = v_{\perp} \left( t = \frac{2\Delta}{v_s} \right) = v_{\perp 0} + \frac{\pi e E_0 k \Delta}{m v_s} e^{i\omega_H \left( t - (\Delta/v_s) \right)} e^{-k(\Delta/\rho_l)}$$

where  $\omega_H[t-(\Delta/v_s)]$  represents a relative phase.

The second term of  $v_{\perp}(t)$  shows that the adiabaticity, given by  $\rho_t \ll \Delta$ , may result in a very small increase to the initial velocity  $v_{\perp 0}$ 

This property resulting from a slow varying (slow in time versus the cyclotron frequency or in space versus the cyclotron length), disturbing force due to the steering electrode, is a characteristic of the "Adiabatic gun". This implies relatively large magnetic fields ( $\rho_l$  small and so  $B_0$  large).

The new gun is quite adiabatic but some resonances have been shown to exist at very low magnetic fields with the use of tracking programs such as EGUN.

## 6. FUNDAMENTAL GUN PARAMETERS

Figure 6.1a shows the proposed gun layout and dimensions. One can see the three electrodes system which consist of: the cathode, the steering electrode and the final electrode (at ground potential) which constitutes the entrance of the drift tube.

The electron-beam diameter will be 2a = 5 cm. Table 6.1 gives the fundamental parameters ( $V_s$  is referred to ground). One can see from it that the steering electrode potential, with respect to ground, may become positive in order to increase the number of electrons extracted for the space-charge cloud.

Electron energy T (keV)	2.3	7.0	20.0	30.0
Beam perveance $p_b$ ( $\mu$ A.V <sup>-3/2</sup> )	0.125-5	0.125-5	0.125-1	0.125-0.5
Beam current, I (A)	0.01-0.53	0.07-2.93	0.35-2.83	0.65-2.6
Steering electrode voltage $V_s$ (kV)	-1.45-+8.1	-4.3-+25.6	-12.5-+11.5	-17.9 -1.0

Table 6.1

The initial thermal velocity, at the cathode output will not be considered in this paragraph. In such case the modulus of the electron transverse velocity is given by :

$$|v_{\perp}| = \sqrt{v_r^2 + v_{\varphi}^2} = \sqrt{V_d^2 + V^2}$$

where:

 $V_d$  is the transverse velocity considering the beam alone i.e. due to the space charge effect,

*V* is the "disturbing" radial velocity introduced by the gun imperfections and more precisely by the electrode transverse-electrical field. *V* is a measure of the e-gun imperfection.

Keeping this in mind, let us define some angles (in rad):

- $\theta = v_{\perp}/v_0$  giving the full angular spread of the beam,
- $\theta_d = V_d / v_0$  the natural drift angle given by Fig 5.1

 $\alpha = V/v_0$  which is a characteristic to the aberration induced by the gun optics.

In the same way one can define the corresponding temperatures:

$$kT_{\perp} = mv_{\perp}^2, kT_d = mV_d^2, kT_{\alpha} = mV^2$$

The electron trajectories and the electrical field on axis are plotted in Fig. 6.1b, when  $U_0 = 30 \text{ kV}$  and  $U_s = 0$ . V.

Figure 6.1c gives the value of the e-beam current  $I_0$  as a function of  $U_t$  for different kinetic energies T. It shows that  $p_g \equiv 0.51 \,\mu A / U_t^{\frac{3}{2}}$  as mentioned in Section 3.2.

Figures 6.2 to 6.3 give the results of simulations made with the SAM program [8]. They represent the above-mentioned parameters as a function of the magnetic field (Fig. 6.2) or of the electron-beam energy for a fixed magnetic field, for different perveances and at a different radius r inside the e-beam (Fig. 6.3). Since  $\theta = \sqrt{\theta_d^2 + \alpha^2}$  or  $T_{\theta} = \sqrt{T_d^2 + T_{\alpha}^2}$  one sees from these curves that the aberration induced by the gun itself is smaller than the perturbation issued by the drift velocity  $V_d$ . A good way to reduce  $\theta$  is therefore to reduce  $V_d$ . This can be performed by a neutralization of the electron beam since then n = 0 (see Eqs. (10) and (11)). This is out of the scope of this report.

Figure 6.3d represents  $\theta$  and  $\alpha$  as a function of the energy T and r when the steering electrode, or, consequently, the perveance is adjusted such as the electron current is fixed at 3 A for any energy. For the purpose of electron cooling  $T\theta$  should not exceed 1 eV.

A caption is given for all figures 6 which are displayed at the end of this report.

## 6.1 Operation With a Linear Relation Between B<sub>0</sub> and p<sub>i</sub>

For operational purposes [7] one may ask for a linear relation between the cooled ion beam momentum  $p_i$  and the solenoidal magnetic field  $B_0$  such as :  $B_0 = k p_i$  (k is a parameter which should not be confused with the Boltzmann constant). In that case the ion orbit will not be perturbed when  $p_i$  is changed.

Since then,  $p_i \cong m_i v_0$ , ( $m_i$  = the ion mass), Eq.(14) can be written as:

$$\theta_d = \frac{1}{4\pi\varepsilon_0 \gamma^2} \frac{r}{a^2} \left(\frac{m}{e}\right)^{3/2} \frac{1}{\sqrt{r}} \frac{p_b}{km_i}$$
(14)

which is, for a fixed perveance  $p_b$ , independent of the ion energy. If we consider the worst case where r = a, we come to:

$$\theta_d \approx 0.01 \frac{p_b}{k m_i}$$

with  $p_b$  expressed in  $\mu A \cdot V^{-3/2}$  and k in gauss/(MeV/c). The present resonant gun used on the LEAR electron cooler uses k = 1.57.

The outcoming results from such a procedure are shown in Figs. 6.4 to 6.9.

In Figs. 6.4 to 6.7 one can observe that  $\theta = \sqrt{\theta_d^2 + \alpha_\alpha^2}$  is quite constant for all energies. This is consistent with our explanations since  $\alpha < \theta_d$  and  $\theta_d$  should be constant according to Eq. (14).

In Figs. 6.8 tp 6.9 the angle  $\theta$  is represented as a function of  $p_b$ . Again, since  $\theta_d$  is a linear function of  $p_b$  (Eq. (14)), and  $\alpha < \theta_d$ ,  $\theta$  is about a linear function of  $p_b$ .

### 6.2 Conclusion

The proposed gun has the properties of adiabaticity and should fit with the electron-cooler requirements mainly if the perveance and the magnetic field are chosen according to Table 6.2 (T, the kinetic energy must not be confused with temperature):

Electron energy T (keV)	7-30	< 7
Beam perveance $p_b$ ( $\mu A \cdot V^{-3/2}$ )	0.125-1	0.125-5
Coefficient k, (Gauss/(MeV/c))	1.57-1.95	4

On the other hand, high-perveance electron beams have to be neutralized. As a matter of fact, since  $n_e$  is large, this will induce large  $V_d$  (see Eq. (11)), and the simplest way to reduce  $V_d$  is to bring  $n \approx 0$  (neutralization.

## 7. TECHNICAL PROBLEMS

#### 7.1 Penning Discharge

Such a type of discharge is possible under certain circumstances and more precisely when the perveance is larger than  $0.5 \,\mu A \cdot V^{-3/2}$  and the steering electrode is at positive potential.

However, a "Penning cathode" does not physically exist since the beam travels free through the drift tube (which constitutes a virtual anode). As a consequence, Penning discharge ignition is highly improbable.

On the other hand, the gun geometry is designed such that we expect no Penning trap of secondary emission electrons since the gun is operating under high vacuum conditions.

## 7.2 Trapping of Slow Electrons

Even if Penning discharge does not occur when the steering anode is positive, the trapping of slow electrons may exist. These electrons are issued from the ionization of the residual gas molecules and may travel along the magnetic field lines into the gun. They are accelerated on their way from the drift space to the steering electrode and decelerated between the steering anode and the cathode where they are reflected at the virtual cathode level. As for these electrons the motion is non-adiabatic, their transverse energy will increase while their longitudinal energy will decrease.

As a result the ionized electrons will be trapped between the cathode and the steering electrode. This cloud of electrons oscillating in the gun may influence the gun properties, since the electron beam density is modified.

The electron cloud density will be comparable to that of the beam after the storage time:

$$\tau = \frac{\Delta I}{I} \frac{L}{V_t} \frac{1}{\alpha^2}$$

where :

*I* is the e-beam current,

 $\Delta I$  is the ionization current:

 $\Delta I = 3.6 \times 10^{16} P \sigma L.(\sigma \text{ the cross section} \cong 2 \times 10^{-17}, L = \text{cooling device length}, P \text{ expressed in Torr}).$ 

 $V_t$  is the slow secondary electron velocity and  $\alpha$  their angular spread.

For a vacuum pressure  $P = 5 \times 10^{-11}$  Torr,  $V_T = 10^8$  cm/s,  $\alpha = 10^{-2}$  rad.

 $\Delta I/I = 10^{-8}$ , then  $\tau \equiv 2 \times 10^4$  s, which is quite acceptable. The problem will be more severe if  $\alpha \sim 0.1$ , which may occur for  $p_b = 5 \,\mu \text{A} \cdot \text{V}^{-3/2}$ , since then  $\tau = 3-5$  min.

One possible way to overcome this disturbance is to bring down to zero the gun current. However, one must mention that high perveances will be used during short periods where we expect to reduce the cooling tune.

### 7.3 Stability of the Anode Voltages

We have seen that the space charge induces, inside the electron beam, a radial electrical field expressed by Eq. (3). Therefore, the potential inside the beam of radius "a" flowing through the drift tube of radius "b" is :

$$U_f = \frac{en(r^2 - a^2)}{4\varepsilon_0} - \frac{ena^2}{2\varepsilon_0} \ln\left(\frac{b}{a}\right)$$

The potential difference between the beam axis and the drift chamber wall when  $n = n_e$  is:

$$U_{f_0} = -\frac{en_e a^2}{4\varepsilon_0} \left[ 1 + 2\ln\left(\frac{b}{a}\right) \right], \quad n_e = \frac{I}{e\beta c\pi a^2}$$

The kinetic energy T of an electron flowing on the axis is therefore not exactly defined by  $U_0$ . The right expression is:

$$T = e(|U_0| - U_{f0}) = eU_a$$

Any ripple or variation of the steering electrode potential will introduce a change in the ebeam intensity; as a consequence  $U_f$  or the electron longitudinal energy will be changed by  $dT = edU_f$  or more generally  $dT = edU_a$  if  $U_0$  is also expected to change.

For a constant  $U_0$  we obtain (see Appendix 2):

For LEAR  $(T = 2 \text{ keV}, U_0 = 2.835 \text{ kV}, U_s = 7840 \text{ V}, I = 0.55 \text{ A}).$ 

$$\frac{dT}{dU_s} \approx -85 \times 10^{-3}$$

which means that if at  $dT/T \le 10^{-4}$  we need  $|dU_s| \le 2.35$  V or  $dU_s/U_s \le 3 \times 10^{-4}$ .

One therefore sees the importance in keeping the ripple in the steering anode voltage as low as possible in order to maintain the cooling energy constant. This effect is obviously more important at higher energies.

Of course, this aspect must also be taken into account when one foresees to change the intensity (or the perveance) when the electron cooling is operating.

#### 7.4 Protection against Power Supply Failures

Since the gun is operating with two independent power supplies, one fixing the final energy and the other determining the intensity I, through  $U_s$ , the gun must be protected against any power supply failure. This is quite important as concerns the steering electrode which is not foreseen to dissipate power.

One possible scheme is shown on Figure 7.1.

More investigations are necessary in this field.

#### 8. CONCLUSIONS

The proposed gun seems to have the flexibility of an adiabatic gun while necessitating relatively low magnetic fields. Many technological problems remain to be solved.

According to the results shown in Chapter 6 it will be necessary to foresee the operational strategies in order to use the full properties and efficiency of this device. This has not been mentioned in this paper.

#### 9. ACKNOWLEDGEMENTS

The authors are very grateful to P. Bourquin, K. Hübner, S. Maury, D. Möhl and A. Skrinsky for their efficient support and help to this work.

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## **APPENDIX 1**

## Computation of the adiabatic disturbance

Let us write 5.8 with a Lorentzian perturbing radial electrical fie

$$v_{\perp}(t) = V_{\perp 0} + e^{i\omega_H t} \int_0^t \frac{eE_0}{m} \frac{e^{-i\omega_H t}}{1 + (vt - \Delta/k\Delta)^2} dt$$

For  $t \leq 2\Delta/v_s$ :

$$B = \int_0^t \frac{e^{-i\omega_H t}}{1 + (vt - \Delta/k\Delta)^2} dt \approx \int_{-\infty}^{+\infty} \frac{e^{-i\omega_H t}}{1 + (vt - \Delta/k\Delta)^2} dt$$
$$B = \frac{k\Delta}{v} e^{-i\frac{\omega_H \Delta}{v}} \int_{-\infty}^{\infty} \frac{e^{-i(\omega_H k\Delta/v)u}}{1 + u^2} du$$

The integrant has a pole at  $\pm i$ . If we use the residue theorem at -i, one obtains:

$$B = -\frac{\pi k\Delta}{\nu} e^{-i(\omega_H \Delta/\nu)} e^{-\omega_H k\Delta/\nu}$$

Since  $v/\omega_H = \rho_l$ , we finally arrive at:

$$v_{\perp}(t) = V_{\perp 0} - \frac{\pi e E_0 k \Delta}{m v} e^{i \omega_H (t - (\Delta/v))} e^{-k \Delta/\rho_l}, \, \mathrm{m \cdot s^{-1}}$$

When  $\rho_i \ll \Delta$ , the additional transverse velocity induced by the electrodes can be very small.

## **APPENDIX 2**

# Influence of the stability of $U_s$ on the kinetic energy T

We use:

(\*) 
$$T = |U_0| - \frac{IC_1}{\beta(t)}$$
 T in volt,  $C_1$  a constant,  $C_1 = 91.\overline{7} \Omega$  for Lear.  
(\*\*)  $I = p_g (|U_0| + U_s)^{3/2}$   $p_g = 5 \times 10^{-7} A \cdot V^{-3/2}$ 

If  $U_0$  is constant,

$$\beta^{2} \frac{dT}{dU_{s}} = -C_{1} \left[ \beta \frac{dI}{dU_{s}} - I \frac{d\beta}{dT} \frac{dT}{dU_{s}} \right]$$
$$\frac{dI}{dU_{s}} = \frac{3}{2} p_{g} \frac{\left( |U_{0}| + U_{s} \right)^{3/2}}{\left( |U_{0}| + U_{s} \right)} = \frac{3}{2} \frac{I}{\left( |U_{0}| + U_{s} \right)}; \quad \frac{d\beta}{dT} = \frac{1}{\gamma(\gamma + 1)} \frac{\beta}{T}$$

then:

$$\beta \frac{dT}{dU_s} \left[ 1 - \frac{C_1 I}{\beta} \frac{1}{T} \frac{1}{\gamma(\gamma+1)} \right] = -\frac{3}{2} C_1 I \frac{1}{(|U_0| + U_s)}$$
$$\frac{dT}{dU_s} = -\frac{3}{2} \frac{(|U_0| - T)}{(|U_0| + U_s)} \frac{1}{[1 - (|U_0| - T/T)(1/\gamma(\gamma-1))]}$$

when using (\*) and (\*\*).

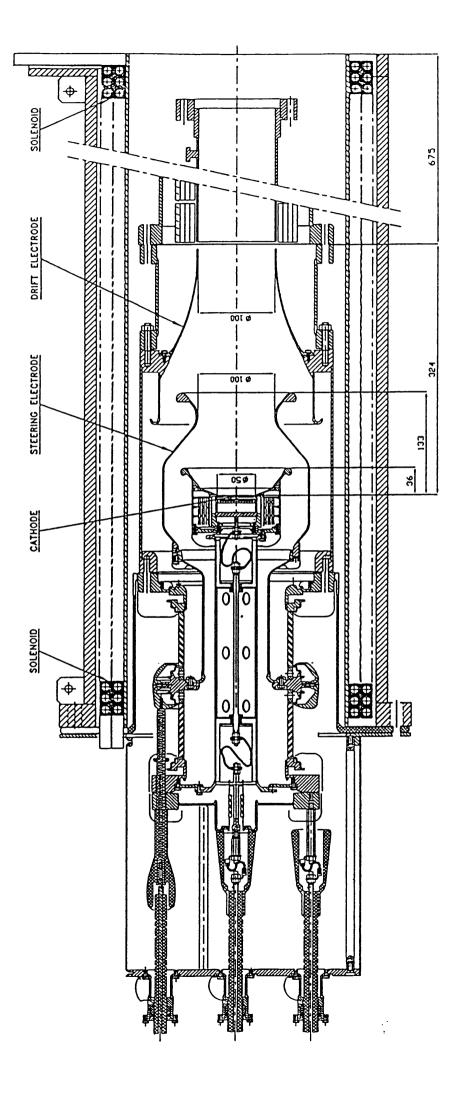
## **CAPTIONS OF FIGURES 6 TO 7**

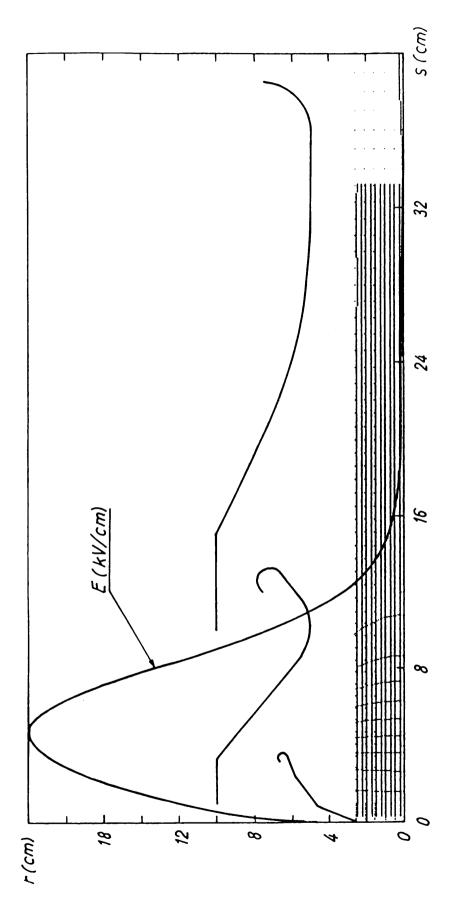
- Figure 6.1aThe electron gun mechanical layoutFigure 6.1bTrajectories, electrical field E(r=0),  $U_0 = 30$  kV,  $U_s = 0$  V,  $I_0 = 2.54$  A<br/> $B_0 = 526$  G,  $E_{max}(r=0) = 3.43$  kV/cmFigure 6.1cElectron beam current as a function of  $U_t$ Figure 6.2Dependence of the electron beam angular spread on the magnetic field<br/>strengthFigure 6.3Dependence of the electron beam angular spread on the electron energy with<br/>constant magnetic field<br/> $\theta$ : full angular spread;  $\alpha$ : radial one,  $p_b$  (in  $\mu A \cdot V^{-3/2}$ ); 0.5 (Fig. 6.3a),<br/>0.125 (Fig. 6.3b), 5.0 (Fig. 6.3c); I = 3 A (Fig. 6.3d)
- Figures 6.4 to 6.7 Dependence of the electron beam angular spread on the electron energy for a variable (with energy) magnetic field (see 6.1)  $\theta$ : full angular spread,  $\alpha$ : radial one,  $m_i = 1$

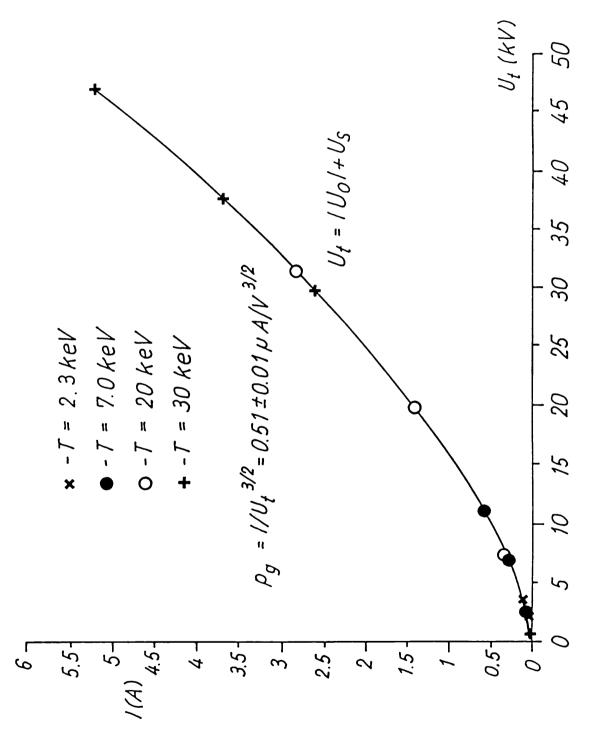
Fig. n <sup>o</sup>	<i>p<sub>b</sub></i> , μΑ·V <sup>-3/2</sup>	k, G / (MeV/c)
6.4a	0.5	1.57
6.4b	0.5	1.69
6.4c	0.5	1.95
6.4d	0.125	1.57
6.5	0.125	1.57
6.6a	1.0	1.57
6.6b	1.0	1.69
6.6c	1.0	1.95
6.7	5.0	1.95

Figures 6.8 to 6.9 Dependence of the electron beam angular spread on pb (pb is expressed in units of 10-6 A·V-3/2)

Electron energy T, keV	k, Gauss / (MeV/c)	B <sub>0</sub> , Gauss
7	1.69	262
7	1.95	302 <sup></sup>
7	4.08	636
20	1.69	445
	keV 7 7 7 7	keV 1.69 7 1.69 7 1.95 7 4.08









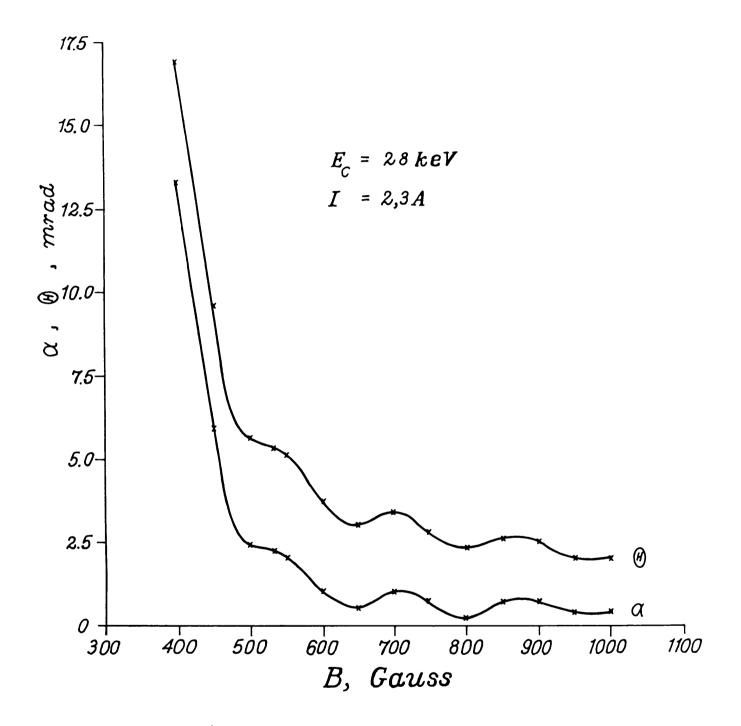
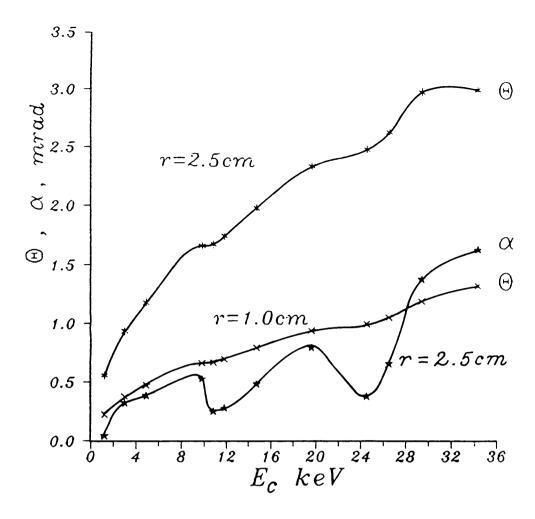


Figure 6.2



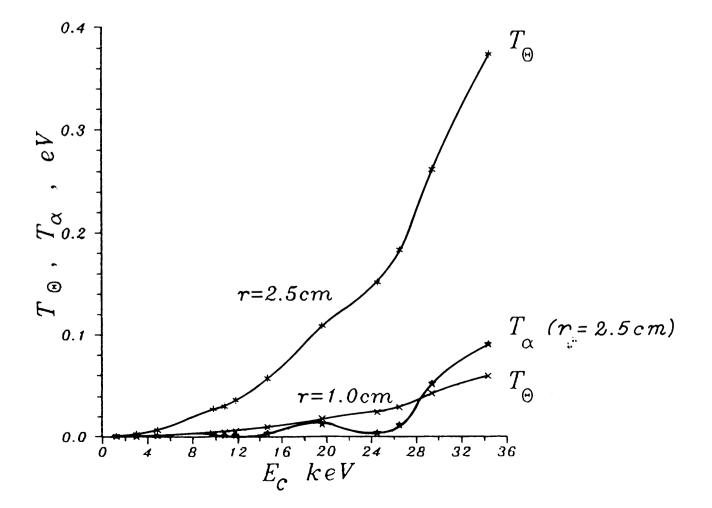


Figure 6.3a

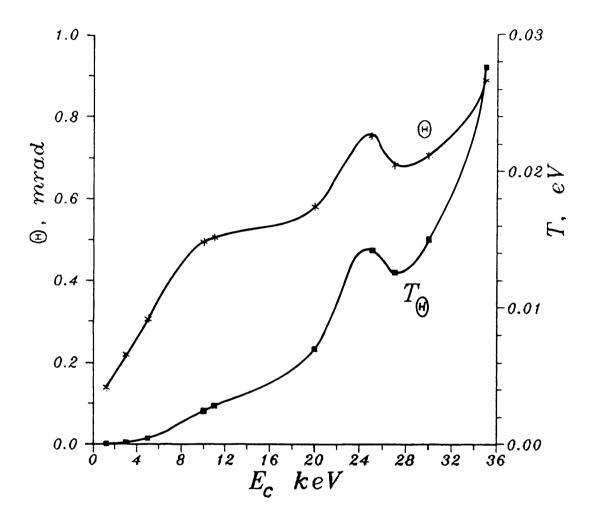
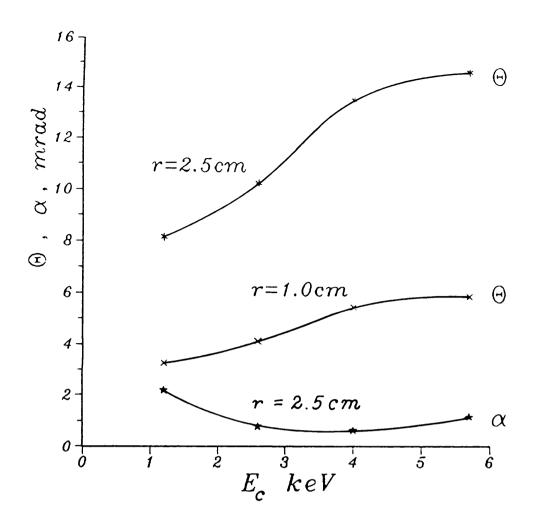


Figure 6.3b



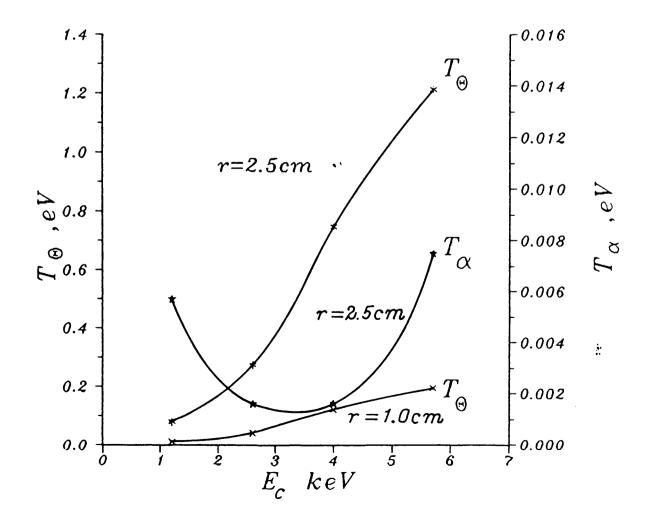
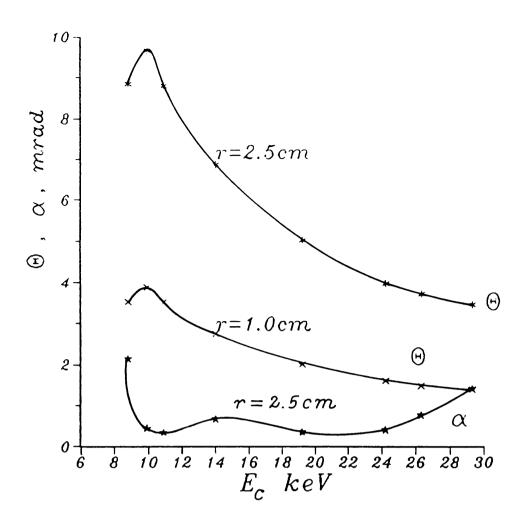


Figure 6.3c



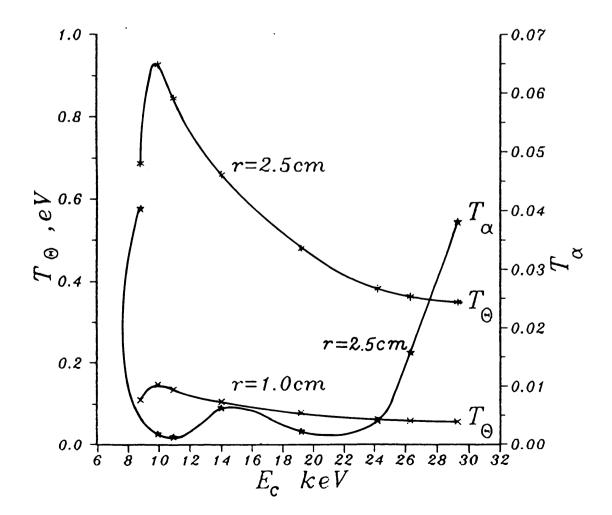


Figure 6.3d

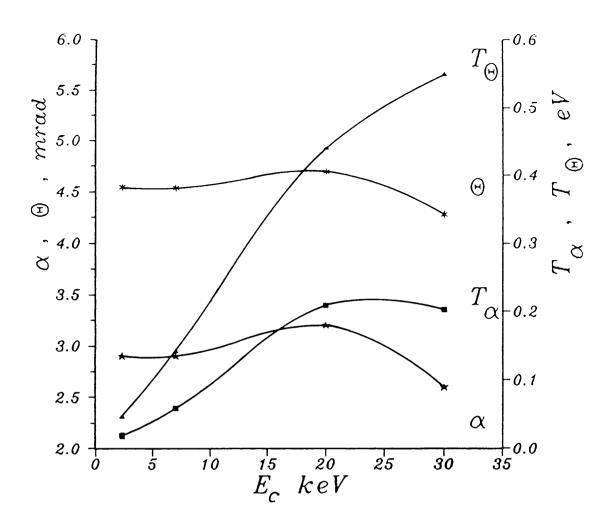


Figure 6.4a

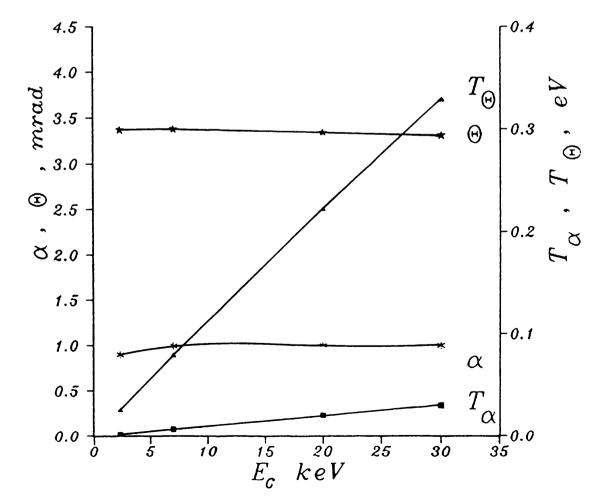
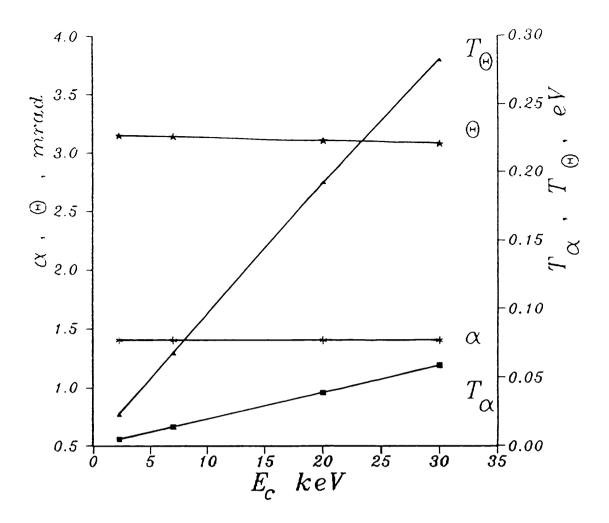


Figure 6.4b





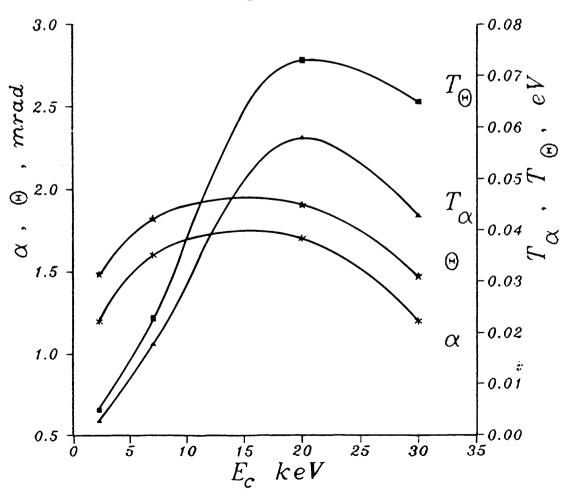


Figure 6.5

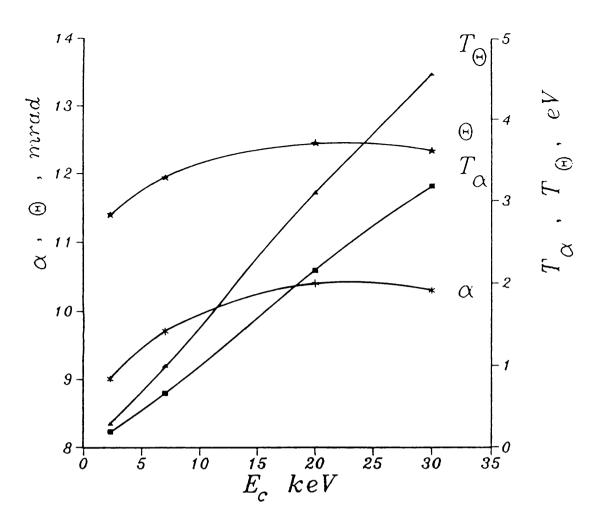


Figure 6.6a

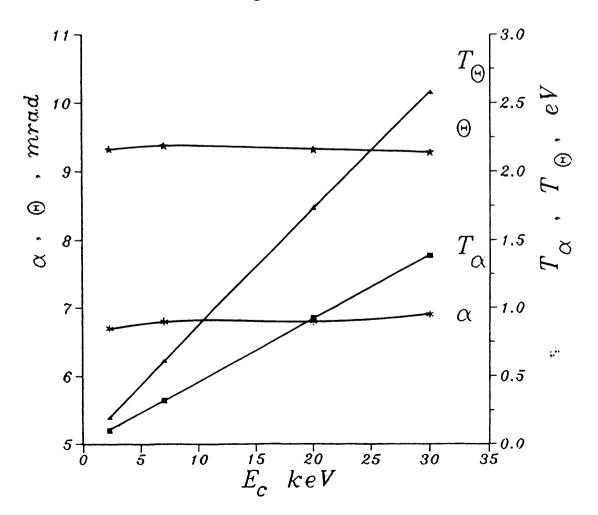


Figure 6.6b

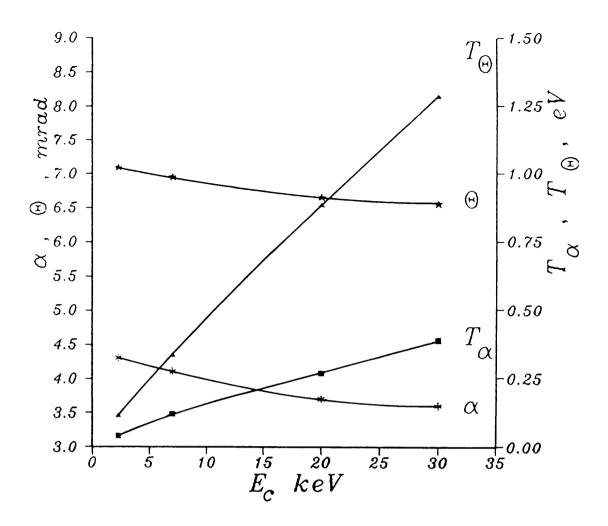


Figure 6.6c

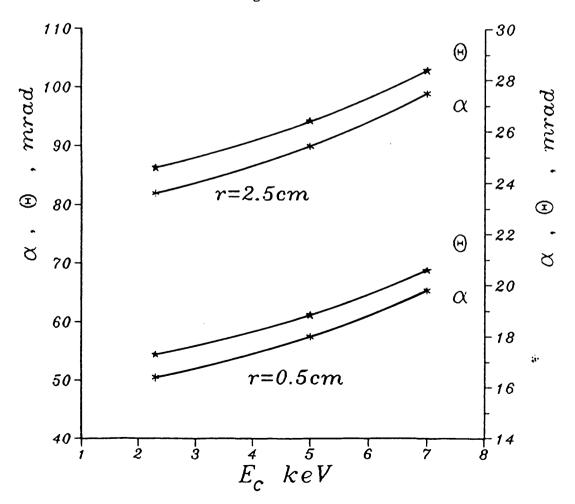


Figure 6.7

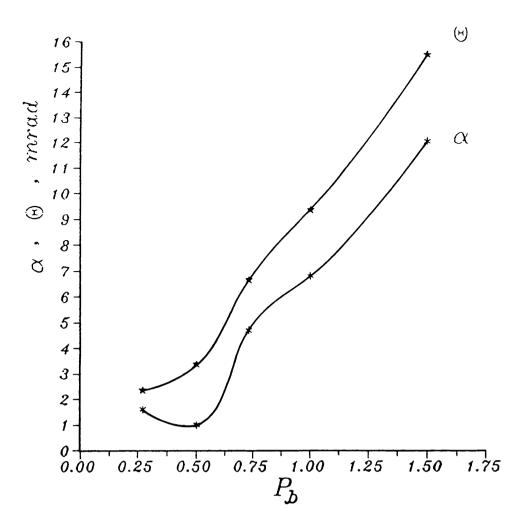


Figure 6.8a

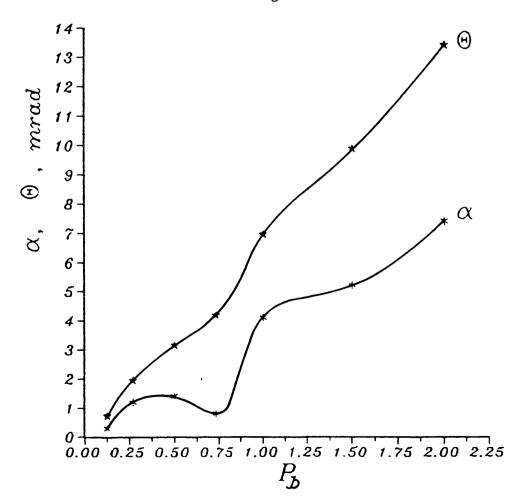
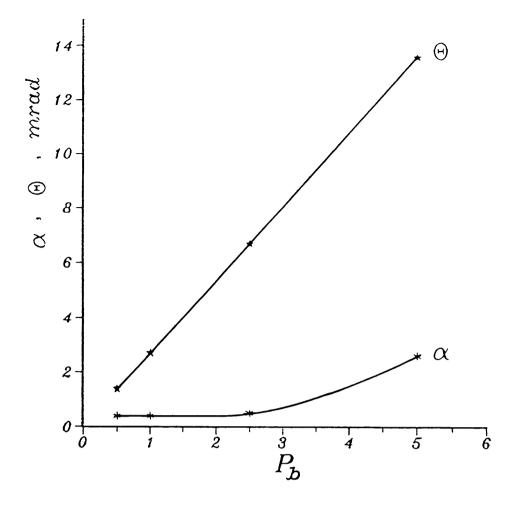


Figure 6.8b





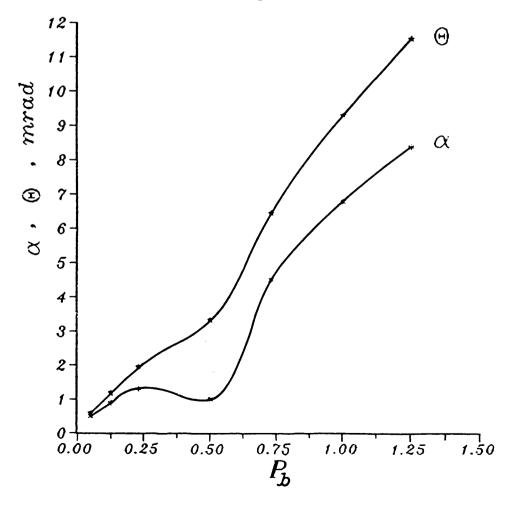


Figure 6.9

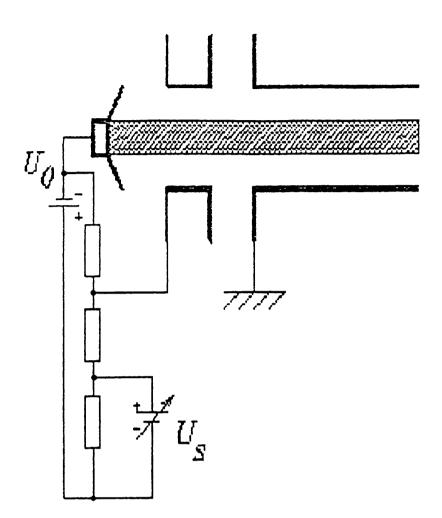


Figure 7.1