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TRANSVERSE IMPEDANCE MEASUREMENTS

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Introduction

To achieve a better understanding of transverse collective effects observed in the Fermilab accelerators (Booster and MR), we propose in this paper two methods to measure the value of the transverse impedance.

These methods are not new [1], the aim of the paper is to briefly recall the theory, provide some handy formulae, suggest experimental procedures and give some numerical examples.

I) $Z_\perp$ MEASUREMENTS WITH UNBUNCHED BEAMS

1. A Reminder of the Basic Theory

The beam-impedance interaction generates a complex shift $\Delta\omega_\beta$ of the betatron frequency $\omega_\beta=(n+Q)\omega_o$ given by (2, 3)

$$\Delta\omega_\beta = j\,\frac{c\,I}{4\pi\, Q\gamma\, E_o/e}\cdot Z_\perp(\omega_\beta + d\omega_\beta)$$

where:

n	=	integer = 0, ±1, ±2,...
Q	=	horizontal or vertical tune
ω_o	=	angular revolution frequency = $\frac{\beta c}{R}$
β	=	relativistic factor
c	=	speed of light
R	=	machine radius
I	=	total beam current
γ	=	relativistic factor
E_o	=	rest energy
$Z_\perp$	=	complex transverse impedance (in Ω / m)

The imaginary part of $Z_\perp$: $\mathrm{Im}\,(Z_\perp(\omega))$ yields a real frequency shift.

The real part of $Z_\perp$: $\mathrm{Re}\,(Z_\perp(\omega))$ yields an imaginary frequency shift, resulting in an exponential growth of the solution $e^{j\omega_\beta t}$ (instability) if $\mathrm{Re}\,(Z_\perp(\omega))<0$ or a damping (stability) if $\mathrm{Re}\,(Z_\perp)>0$.

As an example, Fig. 1 shows the mode spectrum for a machine with $Q \simeq 4.25$, together with the plot of the real part of the resistive wall impedance

$$\mathrm{Re}\ (Z_{\perp rw}) = \mathrm{sign}(\omega)\,.\,\frac{R}{b^3}\sqrt{\frac{2\rho}{\varepsilon_o\,|\omega|}}$$

when:

R = machine radius

b = equivalent chamber radius

ρ = vacuum chamber resistivity

ε_o = free space permissivity

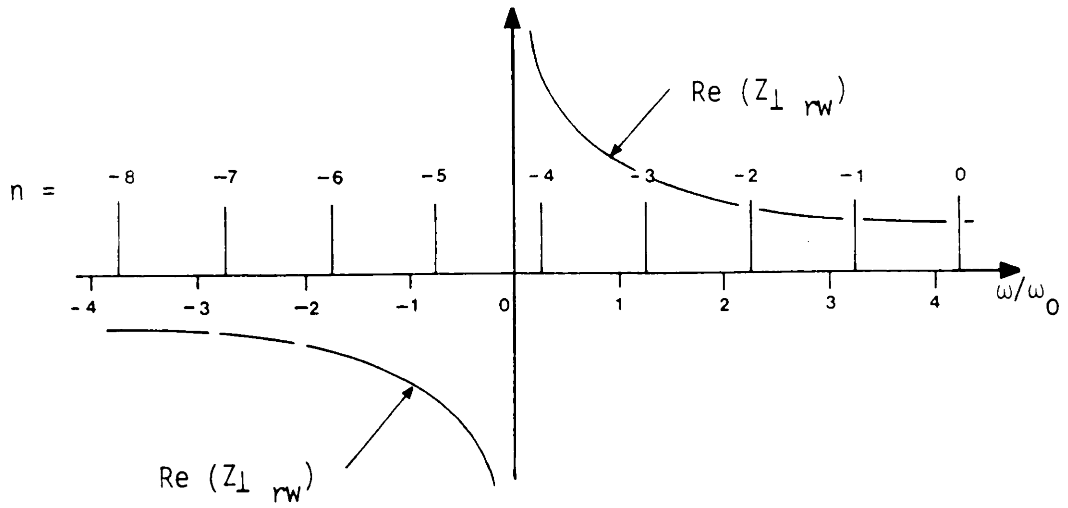

Fig. 1: Transverse mode spectrum and real part of resistive wall impedance

As already mentioned, only the lines "sampling" regions where $(\mathrm{Re}\, Z_\perp) < 0$ are potentially unstable. In our example, only the modes n=-5, -6, -7,..., i.e., the modes corresponding to

$$|n| > Q \quad \text{and} \quad n < 0$$

also named "slow-waves" [4] are unstable.

Remarks:

a) In reality the spectrum of Fig. 1, when measured with a spectrum analyser shows up as shown on Fig. 2 (power spectrum)

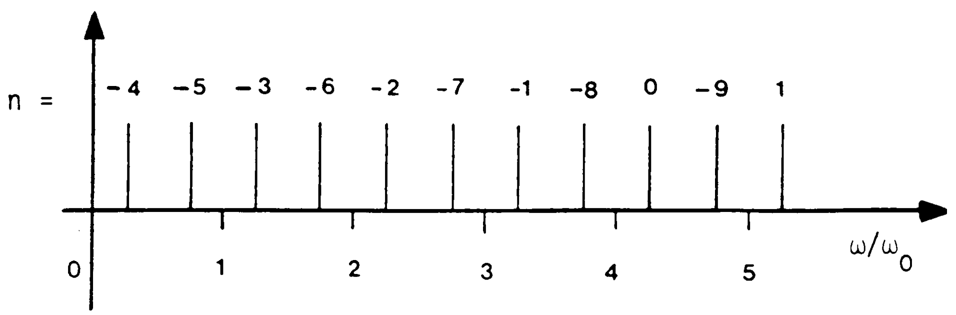

Fig. 2: Mode power spectrum as measured with a spectrum analyser (with frequency spread the lines are widened)

b) The n=-5 mode is the most dangerous as it samples the largest value of $(\mathrm{Re}\, Z_\perp)$.

c) For a narrow band resonator at ω_r, only the modes where $(n+Q)\,\omega_0 \cong \omega_r$ will be excited.

2. Landau Damping

The spectral lines (betatron sidebands) at ω_β have a width given by

$$\delta\omega_\beta = \delta\,[(n+Q)\,\omega_o\,] = (\,Q\,\xi - n\,\eta\,)\,\frac{dp}{p}\,\omega_o$$

where: ξ = chromaticity = $\dfrac{dQ/Q}{dp/p}$

η = frequency slip factor = $-\dfrac{d\omega/\omega}{dp/p} = \gamma_t^2 - \gamma^2$

γ_t = γ at transition

dp/p = relative momentum spread (FWHM)

If the rise time τ_r [where $1/\tau_r = -\,\mathrm{Im}\,(d\omega_\beta)$] of the instability is

$$\tau_r > \frac{1}{\Delta\omega_\beta}$$

or in other words if the coherent frequency shift $\mathrm{Im}\,(d\omega_\beta)$ is smaller than the incoherent frequency spread $\Delta[(n+Q)\omega_o]$ then the instability is Landau damped.

In the approximation where no octupoles effects are present, the incoherent frequency spread is equal to zero (or very small) for

$$n = \frac{Q\xi}{\eta}$$

that is for

$$\omega_\beta = Q\omega_o\,\left(\frac{\xi}{\eta} + 1\right) \cong \frac{Q\omega_o\xi}{\eta} = \omega_\xi$$

there will be no Landau damping and the mode closest to ω_ξ will be easily unstable. This effect can be useful to probe the transverse impedance over a large range of frequencies. The advantages of using a debunched beam is that the unstable modes emerge only at frequencies where $\mathrm{Re}\,(Z_\perp) < 0$. Using bunched beams, aliasing effects can produce ambiguities in the frequency location of the impedance.

The frequency ω_ξ can be varied by changing the chromaticity value. This allows to probe regions of frequencies where $\mathrm{Re}\,(Z_\perp) < 0$.

By measuring the instability rise time τ_r the value of $\mathrm{Re}\,(Z_\perp)$ can be evaluated as

$$\mathrm{Re}\,(Z_\perp(\omega_\beta)) = \frac{4\pi\,Q\,\gamma\,E_o/e}{c\,I\,\tau_r}$$

3. Experimental Procedure

3.1. Set the beam intensity to an average value.

3.2. Debunch the beam by reducing the RF voltage to 0 and if possible successively on each cavity to minimise beam loading effects.

3.3. Switch off octupoles, if any.

3.4. Set the chromaticity to a given value.

3.5. Check on the spectrum analyzer (connected to a wide band transverse pick-up) if any instability appears at $\omega = \omega_\xi$.... or elsewhere.

If instabilities appear elsewhere reduce the beam intensity.

3.6. If an instability appears close to ω_ξ, measure the rise time by tuning the spectrum-analyser, in receiver mode, to the unstable mode. (N.B. : the rise time can easily be measured as the time interval between two points separated by 20 log e ≅ 9 db in amplitude).

3.7. Repeat from 3.4. with another value of chromaticity.

Remarks : In practice, one can expect to probe three main frequency domains, respectively :

I) 0 ÷ 5 MHz II) 5 ÷ 100 MHz III) 100 ÷ 500 Mhz approx.

In the first, low frequency domain, the resistive wall impedance should dominate. In the second, some transverse higher modes in RF cavities (or other resonators) can eventually be discovered. While in the third interval the vacuum chamber broad band impedance can be measured as

$$\mathrm{Re}\,(Z_\perp(\omega)) = \frac{2c}{b^2\omega_o}\,\mathrm{Re}\,\left(\frac{Z_{//}}{n}\right)$$

when $Z_{//}/n$ is the broadband longitudinal impedance of the vacuum chamber divided by

$$n = \omega/\omega_o.$$

II) $Z_\perp$ MEASUREMENTS WITH BUNCHED BEAMS

1. A Reminder of the Basic Theory

As in the previous chapter the real frequency shift $\mathrm{Re}\,(\Delta\omega_{\beta m})$ of the betatron frequency $\omega_{\beta m} = (n+Q)\omega_o + m\omega_s$ for a bunched beam is given by (3)

$$\mathrm{Re}\,(\Delta\omega_{\beta m}) = -\frac{N_b\, e\, c}{4\pi Q\; E/e\; 4\,\sigma_\tau} \cdot \frac{1}{(m+1)} \cdot \mathrm{Im}\,(Z_\perp)$$

where: N_b is the number of particles in the bunch,
σ_τ is the r.m.s. bunch length in s;
m is the mode of oscillation = 0, 1, 2,...
ω_s is the synchrotron angular frequency and
$Z_\perp$ is the transverse wideband impedance given, for simple round structures, by $Z_\perp = \frac{2c}{b^2\omega_o}\,\frac{Z_{//}}{n}$

and $\mathrm{Im}\,(Z_\perp)$ is assumed constant all over the spectrum of the oscillation mode at least for the lowest mode considered herein (m = 0). Such a simplification is generally valid for long (proton) bunches.

Remark: An estimate of the Transverse Mode Coupling instability threshold is given by: $\mathrm{Re}\,(\Delta\omega_{\beta o}) \cong \omega_s / 2$

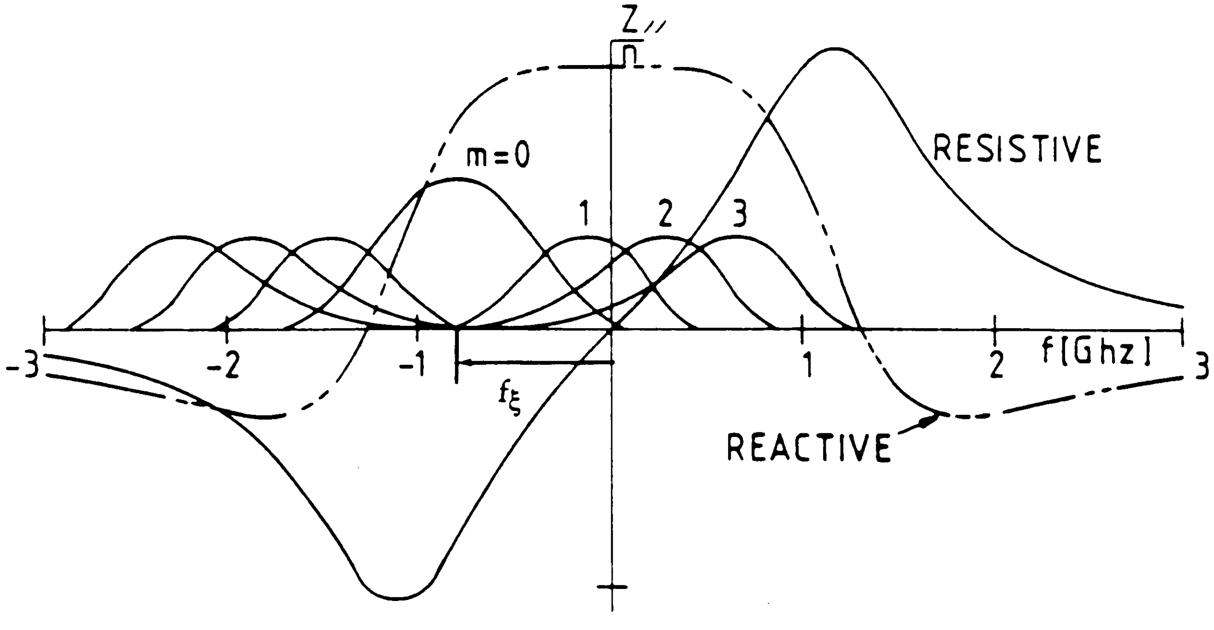

Fig. 3: Shape of the reactive and resistive part of $Z_{//}$ / n (or equiv. $Z_\perp$) versus frequency as well as the shape of the relative amplitude of varius transv. modes for a bunch above transition in a machine with $\xi < 0$.

Measuring the frequency shift of the m = 0 mode as function of N_b/σ_τ one can estimate the value of $\mathrm{Im}\,(Z_\perp)$.

2. Experimental Set-Up

As the expected frequency shifts are rather small (some hundred Hz or less, see for example the numerical example below) one has to adopt FFT techniques to improve the frequency resolution as well as shortening the measurement time. As the m = 0 mode is maximum at $\omega = \omega_\xi$, if $\xi \neq 0$ the signal has to be down converted in order to be FFT analysed at low frequency. See Fig. 4.

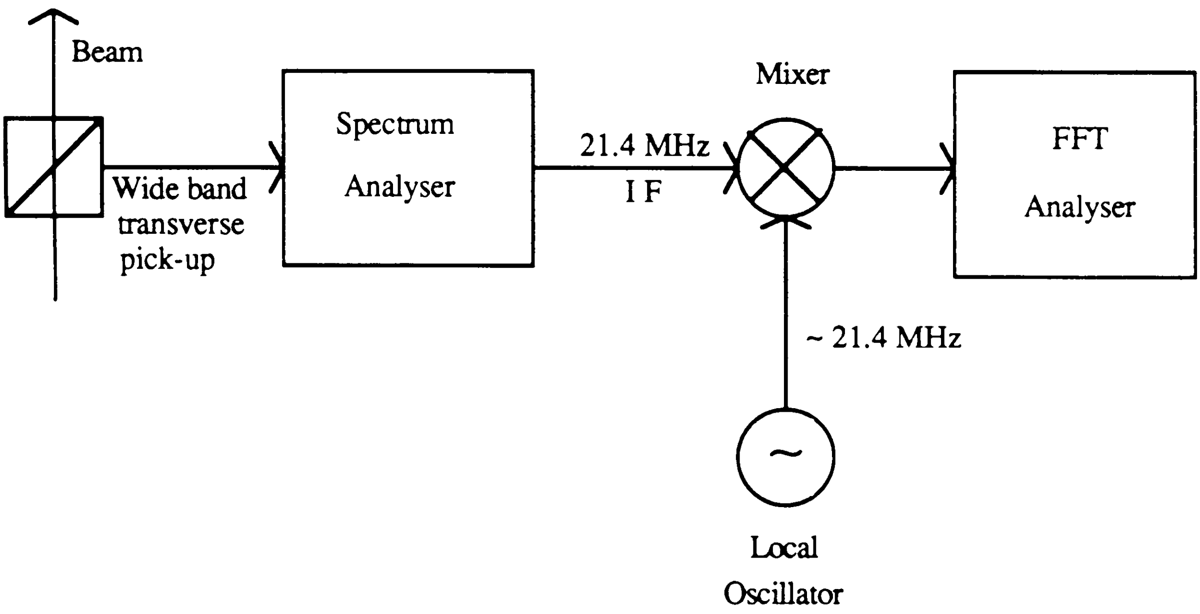

Fig. 4: The signal from a wide band transverse pick-up is first down converted at 21.4 MHz by using a swept-filter spectrum analyser tuned in receiver mode (zero span) to a $\omega_{\beta o} \cong \omega_\xi$. A second local oscillator set at $f \cong 21.4$ MHz mixes the 21.4 IF output of the spectrum analyser down to low-frequency (some KHz) at the input of a FFT analyser (locking the local oscillator to the RF frequency can avoid the frequency shift due to the β variations, if any).

Beam excitation at $\omega_{\beta o}$ as to be provided by powering, for example, a high frequency kicker with a sweeping sinewave synchronised to the measurement.

Numerical example (Machine : Fermilab Booster):

$E = 8$ GeV $R = 75$m $Q = 6.8$ $\beta \cong 1$

$N_b = 3.10^{10}$ p/b $4\sigma_\tau = 10$ ns $b = 3$ cm

this gives: $$Re(\Delta\omega_{\beta o}) \cong 270.10^{-6}\, Z_\perp$$

guessing a $$\frac{Z_{//}}{n} \cong 200\ \Omega \quad \text{then } Z_\perp = \frac{2c}{b^2\omega_o}\frac{Z_{//}}{n} \cong 30\ M\Omega/m$$

so $$Re\ (\Delta\omega_{\beta o})\ /\ 2\pi \cong 1\ KHz$$

In order to measure such a frequency shift with a resolution of ~ 10% (100 Hz) one needs a measurement time of ~ 10 ms.

Numerical example No 2 (Machine : Fermilab MR):

$R = 1000$m $Q = 20$ $n = 10^{-2}$ $b = 3$ cm

$E = 8$ GeV $Nb = 10^{10}$ ppb $4\sigma_t = 10$ ns $Zn/n = 10\ \Omega$

this gives: $$Re\ (\Delta\omega_{\beta o}) = 30.10^{-6}\ Im\ (Z_\perp)$$

if: $$Z_\perp = \frac{2c}{b^2\omega_o}\frac{Z_{//}}{n} = 22\ M\Omega/m$$

so: $$Re\ (\Delta\omega_{\beta o})\ /\ 2\pi \cong 83\ Hz$$

ACKNOWLEDGEMENTS

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3. B. Zotter and F. Sacherer, "Transverse Instabilities of Relativistic Particle Beams in Accelerators and Storage Rings", Proc. 1st Course of International School of Particle Accelerators Erice 1976, CERN 77-13, p. 175-218;

   N.B. : In contrast to the solution $e^{-i\omega t}$ used in these references, we use $e^{j\omega t}$ in the present note.

4. J. Gareyte, "Beam Observation and the Nature of Instabilities", CERN-SPS / 87-18 (AMS).