Exotic nonsupersymmetric gauge dynamics from supersymmetric QCD

Ofer Aharony and Jacob Sonnenschein

*School of Physics and Astronomy, Beverly and Raymond Sackler Faculty of Exact Sciences,
Tel-Aviv University, Ramat Aviv, Tel-Aviv 69978, Israel*

Michael E. Peskin

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Shimon Yankielowicz*
CERN, Geneva, Switzerland

We extend Seiberg's qualitative picture of the behavior of supersymmetric QCD to nonsupersymmetric models by adding soft supersymmetry-breaking terms. In this way we recover the standard vacuum of QCD with N_f flavors and N_c colors when $N_f < N_c$. However, for $N_f \geq N_c$, we find new exotic states—new vacua with spontaneously broken baryon number for $N_f = N_c$, and a vacuum state with unbroken chiral symmetry for $N_f > N_c$. These exotic vacua contain massless composite fermions and, in some cases, dynamically generated gauge bosons. In particular Seiberg's electric-magnetic duality seems to persist also in the presence of (small) soft supersymmetry breaking. We argue that certain, specially tailored, lattice simulations may be able to detect the novel phenomena. Most of the exotic behavior does not survive the decoupling limit of large SUSY-breaking parameters.

PACS number(s): 11.30.Rd, 11.30.Pb, 12.60.Rc

I. INTRODUCTION

When we ask how a gauge theory behaves at strong coupling, we want first of all to understand how the chiral symmetry of this theory is realized. In the familiar strong interactions, we know from experiment that the approximate chiral $SU(3) \times SU(3)$ symmetry is spontaneously broken to a vector $SU(3)$ symmetry. This chiral symmetry breaking allows the quarks to obtain dynamical masses and so justifies the quark model of hadrons. For a long time, physicists have wondered whether this same qualitative behavior should be found in any asymptotically free gauge theory. In a Yang-Mills theory in which the chiral symmetries are not spontaneously broken, these unbroken symmetries can protect composite fermions from obtaining masses [1], leading to a completely new dynamical picture.

In the early 1980s, this same question, which had not been resolved in the case of ordinary Yang-Mills theory, was studied in the supersymmetric extension of Yang-Mills theory. In stages, the qualitative behavior was worked out for supersymmetric pure Yang-Mills theory [2] and for supersymmetric (SUSY) Yang-Mills theory with a small number of quark flavors [3–5]. Recently, Seiberg has returned to this question and, in a remarkable set of papers [6,7], has given a coherent picture of the qualitative behavior of supersymmetric QCD (SQCD) for all numbers of flavors. Seiberg has emphasized that his solution includes dynamical features that are quite exotic, including vacuum states with baryon number violation and massless composite fermions, and he has speculated that these features can potentially also appear in nonsupersymmetric models.

In this paper, we will investigate the extension of Seiberg's vacuum states to nonsupersymmetric models. To do this, we will study how these vacua are perturbed by the addition of soft supersymmetry-breaking terms to the Lagrangian. This method is quantitative only when the soft supersymmetry-breaking masses are much smaller than the strong-coupling scale Λ of the Yang-Mills theory. Despite this limitation, we will show that many of the exotic features found by Seiberg, notably chiral symmetry realizations and duality, do survive in softly broken nonsupersymmetric theories. We will suggest the way in which the supersymmetric limit connects to ordinary Yang-Mills theories of quarks alone.

Soft breaking of supersymmetric Yang-Mills theory was studied previously, with a very different motivation, by Masiero et al. [8,9]. We will follow some of the route uncovered in their papers, but the recent improved understanding of supersymmetric Yang-Mills theory will allow us to obtain a more complete picture.

In addition to the intrinsic interest of exploring nonsupersymmetric extensions of Seiberg's mechanism, this investigation has a broader significance. Today, the most important tool for investigating strong-coupling gauge theories is numerical simulation on the lattice. Up to now, lattice gauge theory simulations have found evidence only for the conventional pattern of chiral symmetry breaking. However, it is by no means clear that the

---

*On leave of absence from the School of Physics, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University.

current simulations have exhausted the possibilities to be discovered. Seiberg's work suggests that lattice gauge theorists should look harder, and in theories with colored fundamental scalars as well as fermions. If supersymmetry were essential to Seiberg's vacuum states, these states would be very difficult to reproduce in simulations, since, in general, there is no known method of ensuring supersymmetry on the lattice.[1] Thus, lattice gauge theorists could reasonably expect success in demonstrating the presence of massless composite fermions and other exotic features only if these phenomena exist in nonsupersymmetric models. Our analysis provides evidence that they do, and it suggests the particular nonsupersymmetric models which are the most promising for finding them.

In this paper, we consider $SU(N_c)$ Yang-Mills theories coupled to N_f flavors of quarks and squarks. In Sec. II, we define our notation and set up a general strategy for analyzing these models. In Sec. III, we consider the case $N_f < N_c$. For this case, we show that soft breaking of supersymmetry leads to the conventional pattern of chiral symmetry breaking, $SU(N_f) \times SU(N_f)$, spontaneously broken to the diagonal $SU(N_f)$. In Sec. IV, we consider the case $N_f = N_c$. In this case, we find that this conventional vacuum state still exists, but that a new vacuum state also appears, with massless composite fermions and spontaneously broken baryon number.

In Sec. V, we consider the case $N_f = (N_c + 1)$. In this case, we find that, for small soft supersymmetry-breaking terms, the chiral symmetry remains unbroken. The vacuum state of this theory contains massless composite fermions with quark and squark constituents; these remain massless even when the squarks have nonzero mass, illustrating a possibility for composite states first discussed by Preskill and Weinberg [11, 12]. In Sec. VI, we discuss the case $N_f \geq (N_c + 2)$. Here the physics of chiral symmetry breaking is quite similar to that found in the previous situation. Seiberg has argued that the supersymmetric limit of these models also possesses a dynamically generated gauge symmetry which, in some circumstances, is weakly coupled. This gauge symmetry is often lost in the nonsupersymmetric case, but we will give some specific models in which it survives. In particular, it seems that the electric-magnetic duality which Seiberg claimed for this region persists in the presence of (small) soft supersymmetry breaking.

Most of our discussion will be carried out for the case $N_c \geq 3$. The case $N_c = 2$ has a number of special complications. However, since this is the case of most interest to people with computers of finite capacity, we discuss this case specifically in Sec. VII. Lattice simulations of gauge theories with scalar fields have a practical difficulty that it may not be possible to reach the continuum limit, due to the presence of a first-order phase transition as a function of the scalar field mass parameter. In Sec. VIII, we discuss how this problem can arise from perturbation of the supersymmetric Lagrangian, and how it

can be avoided. A more detailed analysis of the practical lattice simulation of supersymmetric gauge theories can be found in recent papers of Montvay [13].

In all, these models open a wide variety of new phenomena in nonsupersymmetric models, raising many possibilities for theoretical and numerical investigation and for model building. They confirm Seiberg's intuition that, while supersymmetry is useful for investigating the variety of behaviors possible in strongly coupled gauge theories, it is not a necessary condition for their realization.

II. NOTATION AND STRATEGY

In this paper, we will be concerned with $SU(N_c)$ Yang-Mills theories coupled to N_f flavors of quarks. We will be perturbing about the supersymmetric limit of these theories. In this limit, these theories contain fundamental scalar (squark) fields and a fermion (gluino) in the adjoint representation of the gauge group, in addition to the standard content of Yang-Mills theories with fermions.

A. Fields and symmetries

The quarks and squarks can be grouped into chiral superfields in the N_c and $\overline{N_c}$ representations of $SU(N_c)$. We will refer to these superfields as

$$Q_a^i , \quad \overline{Q}_i^a , \tag{1}$$

where $i = 1, \ldots, N_f$ is a flavor index and $a = 1, \ldots, N_c$ is a color index. When we wish to refer to the individual components of the superfield, we will denote the scalars by Q, $\overline{Q}$ and the fermions by ψ_Q, $\psi_{\overline{Q}}$. The Hermitian conjugate superfields will be denoted $Q^\dagger$, $\overline{Q}^\dagger$. Note that while ψ_Q is a left-handed quark, $\psi_{\overline{Q}}$ is a left-handed antiquark; the right-handed quarks are components of $\overline{Q}^\dagger$. We will reserve the notation q, $\overline{q}$ to denote Seiberg's dual quark superfields, which will appear in Sec. VI. We will denote the gluino as λ^a_b, a matrix in the adjoint representation of $SU(N_c)$.

When $N_c = 2$, the representations N_c and $\overline{N_c}$ become equivalent, and this introduces a number of complications. From this introduction through Sec. VI, we will restrict ourselves to $N_c \geq 3$. In Sec. VII, we will discuss the generalization of our results to $N_c = 2$.

In classical SQCD theory the quark superfields have no interactions beyond their couplings to the gauge supermultiplet. In particular, we will assume that they have zero mass. This implies that the supersymmetric theory has a global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R, \tag{2}$$

where $SU(N_f)_L$ acts on the Q^i, $SU(N_f)_R$ acts on the $\overline{Q}_i$, and $U(1)_B$ denotes baryon number. We will refer to the vectorial flavor group, the diagonal subgroup of the two $SU(N_f)$'s, as $SU(N_f)_V$. The additional factor $U(1)_R$ denotes the anomaly-free combination of the axial $U(1)$ symmetry acting on the quarks and the canonical R symmetry which acts on all fermion fields. Under this

---

[1]See, however, [10], where certain $N = 2$ supersymmetric lattice theories have been considered.

anomaly-free symmetry, the squarks, quarks, and gluinos have the charges

$$Q, \overline{Q} : \frac{N_f - N_c}{N_f}, \quad \psi_Q, \psi_{\overline{Q}} : -\frac{N_c}{N_f}, \quad \lambda : 1. \quad (3)$$

A superpotential W should have R charge 2.

B. Effective Lagrangians

The qualitative behavior of supersymmetric Yang-Mills theory is made most clear by writing an effective Lagrangian in terms of gauge-invariant chiral superfields. As Seiberg especially has emphasized [6], this Lagrangian is strongly constrained by the condition that its superpotential must be a holomorphic function of these fields. For small values of N_f, the only possible gauge-invariant chiral field built from the quark fields is the meson field

$$T^i{}_j = Q^i \cdot \overline{Q}_j. \quad (4)$$

Beginning at $N_f = N_c$, there are also chiral superfields with the quantum numbers of baryons. Let

$$\tilde{N}_c = (N_f - N_c). \quad (5)$$

Then there is a baryon chiral superfield in the $\tilde{N}_c$-index antisymmetric tensor representation of $SU(N_f)_L$,

$$B_{i_1 \cdots i_{N_c}} = \epsilon^{a_1 \cdots a_{N_c}} \epsilon_{j_1 \cdots j_{N_c} i_1 \cdots i_{N_c}} Q^{j_1}_{a_1} \cdots Q^{j_{N_c}}_{a_{N_c}}, \quad (6)$$

and, similarly, an antibaryon chiral superfield $\overline{B}^{i_1 \cdots i_{N_c}}$ built from N_c powers of the field $\overline{Q}$.

Using the gauge supermultiplet, it is possible to build another chiral superfield

$$S = -\text{tr}[W^\alpha W_\alpha] = \text{tr}[\lambda \cdot \lambda] + \cdots . \quad (7)$$

The superfield S has R charge 2 and is neutral under the other global symmetries. In studies of the qualitative behavior of supersymmetric Yang-Mills theory, the component fields of S always acquire mass; these fields are associated with the massive hadrons of the pure glue sector of the theory. However, the dependence of the superpotential on S is still fixed by symmetry arguments [2,3], and S can be inserted or removed in an unambiguous way by Legendre transformations [15]. Though most of our results can be derived without introducing S into the Lagrangian, it will be useful at some points in our analysis to write effective Lagrangians that depend on S as well as T.

C. Soft supersymmetry breaking

In addition to the superpotential, we will need to know the Kähler potential which determines the kinetic energy terms of the fields T, B, and $\overline{B}$. A simple hypothesis, introduced in the work of Masiero and Veneziano [8,9], is that the Kähler potentials of the gauge-invariant fields are canonical:

$$K[T, B, \overline{B}] = A_T \text{tr}[T^\dagger T] + A_B(B^\dagger B + \overline{B}^\dagger \overline{B}). \quad (8)$$

Our main results will rely on weaker assumptions about the Kähler potential, in particular, that it is nonsingular on the space of supersymmetric vacuum states. However, we will support our general remarks by explicit calculations using this simple model. We expect (8) to be the correct form of the Kähler potential near the origin of moduli space, in the cases for which the mesons and baryons give an effective infrared description of the theory.

We will also need to specify the terms by which we break supersymmetry. In this paper, we will break supersymmetry by adding mass terms for the squark fields and for the gaugino,

$$\Delta\mathcal{L} = -m_Q^2\left(|Q|^2 + |\overline{Q}|^2\right) + \left(m_g S + \text{H.c.}\right), \quad (9)$$

where, in (9), Q, $\overline{Q}$, and S are the scalar component fields of the superfields. The scalar mass term is the unique soft supersymmetry-breaking term which does not break any of the global symmetries (2) of the original model. The gaugino mass term breaks only the $U(1)_R$ symmetry, and thus breaks the global symmetry of the supersymmetric model down to that of ordinary Yang-Mills theory with N_f massless flavors. Any other choice for the soft supersymmetry-breaking terms would induce further global symmetry breaking. Because S is a complex field, any sign or phase inserted in front of the gluino mass term could be compensated by a phase rotation of S [or, more generally, by a $U(1)_R$ transformation]. We have chosen the phase of this term so that the potential energy of the broken theory will be minimized when S is real and positive.

Actually, it is not clear whether the "correct" theory of broken supersymmetry should or should not contain the gluino mass term. If this term is included, and then m_Q^2 and m_g are taken to infinity, the theory reverts to the standard Yang-Mills theory with N_f flavors. If this term is omitted, and then m_Q^2 is taken to infinity, the theory becomes a Yang-Mills theory coupled to N_f flavors in the fundamental representation and one extra flavor in the adjoint representation. Both of these are theories whose strong-coupling behavior might be of interest. We will refer to the softly broken theories without and with the m_g term as the R and $\not{R}$ theories, respectively.

Since we will be working in the language of the low-energy effective Lagrangian, we must ask how the supersymmetry breaking term (9) shows up in this Lagrangian. To work this out, rewrite (9) in the superfield form

$$\Delta\mathcal{L} = \int d^4\theta M_Q(Q^\dagger e^V Q + \overline{Q}^\dagger e^{-V^T} \overline{Q})$$
$$+ \int d^2\theta M_g S + \text{H.c.}, \quad (10)$$

where M_Q is a vector superfield whose D component equals $(-m_Q^2)$ and M_g is a chiral superfield whose F component equals m_g. It is straightforward to see that these superfields are gauge invariant and neutral under all of the global symmetries.

The effective Lagrangian description of $\Delta\mathcal{L}$ for $N_f \leq N_c + 1$ is then given by writing the most general La-

grangian built from T, B, $\overline{B}$ and a fixed number of factors of M_Q and M_g. The supersymmetry-breaking terms have an ambiguity related to that of the Kähler potential, because many possible invariant structures can be built from T, B, and $\overline{B}$. In our explicit calculations, we will assume that the coefficient of M_Q is quadratic in these fields; again, this assumption is precise near the origin of moduli space. Then the first-order soft supersymmetry-breaking terms in the effective Lagrangian are

$$\Delta\mathcal{L} = \int d^4\theta (B_T M_Q \mathrm{tr}[T^\dagger T] + B_B M_Q \{B^\dagger B + \overline{B}^\dagger \overline{B}\}$$
$$+ \overline{M_g}\mathcal{M}(T, B, \overline{B}) + \mathrm{H.c.}) + \int d^2\theta M_g \langle S \rangle + \mathrm{H.c.},$$

$$(11)$$

where $\mathcal{M}(T, B, \overline{B})$ is a function of the effective Lagrangian superfields which is neutral under the global symmetries. The quantity $\langle S \rangle$ in (11) should be a combination of the effective Lagrangian chiral superfields which has the quantum numbers of S. In general, this condition restricts that function to be proportional to the expectation value of S as determined from the effective Lagrangian of Refs. [2, 3] which includes S as a basic field. In some of our examples, the symmetry of the vacuum will prohibit S from obtaining a vacuum expectation value; then the only effect of M_g will be from the D term in (11). The appearance of this unknown D term, however, will prevent us from making any quantitative predictions after adding the gluino mass.

The squark mass terms in (11) are not the most general terms that can be written down. As in the Kähler potential (8), higher-order terms in the fields, suppressed by powers of Λ, may appear. However, we expect (11) to be approximately true near the origin of moduli space $T = B = \overline{B} = 0$. Thus, whenever the vacuum which we analyze will be near the origin of moduli space (as will be the case for $N_f \geq N_c + 1$), we expect (8) and (11) to give a good quantitative description of the theory. In other cases, notably for $N_f \leq N_c$ where some expectation values are expected to be of order Λ or higher, higher-order terms cannot be neglected. We expect that the qualitative behavior which we will find when using these simple terms will remain valid also in the exact theory. However, we will not be able to trust the quantitative results.

We will assume throughout this paper that the coefficients B_B and B_T are positive. Their ratio B_B/B_T will be important to our later analysis; however, this ratio cannot be determined from the effective Lagrangian viewpoint. At best, we can argue naively that the coefficient of the mass term of a composite field should be roughly proportional to the sum of the coefficients of the mass terms of the constituents. This would give the relation

$$B_B \approx \frac{N_c}{2} B_T ,$$

$$(12)$$

which the reader might take as qualitative guidance.

To avoid the proliferation of factors Λ^β, where Λ is the nonperturbative scale of the strong interaction theory, we will generally choose units in which $\Lambda = 1$. Then m_Q^2

and m_g will be small dimensionless numbers. We emphasize again that our method makes quantitative sense only for theories with weakly broken supersymmetry, that is, only when m_Q^2 and m_g are much less than Λ, and will not apply directly to models in which the squarks and gluinos are completely decoupled. However, in many of our examples, the qualitative behavior we find in the region $m_Q^2 \ll \Lambda$ will suggest a smooth continuation to the decoupling limit $m_Q^2 \gg \Lambda$. In each case that we study, we will offer at least a plausible conjecture, for both the R and $\not{R}$ cases, of the connection between these two limits.

It is important for our analysis that the behavior of the theory be nonsingular when adding the squark and gluino masses, i.e., that no new nonperturbative effects occur. In general it is not possible to prove this in nonsupersymmetric theories, but a proof of this is possible in softly broken supersymmetric theories, when the soft breaking can be viewed as spontaneous breaking of supersymmetry. For SQCD this was done by Evans et al. [14], who showed how the squark and gluino mass terms may be obtained by spontaneous supersymmetry breaking in a theory which includes some additional chiral superfields. When obtaining the soft breaking terms in this way, from a supersymmetric theory in which we have control over the superpotential, we can show that the form of the SUSY-breaking operators is indeed as in Eq. (11). In fact, in [14], the squark mass is derived from the Kähler term in the original SUSY theory, so that our lack of control of this term in (11) is related to our lack of control over the Kähler term (8), and the two are expected to behave in a similar fashion.

III. $N_f < N_c$

We begin with the simplest situation $N_f < N_c$. In this case, there are no baryon operators; thus, in the supersymmetric limit, the only massless particles are those created by the meson operator T. In this section, we will work out the vacuum and massless spectrum which result when this theory is perturbed by the soft supersymmetry-breaking terms (9).

In this case, the effective theory of the supersymmetric limit is described by the Affleck-Dine-Seiberg superpotential:

$$\int d^2\theta W(T) = \int d^2\theta \frac{(N_c - N_f)}{(\det T)^{1/(N_c - N_f)}},$$

$$(13)$$

where we have set $\Lambda = 1$ as described at the end of Sec. II. To begin, choose the canonical Kähler potential (8). We will comment on other choices of the Kähler potential below.[2] Using (8), we find the potential energy

$$V(T) = \frac{1}{A_T |\det T|^{2/(N_c - N_f)}} \mathrm{tr}\big[(T^{-1})^\dagger T^{-1}\big].$$

$$(14)$$

---

[2] In the limit of large T, to which we will be driven when m_Q^2 is very small, the gauge theory is weakly coupled and the Kähler potential of T takes the semiclassical form $K[T] = \mathrm{tr}[(T^\dagger T)^{1/2}]$. This choice leads to the same qualitative conclusions that we will derive from the canonical form for $K[T]$. We thank N. Seiberg for this comment.

Now add the soft supersymmetry-breaking term (9). Again, we will begin with a simpler situation, choosing the R case where $m_g = 0$. The addition to the potential is

$$\Delta V = B_T m_Q^2 \text{tr}\left[T^\dagger T\right]. \tag{15}$$

To find the vacuum state, we must minimize $V + \Delta V$.

A. Location of the vacuum state

If we use the freedom of $\text{SU}(N_f) \times \text{SU}(N_f)$ to diagonalize T, this potential can be written in terms of the complex eigenvalues t_i of T, as

$$V(T) = B_T m_Q^2 \sum_i |t_i|^2 + \frac{1}{A_T} \frac{1}{|\prod t_i|^{2/(N_c - N_f)}} \sum \frac{1}{|t_i|^2}. \tag{16}$$

The minimization equation is

$$0 = B_T m_Q^2 t_i$$
$$- \frac{1}{A_T} \frac{1}{\mathcal{D}^{2/(N_c - N_f)}} \left[\frac{1}{t_i^* |t_i|^2} + \frac{1}{(N_c - N_f) t_i^*} \mathcal{T}\right], \tag{17}$$

where

$$\mathcal{D} = \left|\prod_i t_i\right| \quad , \quad \mathcal{T} = \sum_i \frac{1}{|t_i|^2}. \tag{18}$$

Multiplying through by t_i^*, we find an equation of the form

$$0 = B_T m_Q^2 |t_i|^2 - F(|t_i|^2, \mathcal{D}, \mathcal{T}), \tag{19}$$

where, for fixed $\mathcal{D}$ and $\mathcal{T}$, the function F decreases monotonically as the first term increases monotonically from 0 to infinity. This equation has a unique solution for t_i; thus, all of the t_i are equal at the minimum of the potential, up to phases removable by global symmetry transformations.

Thus, we may set $t_i = t$ for all i. This gives the expression

$$V + \Delta V = B_T m_Q^2 N_f |t|^2 + \frac{1}{A_T} \frac{N_f}{|t|^{2N_c/(N_c - N_f)}}. \tag{20}$$

It is easy to see that this expression is minimized for

$$|t| = t_* = \left[\frac{N_c}{(N_c - N_f)} \frac{1}{B_T A_T m_Q^2}\right]^{\frac{(N_c - N_f)}{2(2N_c - N_f)}}. \tag{21}$$

The potential $V(t)$ is shown for the case $N_f = 2$, $N_c = 3$ in Fig. 1.

The minimum of the potential can be brought by global symmetry transformations into the form

$$\langle T \rangle = t_* \mathbf{1} \ , \tag{22}$$

where $\mathbf{1}$ is the unit matrix. This expectation value spontaneously breaks (2) to $\text{SU}(N_f)_V \times \text{U}(1)_B$.
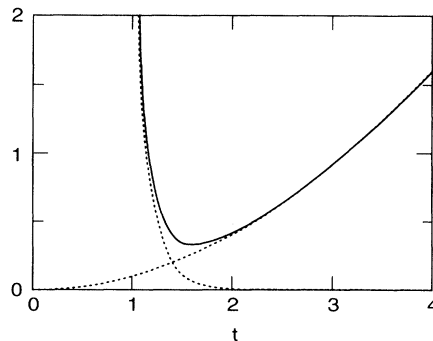

FIG. 1. The potential $V(t)$ for softly broken supersymmetric Yang-Mills theory with $N_c = 3$, $N_f = 2$.

B. Spectrum of the R model

It is straightforward to work out the spectrum of the model by expanding about the minimum of V. Consider first the bosons of the model. A general $N_f \times N_f$ complex matrix T can be parametrized in terms of real-valued component fields as

$$T = t_* e^{(t_V + i t_A)/\sqrt{2N_f}} V U \ , \tag{23}$$

where

$$V = e^{t_{Vi}\lambda^i} \ , \quad U = e^{it_{Ai}\lambda^i} \ , \tag{24}$$

and the λ^i are $\text{SU}(N_f)$ matrices, normalized to $\text{tr}[\lambda^i \lambda^j] = \frac{1}{2}\delta^{ij}$. In this parametrization, $|\det T|$ is a function of t_V only, and the various real-valued components all have kinetic energy terms of the form

$$\mathcal{L} = \sum_I \frac{1}{2} A_T (\partial_\mu t_I)^2 + \cdots. \tag{25}$$

The fields t_{Ai} and t_A drop out of the potential completely. This is natural, because they are the Goldstone bosons of the spontaneously broken $\text{SU}(N_f)$ and $\text{U}(1)_R$ symmetries. The fields t_{Vi}, which form an adjoint representation of the unbroken $\text{SU}(N_f)$ flavor group, obtain the mass

$$m_{Vi}^2 = \left(\frac{2N_c - N_f}{N_c - N_f}\right) \frac{2}{A_T^2} \left(\frac{1}{t_*}\right)^{2N_c/(N_c - N_f)} \ , \tag{26}$$

and the singlet field t_V obtains the mass

$$m_V^2 = \left(\frac{N_c^2 - N_c N_f + N_f^2}{(N_c - N_f)^2}\right) \frac{2}{A_T^2} \left(\frac{1}{t_*}\right)^{2N_c/(N_c - N_f)}. \tag{27}$$

The fermion masses can be read directly from the superpotential (13). Expanding this formula about the minimum according to

$$T = t_* \mathbf{1} + \theta \left(\psi_T \frac{1}{\sqrt{2N_f}} + \psi_{Ti} \lambda^i\right) + \cdots, \tag{28}$$

we find mass terms for the flavor-singlet and -adjoint fermions:

$$m_\psi = \frac{N_c}{(N_c - N_f)} \frac{1}{A_T} \left(\frac{1}{t_*}\right)^{(2N_c - N_f)/(N_c - N_f)} ,$$

$$m_{\psi i} = \frac{1}{A_T} \left(\frac{1}{t_*}\right)^{(2N_c - N_f)/(N_c - N_f)} . \qquad (29)$$

No fermions remain massless.

C. $\not\!\!R$ Model

Now we introduce the more general supersymmetry-breaking term with m_g nonzero. Though it is possible to discuss this term from the beginning with m_Q^2 and m_g treated on the same footing, it is simpler, and one obtains qualitatively the same results, if we treat m_g as a perturbation on the R model just described.

The superpotential term involving m_g requires $\langle S \rangle$. Quite generally, we can obtain the expectation value of S from the superpotential of a supersymmetric effective Lagrangian by using the formula

$$\langle S \rangle = \frac{\partial}{\partial \ln \Lambda^{(3N_c - N_f)}} W . \qquad (30)$$

This equation can be derived by starting from the effective Lagrangian which includes S explicitly [2, 3], or directly from considerations of anomalies [15].

Restoring Λ to (13) and applying (30), we find for the supersymmetry-breaking potential

$$-m_g S = -\frac{m_g}{(\det T)^{1/(N_c - N_f)}} . \qquad (31)$$

This potential depends on the phase of $\det T$, and thus it induces a mass for the field t_A in (23). We find

$$m_A^2 = \frac{N_f}{(N_c - N_f)^2} \frac{m_g}{A_T} \left(\frac{1}{t_*}\right)^{(2N_c - N_f)/(N_c - N_f)} . \qquad (32)$$

The appearance of this mass term is expected: The gluino mass term explicitly breaks the $U(1)_R$ global symmetry and so should give mass to the corresponding Goldstone boson.

It is not difficult to work out the general formulas for the other particle masses to first order in m_Q^2 and m_g. However, there are no surprises. The vacuum remains unique up to global symmetry transformations, and all of the particles except the $SU(N_f)$ Goldstone bosons remain massive.

We can now discuss the extension of our results to more general forms for the Kähler potential. Because the spectrum we have found is the generic spectrum for the symmetry-breaking pattern we have observed, sufficiently small perturbations of the Kähler potential do not affect the qualitative physics. It is possible to choose Kähler potentials which decrease sufficiently strongly as the t_i increase that the potential has more than one minimum. In this situation, it is formally possible to have a minimum of V in which the eigenvalues of T take distinct values. In such a case, the vectorial flavor $SU(N_f)$ symmetry is also partially broken. We do not consider this scenario likely, but we cannot rule it out. Nevertheless, we will disregard this possibility in the rest of our discussion.

D. Decoupling of superpartners

In the arguments just concluded, we have calculated the symmetry-breaking pattern and the spectrum of supersymmetric Yang Mills theory perturbed to first order in soft supersymmetry-breaking terms. It is interesting that our results for the global symmetry and the massless particles reproduce the standard expectations for chiral symmetry breaking in N_f-flavor QCD. The final symmetry-breaking pattern leaves a global symmetry $SU(N_f)_V \times U(1)_B$, and the only massless particles are the Goldstone bosons corresponding to this symmetry breaking. In QCD, this expectation is not particularly well supported for large values of N_f, but it is known to hold in the case which has been studied experimentally, $N_c = 3$, $N_f = 2$, and in the limit $N_c \to \infty$, N_f fixed [16].

Thus, we feel confident in conjecturing that the results we have obtained, at first order in supersymmetry breaking, are smoothly connected to the limit $m_Q^2, m_g \to \infty$, in which the superpartners decouple and the system reverts to an ordinary Yang-Mills theory with fermions. It is reasonable that this smooth extrapolation should apply quite generally for $N_f < N_c$. We will need to explore case by case whether a similar extrapolation can hold for larger numbers of flavors.

There are two features of this extrapolation which deserve further comment. First, in QCD, chiral symmetry breaking is characterized by a nonzero vacuum expectation value of the quark-antiquark bilinear, $\psi_Q^i \psi_{\overline{Q}j}$ in our present notation. In the language of the supersymmetric effective Lagrangian, this operator is a part of the F term of the superfield $T^i{}_j$. The expectation value of this term may easily be found to be proportional to

$$t_*^{-\frac{N_c}{N_c - N_f}} \sim m_Q^{\frac{2N_c}{2N_c - N_f}} . \qquad (33)$$

Thus, the F term of T does obtain an expectation value in the vacuum state that we have found. This expectation value naturally becomes a nonzero expectation value for the quark bilinear in the decoupling limit. As m_Q increases, the quark bilinear becomes larger while the squark bilinear becomes smaller, in exact accordance with our expectations.

When m_Q^2 is small, the vacuum we have identified occurs at a very large value of $\langle T \rangle$. When $\langle T \rangle$ is large, the behavior of supersymmetric Yang-Mills theory can be described classically, as the spontaneous breaking of the $SU(N_c)$ gauge symmetry by squark field vacuum expectation values. In other words, the gauge symmetry is realized in the Higgs phase. However, since the matter fields belong to the fundamental representation, there is no invariant distinction between the Higgs and confinement phases of this model, and so there is no impediment to the Higgs phase at small m_Q^2 being smoothly connected to a confinement phase at large m_Q^2.

IV. $N_f = N_c$

In the case $N_f < N_c$, we have found a very natural connection between the physics of the theory with weak

supersymmetry breaking and the physics of the theory after the supersymmetric partners have been decoupled. For larger numbers of flavors, however, this connection will become increasingly tenuous.

We next consider the case $N_f = N_c$. Here the low-energy effective Lagrangian of the supersymmetric limit contains both meson and baryon superfields. In this special situation, the baryon fields B, $\overline{B}$ are flavor singlets, and both the meson fields $T^i{}_j$ and the baryon fields have zero R charge. Seiberg has argued [6] that this model has a manifold of supersymmetric ground states, in which the meson and baryon fields satisfy the relation (in units where $\Lambda = 1$)

$$\det T - B\overline{B} = 1 . \tag{34}$$

Many forms for the superpotential are consistent with this relation. The S-dependent superpotential, for example, has the form

$$W = S\ln\left(\det T - B\overline{B}\right). \tag{35}$$

Note that this superpotential leads to conditions for a supersymmetric vacuum state which imply not only (34) but also the constraint $\langle S \rangle = 0$, so that the $U(1)_R$ symmetry is not spontaneously broken.

A. Location of the vacuum states

The presence of a manifold of degenerate vacuum states not related by a global symmetry is necessarily accidental unless it is a a consequence of supersymmetry. Thus, any such degeneracy should be broken as soon as supersymmetry-breaking terms are added to the Lagrangian. At first order, this is the main effect of the soft supersymmetry-breaking perturbation. To analyze this effect, we should restrict our attention to the values of T, B, and $\overline{B}$ obeying the constraint (34), for which the vacuum energy vanishes in the supersymmetric limit, and study the behavior of the supersymmetry-breaking potential over this space.

For simplicity, we begin with the R models, for which $m_g = 0$. Then the soft supersymmetry-breaking terms (11) lead to the potential

$$\Delta V = B_T m_Q^2 \text{tr}\left[T^\dagger T\right] + B_B m_Q^2\left(B^\dagger B + \overline{B}^\dagger \overline{B}\right). \tag{36}$$

Using $SU(N_f) \times SU(N_f)$, we can diagonalize T to complex eigenvalues t_i. Parametrize the baryon fields as

$$B = xb , \quad \overline{B} = -\frac{1}{x}b , \tag{37}$$

with x and b complex. Then b obeys the constraint

$$\prod_i t_i + b^2 = 1. \tag{38}$$

The variable x appears in the potential only through the baryon mass term

$$\Delta V = \cdots + B_B m_Q^2\left(|x|^2 + \left|\frac{1}{x}\right|^2\right)|b|^2 , \tag{39}$$

and this is minimized at $|x| = 1$ for any b. Thus, we may set $|x| = 1$.

The problem becomes that of minimizing

$$\Delta V = B_T m_Q^2 \sum_i |t_i|^2 + 2B_B m_Q^2 |b|^2 \tag{40}$$

subject to the constraint (38). There are three types of stationary points of this potential:

(1) If $b = 0$, ΔV is stationary when $|t_i|$ are all equal:

$$|t_i| = 1 , \quad \prod_i t_i = 1 , \quad b = 0 . \tag{41}$$

(2) If $T = 0$, ΔV is stationary:

$$T = 0 , \quad b = \pm 1 . \tag{42}$$

(3) If neither T nor b vanish, there can be an additional stationary point with $|t_i|^{(N_f - 2)} = (B_T/B_B)$ for all i. This point is always unstable with respect to the other vacuum states.

The shape of the potential ΔV, for three choices of (B_T/B_B), is shown in Fig. 2. Notice that the vacuum at $b = 0$ is the absolute minimum for sufficiently large values of (B_B/B_T), but that the vacuum at $T = 0$ is always a local minimum.

The method of effective Lagrangians cannot tell us which of the two vacuum states at $b = 0$ and $T = 0$ is the preferred one. This depends on the ratio B_B/B_T, which is a phenomenological input to the effective Lagrangian analysis. We will see below that the vacuum at $T = 0$ is locally stable if $B_B > B_T$ and is globally stable if $B_B > (N_f/2)B_T$. In (12), we attempted to estimate the ratio of B_B and B_T. Our naive estimate puts the theory just at the boundary at which the two vacuum states have equal energy. Probably, this question can only be decided by computer simulations. We note, however, that if the vacuum structure of this theory were being studied in a lattice simulation, one could bias the simulation in favor of one vacuum or the other by adding an explicit B_B or B_T term to the Lagrangian. In the
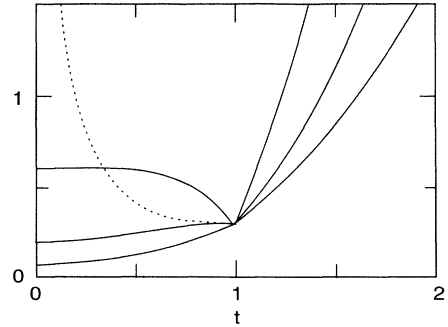

FIG. 2. The potential ΔV for softly broken supersymmetric Yang-Mills theory with $N_c = 3$, $N_f = 3$. The potential is shown on the subspace $T = t \cdot 1$, as a function t. The three curves correspond to $(B_B/B_T) = \frac{1}{3}, 1, 3$, from bottom to top. The dotted line shows the location of the stationary point (3) referred to in the text.

discussion to follow, we will treat each locally stable vacuum state as if it could be separately realized in a such a computer experiment.

Up to now, we have ignored the possible effects of the $U(1)_R$-violating supersymmetry breaking term proportional to m_g. However, these effects cannot change the qualitative picture when m_g is small. We showed earlier that the superpotential (35) implies that, in the manifold of supersymmetric vacuum states about which we are perturbing, $\langle S \rangle = 0$. Thus, the superpotential term proportional to M_g does not contribute to the vacuum energy. More generally, since M_g, T, B, and $\overline{B}$ are all invariant under $U(1)_R$, while a superpotential has R charge 2, this term does not contribute to the superpotential to any order in m_g. There are possible Kähler potential terms involving M_g. (The simplest one will be discussed in a moment.) However, near the vacuum with $b = 0$, these will be polynomials in B and $\overline{B}$ of order at least 2, and near the vacuum with $T = 0$ they will be polynomials in T of order at least 2. Thus, these terms will not affect the presence of stationary points of the vacuum energy at these positions in the field space. These terms may alter the details of the mass spectrum computed below, but they will not alter the qualitative physical picture of vacuum stability which follows from this calculation.

B. Spectrum at $b = 0$

We will now work out the spectrum of particle masses at the two candidate vacuum states that we have identified. The boson masses can be found by expanding the potential (36) about the two vacuum states, with fields subject to the constraint (34). At this level, the fermionic partners of these fields remain massless. Fermion masses will be induced when we include effects of first order in m_g.

To expand about the vacuum at $b = 0$, parameterize T as in (23), with $t_* = 1$, and parametrize

$$B = b + c , \quad \overline{B} = -(b - c) . \tag{43}$$

The complex fields b and c have kinetic energy terms proportional to the factor $(2A_B)$, which must be divided out in computing masses. The fields t_V and t_A in (23) are removed by the constraint. To leading order, (34) implies

$$t_V + it_A = -\sqrt{\frac{2}{N_f}}(b^2 - c^2) \tag{44}$$

to quadratic order in baryon fields. Now we simply expand ΔV and read off the spectrum of masses. We find, respectively for the masses of t_{Vi}, the real part of b and the imaginary part of c, and the imaginary part of b and the real part of c:

$$m_{Vi}^2 = \frac{2}{A_T}B_T m_Q^2,$$

$$m_-^2 = \frac{2}{A_B}(B_B - B_T)m_Q^2,$$

$$m_+^2 = \frac{2}{A_B}(B_B + B_T)m_Q^2 . \tag{45}$$

Notice that, just at $B_B = B_T$, when the unstable stationary point (3) interposes itself between the $b = 0$ and $T = 0$ vacuum states, the $b = 0$ vacuum becomes locally stable with respect to baryon-number-violating fields. Since the energies of the $b = 0$ and $T = 0$ vacua are $(N_f B_T m_Q^2)$ and $(2B_B m_Q^2)$, respectively, the vacuum at $T = 0$ remains the global minimum of the potential as long as

$$B_B < \frac{N_f}{2}B_T , \tag{46}$$

as we claimed at the end of the previous section.

The expectation value of T in this vacuum spontaneously breaks $SU(N_f) \times SU(N_f)$ to $SU(N_f)_V$. The fields t_{Ai}, which are the Goldstone bosons corresponding to this symmetry breaking, remain at zero mass. Since $U(1)_R$ is not spontaneously broken, we expect no singlet Goldstone boson in the spectrum, and, indeed, none appears.

Since (34) is a superfield constraint, it also removes one fermion from the theory, specifically, the fermionic partner of $\mathrm{tr}[T]$. The other fermionic components of T, B, and $\overline{B}$ remain at zero mass at this level of the analysis. This is natural, because the mass terms for these fields violate $U(1)_R$ by 2 units. Thus, these mass terms can only be induced when the R-charge-breaking term proportional to m_g is added. We have noted above that this term cannot induce a superpotential. However, it can induce a D-term contribution of the form indicated in (11). There are many possibilities for such a term; a set of simple examples is given by

$$\Delta\mathcal{L} = \int d^4\theta \overline{M}_g\left(C_T \det T + C_B B\overline{B}\right) + \mathrm{H.c.} , \tag{47}$$

where C_T and C_B are some constants. If one begins from the effective Lagrangian including S, with the canonical superpotential and Kähler terms,

$$\mathcal{L} = \int d^4\theta S^* S + \int d^2\theta (S\ln(\det T - B\overline{B}) + M_g S) + \mathrm{H.c.} , \tag{48}$$

and integrates out S, one finds

$$\Delta\mathcal{L} = \int d^4\theta \overline{M}_g \ln(\det T - B\overline{B}) + \mathrm{H.c.}, \tag{49}$$

which gives qualitatively similar results. In the following discussion, we will work with (47).

To obtain baryon masses from (47), expand the superfields about the vacuum state $T = 1$, $b = 0$ according to (28) and

$$B = \theta \cdot \psi_B , \quad \overline{B} = \theta \cdot \psi_{\overline{B}} . \tag{50}$$

We find, for the flavor adjoint and baryonic fermions, the masses

$$m_{\psi i} = \frac{1}{2}\frac{C_T}{A_T}m_g,$$

$$m_{\psi B} = \frac{C_B}{A_B}m_g . \tag{51}$$

No zero-mass fermions remain.

C. Spectrum at $T = 0$

Using the same techniques, we can work out the spectrum of masses in the vacuum at $T = 0$. For the scalars, parametrize B and $\overline{B}$ by

$$B = (1 + b + c) , \quad \overline{B} = -(1 + b - c) . \tag{52}$$

The constraint (34) allows us to eliminate b:

$$b = -\frac{1}{2} \det T + \frac{1}{2} c^2 . \tag{53}$$

The contribution from T is higher order than quadratic and so does not affect the mass spectrum. Inserting (52) and (53) back into ΔV and expanding to quadratic order, we find the following masses for the components of T and the real part of c:

$$m_T^2 = \frac{B_T}{A_T} m_Q^2 , \quad m_{cR}^2 = \frac{B_B}{A_B} m_Q^2 . \tag{54}$$

The imaginary part of c remains at zero mass, which is expected, because this field is the Goldstone boson of spontaneously broken baryon number symmetry $U(1)_B$.

The constraint (34) removes one linear combination of the fermionic components of the baryon fields. Otherwise, no fermion masses appear until we add the R-symmetry-breaking terms involving m_g. Then the term (47) gives mass to the remaining baryonic fermion,

$$m_{\psi B} = \frac{2C_B}{A_B} m_g , \tag{55}$$

but it leaves the fermionic components of T massless.

D. Toward the decoupling limit

In the vacuum state at $b = 0$, when we include a nonzero gluino mass m_g, we find again the standard symmetry-breaking pattern expected in QCD. The global group $SU(N_f) \times SU(N_f) \times U(1)_B$ is broken spontaneously to $SU(N_f)_V \times U(1)_B$, leaving no massless particles except for the required Goldstone bosons. It is reasonable to expect that here, as in the cases considered in Sec. III, there is a smooth transition from the situation of weak supersymmetry breaking to the decoupling limit $m_Q^2, m_g \to \infty$. The symmetry-breaking term (47) also induces a nonzero F term for the $SU(N_f)_V$ singlet part of T. This term should go naturally, in the decoupling limit, into the chiral symmetry-breaking expectation value of the quark-antiquark bilinear.

However, all of the other vacuum states that we have identified are unusual and unexpected. All of them contain massless composite fermions. The vacua at $T = 0$ have restored chiral symmetry and spontaneously broken baryon number. Could these vacuum states survive to large values of the supersymmetry-breaking parameters?

To answer this question, we must first understand why these vacua contain massless fermions. In general, in a strongly coupled gauge theory, chiral symmetries with nonzero anomalies generate sum rules over the spectrum of zero mass particles. These sum rules can be saturated either by Goldstone bosons, if one of the symme-

tries is spontaneously broken, or by massless composite fermions, if the symmetries remain exact. In the latter case, the anomalies computed from the composite fermions must match the anomalies of the original fermions; this is the 't Hooft anomaly matching condition [1, 17].

The three unusual vacua discussed in this section, the $b = 0$ vacuum of the R model and the $T = 0$ vacua of the R and $\not{R}$ models, all have unbroken anomalous chiral symmetries. In all cases, the fermionic content of the supersymmetric model is known to provide a solution to the 't Hooft anomaly conditions associated with these symmetries [6, 18]. In fact, one might say that the fermions are protected from obtaining masses by the 't Hooft anomaly conditions, because providing masses for a subset of the multiplet of fermions would leave over a set of fermions which violates the 't Hooft conditions and is therefore inconsistent, unless the chiral symmetry is broken.

An interesting illustration of this argument is found by comparing the spectra of massless fermions in the two vacuum states at $T = 0$ in the R and $\not{R}$ models. In the R model, we have massless fermions in the following representations of the unbroken symmetry group $SU(N_f) \times SU(N_f) \times U(1)_R$:

$$(N_f, \overline{N_f}, -1) + (1, 1, -1) , \tag{56}$$

corresponding to the fermions in T and a linear combination of the fermions in B and $\overline{B}$. Both multiplets are necessary to satisfy the anomaly conditions involving $U(1)_R$. When $U(1)_R$ is broken explicitly by m_g, these conditions no longer need to be satisfied, and so the baryonic fermions can obtain mass. According to (55), they do.

Because the massless composite fermions in these vacuum states exist in order to satisfy the 't Hooft anomaly conditions, the qualitative properties of these vacuum states are quite rigid. We should recall that the $T = 0$ vacuum and the $b = 0$ vacuum, for $B_T < B_B$, are locally stable minima of the energy for sufficiently small m_Q^2; thus, there is a finite range of m_Q^2 for which the pattern of symmetry breaking remains unchanged. Given this pattern of symmetry breaking, the multiplet of composite fermions cannot obtain mass. Even if the composite fermions contain as constituents bosons Q or $\overline{Q}$ which obtain mass from the m_Q^2 term, the composites are bound rigidly to remain at zero mass. This idea, that composites of massive constituents may be forced to remain massless in order to satisfy the 't Hooft condition, was formulated by Preskill and Weinberg [11].

Even if a vacuum with unbroken chiral symmetry is globally unstable to tunneling processes, the 't Hooft argument applies as long as it is locally stable. Thus, a vacuum with unbroken chiral symmetry can only disappear, as $m_Q^2, m_g \to \infty$, through a second-order phase transition.

With this introduction, we can speculate on the evolution of these vacuum states as m_Q^2 and m_g are increased from zero. Consider first the $b = 0$ vacuum of the R model. As m_Q^2 is taken to infinity, the squarks decou-

ple, and the model becomes a purely fermionic Yang-Mills theory with N_f quark flavors plus one fermion flavor in the adjoint representation of the gauge group. For small values of the supersymmetry-breaking mass m_Q^2, this vacuum contains massless fermions corresponding to the fermionic components of the superfields T, B, and $\overline{B}$. We might think of these as being built out of scalars, with one squark replaced by a quark to give the composite spin $\frac{1}{2}$. But it is also possible to build objects with the same quantum numbers purely out of fermions, by replacing

$$Q^i \rightarrow \lambda^\alpha \psi_{Q\alpha}^i \; , \quad \overline{Q}_j \rightarrow \lambda^\alpha \psi_{\overline{Q}\alpha j} \; , \tag{57}$$

where α is a two-component spinor index and the gauge indices are implicit. Notice that this combination has the same quantum numbers as the squark, including zero R charge. Then, for example, the fermion created by $T^i{}_j$ could be constructed as

$$\psi_{T\alpha}{}^i{}_j \rightarrow \psi_{Q\alpha}^i \lambda^\beta \psi_{\overline{Q}\beta j} \; . \tag{58}$$

With this replacement, the composite fermions are built only out of constituents which remain massless as the squarks are decoupled. Thus, it is *a priori* reasonable that the $b = 0$ vacuum of the R model could go smoothly into a vacuum of the purely fermionic Yang-Mills theory described above. This vacuum would have broken SU(N_f) × SU(N_f) but unbroken chiral U(1)$_R$, zero values for the vacuum expectation values of quark-antiquark bilinears, massless composite fermions in the adjoint representation of flavor SU(N_f)$_V$, and massless baryons. We will refer to this scenario as "option 1." It will have analogues in the models to be discussed later; however, these analogous phases will be less well motivated. It is easy to see that the replacement (57) can formally be used to build composite fermions with only fermionic constituents in any model with unbroken R symmetry.

The other possibility for this model is that, after the squarks decouple, the gluino fields pair condense, in a second-order phase transition at some value of m_Q^2, and the nonzero value of the condensate $\langle \lambda \cdot \lambda \rangle$ spontaneously breaks U(1)$_R$. In this case, the physics would revert to the usual symmetry-breaking pattern of QCD, and the composite fermions would become massive. The gluino condensate would make itself felt only by providing an extra SU(N_f)-singlet Goldstone boson. We will refer to this scenario as "option 2."

One way to understand the physical distinction between options 1 and 2 is to consider a question raised some time ago but never answered in a satisfactory way: If a fermion in a large color representation is added to QCD, does its pair condensation to chiral symmetry breaking occur at the usual QCD scale or at much shorter distances [19, 20]? If gluinos pair condense at very short distances, before normal quarks feel the full forces of the strong interactions, then option 2 would be favored. If gluinos feel strong interactions at more or less the same scale as quarks, option 1 is a reasonable possibility. An extreme model in which gluinos condense and decouple at a very high scale, as suggested in the papers just cited, appears unlikely as a result of our analysis, because we

know that option 1 is actually realized when squarks are added back to the model.

Consider next the vacuum state at $T = 0$, first in the $\not{R}$ case. In this model, the massless composite fermions belong to the $(N_f, \overline{N_f})$ representation of an unbroken flavor group SU(N_f) × SU(N_f). There are no constraints from the U(1)$_R$ symmetry, which is explicitly broken, or from baryon number, which is spontaneously broken. With this freedom, can we build these fermionic composites out of fields that survive in the decoupling limit $m_Q^2, m_g \rightarrow \infty$? For N_c even, it is impossible, because the only constituents available are the quarks ψ_Q^i, $\psi_{\overline{Q}j}$, and gauge-invariant states must contain an even number of these. For N_c odd, however, it is possible to build composites with the correct quantum numbers, as follows:

$$\psi_{T\alpha}{}^i{}_j \rightarrow \epsilon^{ab\cdots d} \psi_{Q\alpha a}^i \, \epsilon_{jk\cdots \ell} \overline{\psi}_{\overline{Q}b}^{\dot\beta k} \cdots \overline{\psi}_{\overline{Q}\dot\gamma d}^{\ell} \; , \tag{59}$$

where $\overline{\psi}_{\overline{Q}}$ is the right-handed fermion field in $(\overline{Q})^*$. The $(N_c - 1)$ right-handed fermion fields must be contracted into a Lorentz scalar combination. For the case $N_f = N_c = 3$, eight of the nine fermions in (59) have the quantum numbers of the baryon octet in QCD.

However, in this case, there are two compelling arguments that the spectrum which we find cannot survive to the decoupling limit. In the limit $m_Q^2 \rightarrow \infty$, even without introducing m_g, we have a vectorlike gauge theory of fermions. For such theories, the QCD inequalities of Weingarten [21] and Vafa and Witten [22] apply. In the Appendix, we use Weingarten's method to prove that, in the decoupling limit, flavor-nonsinglet composite fermions must be heavier than the pions, which are massive in the $T = 0$ vacuum. Alternatively, we can apply the theorem of Vafa and Witten in the decoupling limit to show that vectorlike global symmetries, in particular, baryon number, cannot be spontaneously broken.

By either argument, the $T = 0$ vacuum state must disappear in a second-order phase transition at a finite value of m_Q^2. Most likely, this vacuum becomes locally unstable with respect to a decrease in the expectation value of b, driving the theory back to the more familiar vacuum at $b = 0$.

Finally, we may consider the $T = 0$ vacuum in the R models. The arguments that we have just presented for the $T = 0$ vacuum in the $\not{R}$ models apply equally well to the R case. Again, we must have a second-order transition, probably with an instability to the $b = 0$ vacuum. There are then two possible end points, depending on which option is chosen for the $b = 0$ vacuum. If the option 1 for the $b = 0$ vacuum is correct, it is not necessary that U(1)$_R$ be spontaneously broken in this transition.

V. $N_f = (N_c + 1)$

So far we have considered separately models with $N_f < N_c$ and models with $N_f = N_c$. The cases where the number of flavors exceeds the number of colors fall into two classes, those of $N_f = N_c + 1$ and those of $N_f > N_c + 1$. These two classes of theories have qualitatively similar physics, in both cases much simpler than that of

$N_f = N_c$.

In the case of $N_f = N_c + 1$, like in the case of $N_f = N_c$, the low-energy effective Lagrangian in the supersymmetric limit is expressed in terms of the baryon, antibaryon, and meson superfields. However, now these superfields are not R neutral, and the baryons are not flavor singlets. Rather, they transform in the representations of the global symmetry (2),

$$B: \; (\overline{N_f}, {}^1)_{1,1-\frac{1}{N_f}}, \quad \overline{B}: \; (1, N_f)_{-1, 1-\frac{1}{N_f}},$$

$$T: \; (N_f, \overline{N_f})_{0, \frac{2}{N_f}}, \tag{60}$$

where the second subscript is the R charge of the scalar component of the superfield. In the supersymmetric theory the low-energy effective theory is described (at least near the origin of moduli space) by the Kähler potential given by (8) and by the following superpotential [6]:

$$W = B_i T^i_j \overline{B}^j - \det T. \tag{61}$$

The supersymmetric vacuum is, thus, described by a moduli space characterized by

$$B_i T^i_j = 0, \quad T^j_i \overline{B}^i = 0,$$

$$\frac{1}{N_c!} \epsilon_{i_1, \ldots, i_{N_f}} \epsilon^{j_1, \ldots, j_{N_f}} T^{i_1}_{j_1} \cdots T^{i_{N_c}}_{j_{N_c}} - B_{i_{N_f}} \overline{B}^{j_{N_f}} = 0. \tag{62}$$

As was argued in [6], these equations correctly describe the moduli space of vacuum states in the full quantum theory. At the origin of the moduli space, $\langle T \rangle = \langle B \rangle = \langle \overline{B} \rangle = 0$, where the full global symmetry (2) remains unbroken, there is a further consistency check for the low-energy behavior. The fermionic components of the low-energy superfields (60) match the global anomalies of the underlying theory.

A. Vacuum

When we break supersymmetry by squark and gluino masses, we add to the effective Lagrangian the mass terms for T, B, and $\overline{B}$ indicated in (11). Since we are adding terms to the potential which are positive and vanish at the origin of moduli space, it is obvious that the origin becomes the only vacuum state of the theory. All of the scalar particles in the effective theory obtain mass terms proportional to $B_T m_Q^2$ or $B_B m_Q^2$.

Though all of the scalars obtain mass, all of the fermions remain massless. The superpotential (61) is at least cubic in fields, and so any mass term derived from this superpotential vanishes at the origin. Similarly, in the $\not{R}$ case, the M_g term in (11) requires a function of T, B, and $\overline{B}$ which is neutral with respect to the global group; the only such functions quadratic in fields are $\text{tr}[T^\dagger T]$, $B^\dagger B$, and $\overline{B}^\dagger \overline{B}$, and these do not give fermion masses when integrated with M_g. In fact, it is required that no fermions should obtain mass, since the full multiplet of fermions in T, B, and $\overline{B}$ is needed to satisfy the 't Hooft anomaly conditions for the remaining global symmetry group $SU(N_f) \times SU(N_f) \times U(1)_B$.

B. Toward the decoupling limit

The analysis of the previous section indicates that in a finite region of small m_Q and m_g, the ground state of the theory is a smooth continuation of the origin of the moduli space of supersymmetric vacuum states. In this region of soft supersymmetry-breaking parameters, chiral symmetry is unbroken, and the full complement of fermions is kept massless by the requirement that the 't Hooft anomaly conditions be satisfied. In the $\not{R}$ case, since both gluinos and squarks are massive, at least some of the massless composite fermions must have massive constituents. As in our earlier examples, these particles are protected from receiving mass by the 't Hooft conditions.

In neither the $\not{R}$ nor the R case, however, can this spectrum of particles be correct in the decoupling limit. In that limit, the Weingarten inequality proved in the Appendix prohibits a composite fermion which is nonsinglet in flavor from remaining massless while the pion is massive. In both cases, then, the phase we have found at small m_Q must disappear at a second-order phase transition when m_Q reaches a critical value. In the $\not{R}$ case, the theory has no option but to revert to the conventional pattern of symmetry breaking in which the chiral symmetry group is broken to $SU(N_f)_V \times U(1)_B$ and all fermions become massive.

For the R case, however, there are still two options, corresponding to options 1 and 2 described in Sec. IV D. Option 2 is the scenario just described for the $\not{R}$ case, with symmetry breaking to $SU(N_f)_V \times U(1)_B$ and one extra Goldstone boson. Option 1 is the breakdown of the chiral symmetry group only to $SU(N_f)_V \times U(1)_B \times U(1)_R$. In order to satisfy the 't Hooft anomaly conditions associated with the $U(1)_R$, all of the fermionic components of T, B, and $\overline{B}$ must remain massless. As in the case considered in Sec. IV D, we can build all of the required massless fermions out of quarks and gluinos by using the replacement (57). In this case, as opposed to that of Sec. IV D, the partial symmetry breaking required in option 1 is not particularly well motivated. However, we have not been able to rule it out as a possibility. We should also note that, even if this case is realized in the more conventional option 2, the case $N_f = N_c$ could be realized in option 1. There is no theorem that, when one quark becomes very heavy, fermions not containing that quark cannot become massless.

VI. $N_f \geq (N_c + 2)$

No solution of the 't Hooft anomaly matching conditions for SQCD involving gauge-invariant bound states is known for $N_f > (N_c + 1)$. However, Seiberg has suggested a compelling solution to these constraints in terms of new gauge degrees of freedom which are dual to the original quarks and gluons [7]. In this picture, the theory is equivalent in the infrared to an SQCD theory with gauge group $SU(N_f - N_c)$, N_f dual quark flavors, and additional singlet fields T^i_j identified with the mesons of the original theory. The original SQCD theory is infrared

free for $N_f \geq 3N_c$, so that in that case the low-energy description of the theory is in terms of the original quarks and gluons. For $N_f \leq \frac{3}{2}N_c$, the dual "magnetic" theory is infrared free, and then the low-energy description of the theory should be in terms of the dual quarks, gluons, and the singlet meson fields. In the intermediate range $\frac{3}{2}N_c < N_f < 3N_c$ both theories are asymptotically free. Seiberg suggested that, in this region, the theory has a nontrivial infrared fixed point, and the theory has dual descriptions in the infrared as interacting gauge theories with superconformal global symmetry. While the origin of this dynamically generated gauge symmetry is still unclear, there is ample evidence that Seiberg's description of the SQCD theory is correct, and we will assume it throughout this section.

If we break supersymmetry by giving masses to some of the fields of SQCD, the leading term of the β function will change for distances greater than the scale of the masses. The long-distance gauge theories will be asymptotically free in a larger range of N_f, for $N_f < \frac{9}{2}N_c$ after adding squark masses, and for $N_f < \frac{11}{2}N_c$ after adding squark and gluino masses. Beyond the point where the theory is asymptotically free, we expect the effects of adding soft SUSY-breaking mass terms to be trivial. The massless particles are expected to be infrared free, and there is no reason for the chiral symmetry to break. We will concentrate our analysis, then, on the cases of N_f relatively close to the boundary $(N_c + 2)$, where the original gauge theory becomes strongly coupled and the dual description is appropriate in the infrared. In the next subsection we will analyze the effect of adding soft SUSY-breaking mass terms on the dual description of the theory. In the second subsection we will discuss in what range of N_f we expect this dual description to be relevant, and speculate on the infrared behavior of the theory for different values of N_f.

A. Spectrum and vacuum of the dual theory

Seiberg's dual description of SQCD has an $SU(\tilde{N}_c)$ local gauge symmetry, where $\tilde{N}_c = (N_f - N_c)$ as in (5). The elementary fields in the dual theory are an $SU(\tilde{N}_c)$ supergauge multiplet, N_f flavors of dual quarks q_i^a and antiquarks $\bar{q}_a^j$ in the fundamental and antifundamental representations of $SU(\tilde{N}_c)$, respectively, and meson fields $T^i{}_j$. The quark fields are in the $(\{\overline{N_f}, 1)$ representation of the $SU(N_f) \times SU(N_f)$ flavor group, the antiquark fields are in the $(1, N_f)$ representation, and the meson fields are in the $(N_f, \{\overline{N_f})$ representation. It is useful to think that the dual quarks are obtained by dissociating a baryon (6) into $\tilde{N}_c$ components, and that the new gauge fields parametrize a constraint which gives these baryons as its solutions. Seiberg also requires a superpotential

$$W = T^i{}_j q_i^a \bar{q}_a^j, \tag{63}$$

so that the scalar potential, including the F and D terms, is

$$V(T, q, \bar{q}) = \frac{1}{A_T}(q^\dagger)_i^a(\bar{q}^\dagger)_a^j q_i^b \bar{q}_b^j$$

$$+ \frac{1}{A_q}\left[(T^\dagger)^i{}_j(\bar{q}^\dagger)_a^j T^i{}_k \bar{q}_a^k + (T^\dagger)^i{}_j(q^\dagger)_i^a T^l{}_j q_l^a\right]$$

$$+ \frac{g^2}{2}[(q^\dagger)_i \tau^A q_i - (\bar{q}^\dagger)^j \tau^A \bar{q}^j]^2, \tag{64}$$

where g is the $SU(\tilde{N}_c)$ gauge coupling, and τ^A are the $SU(N_f - N_c)$ generators. A_T and A_q are the coefficients of the corresponding (canonical) kinetic terms. This scalar potential has a moduli space of vacua, which includes the point $\langle T \rangle = \langle q \rangle = \langle \bar{q} \rangle = 0$ at which the chiral symmetry is unbroken [7].

Now add squark masses to the theory. Their effect should be seen in the effective Lagrangian, and we can represent it by applying the logic of Sec. II C to the dual theory. That is, we should add to the effective Lagrangian of the dual theory the term

$$\Delta V = B_T m_Q^2 \text{tr}(T^\dagger T) + B_q m_Q^2(|q|^2 + |\bar{q}|^2), \tag{65}$$

at least near the origin of moduli space. After we add this perturbation, the only minimum of the potential is at $\langle T \rangle = \langle q \rangle = \langle \bar{q} \rangle = 0$. Thus, adding a squark mass leaves the theory in the phase in which the chiral symmetry is unbroken. All scalars get masses (originating only from ΔV, since the original scalar potential is quartic in the fields), while all fermions remain massless. As in the original supersymmetric theory, this complement of massless fermions has just the right quantum numbers to satisfy the 't Hooft anomaly conditions for completely unbroken chiral symmetry. Thus, our picture of the effect of soft supersymmetry breaking in this case is just the same as in the case $N_f = (N_c + 1)$ considered in the previous section, except that the baryons of that case are replaced here by their constituent dual quarks.

The glueball operator $\text{tr}(W_\alpha^2)$ is identified (up to a sign) between the original and the dual theory [23]. Thus, to leading order in m_g, a gluino mass in the original theory is just equal to a gluino mass in the dual theory. Adding this term breaks the $U(1)_R$ symmetry, but the $SU(N_f) \times SU(N_f)$ global symmetry still remains and protects the dual quarks from getting a mass. Thus, we find the same spectrum in the R and $\tilde{R}$ cases, except that in the latter case the dual gluino, which can be an asymptotic particle, becomes massive.

B. Toward the decoupling limit

Let us discuss now the infrared description of the theory. We consider first the case of small m_Q (and small m_g, in the $\tilde{R}$ case). We have already remarked that, for $N_f > \frac{11}{2}N_c$ ($N_f > \frac{9}{2}N_c$ in the $\tilde{R}$ case), the theory becomes free in the infrared and is well described in terms of the original variables—gluons and quarks (and gluinos in the $\tilde{R}$ case). This statement applies equally well to the dual version of the theory. Thus, for $N_f > \frac{11}{2}\tilde{N}_c$, or $N_f < \frac{11}{9}N_c$, the dual theory is free in the infrared. For the $\tilde{R}$ theory, the corresponding criterion is $N_f < \frac{9}{7}N_c$. In this range of N_f, the spectrum of the theory contains

massless dual quarks interacting weakly through a dual gauge field which becomes asymptotically weak at large distances. Unfortunately, this range of N_f is rather narrow; the first example requires an SU(8) gauge group and ten flavors, even in the $\not{R}$ case.

However, it is likely that Seiberg's duality would still hold in the intermediate range of N_f: $\frac{11}{9}N_c < N_f < \frac{11}{2}N_c$ in the R case and $\frac{9}{7}N_c < N_f < \frac{9}{2}N_c$ in the $\not{R}$ case. As in the supersymmetric case, we can prove the existence of an infrared fixed point for values of N_f very close to the boundary of this region by using the fact that the second coefficient of the QCD β function is positive when the first coefficient vanishes [24]. Thus, some part of this intermediate range is controlled by a nonsupersymmetric infrared fixed point. At least when the fixed point coupling is sufficiently small, the chiral symmetries of the theory remain unbroken and the spectrum still contains massless quarks or dual quarks. If at some value of N_f, the massless fermions are no longer asymptotic states, and then also the solution to the 't Hooft anomaly conditions is lost and the theory reverts to a scenario with broken chiral symmetry.

The discussion of the decoupling limit for these theories is very similar to that for the $N_f = (N_c + 1)$ theory. If the full chiral symmetry group remains unbroken for small values of m_Q, the fermions in the supermultiplet T still cannot remain massless in the decoupling limit where we have the QCD inequality, precisely as discussed in Sec. V B. Thus, those values of N_f which have massless fermions for small values of m_Q must have a second-order phase transition as m_Q is increased. It is not clear how the theory behaves on the other side of this phase transition. In the $\not{R}$ case obviously only a $SU(N_f)_V \times U(1)_B$ symmetry remains, with no massless fermions. However, in the R case, we can use the dual fermions in T, q, and $\bar{q}$ to solve the 't Hooft anomaly equations associated with $U(1)_R$. Thus, in this case, we have available both option 1, in which the chiral group is broken to $SU(N_f)_V \times U(1)_B \times U(1)_R$ and all fermions remain massless, and option 2, in which the chiral group is broken to $SU(N_f)_V \times U(1)_B$ and all fermions obtain mass.

VII. $N_c = 2$

For $N_c = 2$ there is no distinction between massless quarks and antiquarks, so that the global symmetry in the supersymmetric limit is $SU(2N_f) \times U(1)_R$ instead of $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$. This changes some of the details in the discussions above, but does not change the qualitative picture. The meson is now given by $T^{ij} = Q^i Q^j$, in the antisymmetric representation of $SU(2N_f)$, and the superpotential generally involves $Pf(T)$ instead of $det(T)$. There are no baryon operators in this case; rather, the baryons of the previous examples are absorbed into the extended meson multiplet. In the usual QCD theory with two colors, the global symmetry breaks from $SU(2N_f)$ to $Sp(2N_f)$ [we denote by $Sp(2N_f)$ the Sp group whose fundamental representation is of size $2N_f$]. We shall now discuss the picture

after soft SUSY breaking for each relevant value of N_f.

For $N_f = 1$, the behavior is similar to the other cases $N_f < N_c$. The effective Lagrangian has a superpotential of the form $W = 1/Pf(T)$. There is just one vacuum, in which T^{12} obtains an expectation value, breaking the flavor symmetry from $SU(2) \times U(1)_R$ to $Sp(2)$ [which is isomorphic to $SU(2)$]. The meson T^{12} is the Goldstone boson for the breaking of the $U(1)_R$ symmetry in the R case; this particle obtains a mass when we add a gluino mass. A smooth transition is expected to the decoupling limit, as for $N_c > 2$.

For $N_f = 2$, the moduli space of supersymmetric vacuum states is constrained by the equation $Pf(T) = 1$ [6]. As in Sec. IV, the potential from the soft supersymmetry-breaking terms can be considered on the space satisfying this constraint. Then, up to global symmetry transformations, there is just one stable vacuum, for which

$$T = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}. \tag{66}$$

This breaks the SU(4) flavor symmetry to Sp(4). In the R case, the $U(1)_R$ symmetry is left intact. The fermionic fluctuations around the vacuum (66), which transform as 6_1 under $SU(4) \times U(1)_R$, decompose under $Sp(4) \times U(1)_R$ as $(5 + 1)_1$. For small values of the SUSY-breaking parameters, the fermions in 5_1 remain massless and satisfy the 't Hooft anomaly matching conditions for the unbroken symmetry group $Sp(4) \times U(1)_R$ [6]. In the R case, there are two options for the decoupling limit, as in the $b = 0$ vacuum of Sec. IV. In option 1, this spectrum continues smoothly to the decoupling limit. In option 2, the $U(1)_R$ symmetry is spontaneously broken and the fermions in the 5_1 obtain mass. In the $\not{R}$ case, as in the discussion of Sec. IV, the vacuum state obtained for small supersymmetry breaking has no massless fermions and can smoothly become the standard QCD vacuum as $m_Q \to \infty$.

For $N_f = 3$, the effective description of the SQCD theory has a superpotential of the form $W = -Pf(T)$ [6]. In the supersymmetric case there is a moduli space of vacua, but adding the squark masses leaves only the vacuum at $T = 0$, as for $N_c > 2$. At this vacuum the chiral symmetry is unbroken. All of the fermions, which are in the $15_{-1/3}$ representation of the global symmetry, remain massless, and this multiplet satisfies the 't Hooft anomaly matching constraints [6]. As in Sec. V, in the decoupling limit we expect a second-order phase transition, breaking the global symmetry from $SU(6) \times U(1)_R$ to $Sp(6) \times U(1)_R$ or to $Sp(6)$.

For $N_f \geq 4$, SQCD theory has a description in terms of dual gauge variables. For $N_f < 6$, the theory is conjectured to be described by an infrared fixed point. As in Sec. VI, we expect the theory near the supersymmetric point to be either at some nontrivial infrared fixed point with the chiral symmetry unbroken or to be in a QCD-like phase in which the chiral symmetry breaks to $Sp(2N_f)$. In these cases, the dual gauge group is always asymptotically free, and so we do not expect a phase in which the dual gauge symmetry is weakly coupled. Thus, the dynamically generated gauge symmetry suggested by

Seiberg should be difficult to identify in simulations with the gauge group SU(2).

VIII. PROBLEMS OF APPROXIMATE SUPERSYMMETRY ON THE LATTICE

Can the phenomena we have discussed in this paper be seen in lattice gauge theory simulations? Throughout this paper, we have considered only soft supersymmetry-breaking perturbations. However, since, in general, gauge theories on the lattice cannot be made supersymmetric at the fundamental level, we expect that lattice simulations of these theories will also contain small dimension-4 perturbations which violate supersymmetry. Our analysis has been based on the assumption that, if the phenomena discussed by Seiberg survive perturbations which are relevant in the infrared, they should also survive small marginal perturbations.

However, there is a serious difficulty with this logic. Our argument does not apply unless we can reach the continuum limit. But typically in lattice gauge theory simulations with scalar fields, there is no continuum limit; instead, one finds a first-order phase transition as a function of the scalar field mass parameter [25]. This fact is understood using the mechanism discovered by Coleman and Weinberg [26]: Renormalization effects in a gauge theory can induce an unstable potential for a scalar field coupled to the gauge bosons, leading to a "fluctuation-induced first-order phase transition." We must ask whether there is a possibility of such first-order phase transitions in approximately supersymmetric models, and, if so, how they can be avoided.

To analyze this question, consider the renormalization group equations for an approximately supersymmetric gauge theory. Viewed as a conventional renormalizable gauge theory, SQCD has three coupling constants, the gauge coupling g, the quark-squark-gluino coupling g_λ, and the four-scalar coupling g_D. The scalar potential has the specific form

$$V = \frac{g_D^2}{2}\left[Q^\dagger \tau^A Q - \overline{Q}\tau^A \overline{Q}^\dagger\right]^2 ,\qquad (67)$$

where τ^A is an SU(N_c) matrix. If we relax the constraint of supersymmetry, there are four possible invariants under the symmetries of the problem, including the continuous global symmetries and parity $Q \leftrightarrow \overline{Q}^\dagger$. The most general linear combination of these invariants can be generated by the renormalization group flow.

We will view the lattice theory as providing a finite cutoff for the quantum field theory, which does not respect supersymmetry. In this cutoff field theory we will choose the bare couplings to obey the supersymmetry relations, at least approximately. In particular, we will choose the bare scalar potential to be given by (67). The radiative corrections will cause a finite renormalization of the couplings, which will violate supersymmetry and generate other scalar potential terms. We expect the generated terms to be smaller than the original terms. Our analysis of the renormalization group flow of the theory will, therefore, be performed near the supersymmetric point.

In particular we will restrict our analysis to the surface given by the three couplings g, g_λ, and g_D. We assume that our initial conditions lie near this surface, and we are interested in the flow of the couplings towards the infrared. Note that since we we are not interested in scaling towards the continuum limit, we do not analyze here the flow of the couplings towards the ultraviolet. It is not possible to ensure that all couplings tend smoothly to zero in the ultraviolet without fine adjustment of their initial values.

In the surface given by g, g_λ, and g_D, the β functions of the three couplings are given (to leading order in perturbation theory) by

$$\beta_g = -\frac{1}{(4\pi)^2}\left[3N_c - N_f\right]g^3,$$

$$\beta_{g_\lambda} = -\frac{1}{(4\pi)^2}\{g_\lambda g^2\left[3N_c + 3C_2(N_c)\right]$$
$$-g_\lambda^3\left[3C_2(N_c) + N_f\right]\},$$

$$\beta_{g_D^2} = -\frac{1}{(4\pi)^2}\{4g_\lambda^4 N_c + 2g_D^4\left[N_c - N_f - 2C_2(N_c)\right]$$

$$+12g_D^2 g^2 C_2(N_c) - 8g_D^2 g_\lambda^2 C_2(N_c)\} ,\qquad (68)$$

where $C_2(N_c) = (N_c{}^2 - 1)/2N_c$. These three functions all reduce to the standard SQCD β function on the supersymmetric subspace; for $g^2 = g_\lambda^2 = g_D^2$, $\beta_g = \beta_{g_\lambda} = \beta_{g_D^2}/2g$. Note that, for $N_c \sim N_f$ and the three couplings in reasonable ratio, all three couplings are infrared unstable. In particular, g_D^2 is renormalized toward larger positive values.

The potential instability to a first-order phase transition arises because a new structure in the potential is induced by the renormalization group flow. To lowest order, the form of the potential induced is

$$V_E = \frac{g_E^2}{2}[Q^\dagger\{\tau^A, \tau^B\}Q + \overline{Q}\{\tau^A, \tau^B\}\overline{Q}^\dagger]^2 .\qquad (69)$$

On the surface $g_E^2 = 0$, the β function for g_E^2 is

$$\beta_{g_E^2} = -\frac{1}{(4\pi)^2}[4g_\lambda^4 - 3g^4 - g_D^4].\qquad (70)$$

This equation implies that, if one leaves out the gluinos, g_E^2 becomes negative in the infrared, leading to a fluctuation-induced first-order phase transition. According to (70), this effect is removed if the lattice simulation includes gluinos, and if the gluino coupling g_λ is large enough. If we choose initial conditions in which g_λ is slightly larger than g, Eq. (68) guarantees that this condition will be preserved along the renormalization group flow. Equation (70) then shows that no instability is generated in the perturbative region. Hopefully, this perturbative result remains valid as we flow towards the infrared.

With this provision to avoid possible first-order phase transitions, we expect that lattice simulations with an approximately supersymmetric action can reach the continuum limit and test our predictions for softly broken supersymmetric QCD.

IX. SUMMARY AND CONCLUSIONS

In this paper we investigated softly broken $N = 1$ supersymmetric QCD. We considered two types of soft breaking terms, associated with squark masses m_Q with or without additional gluino masses m_g. We denoted these cases by $\not{R}$ and R, respectively. In the limit of $m_Q, m_g \rightarrow \infty$ the $\not{R}$ case should go over to ordinary QCD, while in the R case, in the limit $m_Q \rightarrow \infty$, we recover QCD with an additional massless adjoint fermion. The two main questions that we addressed are the following:

To what extent do the results which were recently obtained for $N = 1$ SU(N_c) SQCD [6, 7], and for other $N = 1$ supersymmetric gauge theories as well, carry over to the nonsupersymmetric case? Is supersymmetry an essential prerequisite for those exotic phenomena?

How does the theory behave in the decoupling limit, in which we take the soft breaking terms (m_Q in the R case and m_Q, m_g in the $\not{R}$ case) to be very large compared to the dynamically generated scale Λ?

Our main results are the following:

(i) *All the "exotic" phenomena that characterize the supersymmetric theory continue to exist for small values of the soft breaking mass parameters.*

It seems that the appearance of the exotic behavior is not related to supersymmetry, though it probably is related to the presence of fundamental scalar fields. Theories which include scalar fields generally do not possess a positive definite measure for the gauge fields; this is the case in particular for supersymmetric gauge theories as well as for the softly broken supersymmetric theories. In these cases we cannot apply the QCD inequalities method, as used in the Appendix, to obtain information about the theory. We recall that in QCD the inequalities imply chiral symmetry breaking.

The presence of massless composite fermions in the supersymmetric case has a natural explanation in terms of supersymmetry. For $N_f \geq N_c$, SQCD contains a manifold of degenerate vacuum states. The fluctuations along the flat directions of the potential are described by effective scalar fields, and these scalar fields must have supersymmetric partners, which are massless fermions. Soft supersymmetry breaking removes the vacuum degeneracy and the flat directions of the scalar potential. Nevertheless, we saw that, in all cases except for the baryon-number-conserving vacuum of the $\not{R}$ case for $N_f = N_c$, the massless composite fermions of the supersymmetric limit remain massless after soft supersymmetry breaking.

For the $N_f = N_c$ and $N_f = N_c + 1$ cases, the massless fermions are gauge-invariant composite states. They are required to remain massless in order to satisfy the 't Hooft anomaly matching conditions corresponding to unbroken chiral symmetries in the energetically preferred vacuum state. This requirement is strong enough to keep the composite fermions massless even though their squark constituents obtain mass from soft supersymmetry breaking.

For $N_f > N_c + 1$, Seiberg argued that the $N = 1$ SQCD theory admits a dual description in the infrared. This dual theory contains a dynamically generated gauge symmetry which is infrared free for $N_f \leq \frac{3}{2} N_c$ and possesses a

nontrivial infrared fixed point for $3N_c > N_f > \frac{3}{2} N_c$. The dual theory contains massless composite fermions which belong to nontrivial representations of the dual gauge symmetry. We have argued that such a dual description will also exist for the softly broken theories, for some values of N_f and for small enough m_Q and m_g. The dual gauge theory is infrared free for $N_f < \frac{11}{9} N_c$ in the R case and for $N_f < \frac{9}{7} N_c$ in the $\not{R}$ case. For $\frac{11}{9} N_c < N_f < \frac{11}{2} N_c$ in the R case, and $\frac{9}{7} N_c < N_f < \frac{9}{2} N_c$ in the $\not{R}$ case, we expect to find a situation in which the theory is controlled by a nontrivial infrared fixed point, with weak coupling for the dual theory at the low-N_f boundary and weak coupling for the original theory at the high-N_f boundary. As in the supersymmetric case, the existence of this fixed point can be proved near the boundary, that is, for large N_f and N_c approximately in the boundary ratios. It is likely that a single infrared fixed point interpolates between these two boundary situations.

The prospect of finding this kind of *infrared duality for nonsupersymmetric gauge theories* is quite exciting. In the supersymmetric case we have several arguments and cross-checks which support the presence of the duality. These include satisfaction of the 't Hooft anomaly matching conditions, identification of all the gauge-invariant operators in the chiral ring, identification of all flat directions, and verification of the behavior under mass perturbations [7]. So far the evidence for duality in the softly broken theories relies only on the fact that the 't Hooft anomaly matching conditions are satisfied, and on their connection with the SQCD theory. For small supersymmetry-breaking parameters, the identification of those gauge-invariant operators which were identified in SQCD still goes through. However, some operators which were not identified in SQCD (such as the mesons made from the dual quarks) apparently should be identified after soft supersymmetry breaking. It seems that the low-energy spectrum after soft supersymmetry breaking should remain the same as in SQCD, except for the splitting between the states in a supermultiplet. Hence, naively, we would expect the operator identification to work in the same way. Clearly, we would like to have more support for the nonsupersymmetric duality conjecture. This is not easy in view of the fact that we have few tools for analyzing the nonperturbative behavior of the theory in the nonsupersymmetric case.

(ii) *In the decoupling limit most of the "exotic" phenomena disappear.*

As we move towards the decoupling limit in which we take m_Q (and also m_g in the $\not{R}$ case) to be large, it seems that most of our "exotic" phenomena disappear. Typically, in these cases we encounter a second-order phase transition to the chirally broken phase of QCD. This behavior is dictated by arguments that generalize mass inequalities of vector like gauge theories [21, 22]. For $N_f < N_c$, and for the baryon-number-conserving vacuum in the $\not{R}$ case for $N_f = N_c$, the decoupling limit to QCD is achieved through a smooth transition from a softly broken vacuum which already exhibits the QCD chiral symmetry breaking. The corresponding R case in this last model is ambiguous, as described below. In the

other cases that we considered, the decoupling limit is reached by a second-order phase transition at some finite value of m_Q in which chiral symmetry is broken. Investigating this phase transition is another interesting problem which we leave for future research.

In the R cases, it is possible that some exotic phenomena might survive the decoupling limit. For these theories, we presented two options for the decoupling limit, option 2, with a conventional chiral symmetry-breaking pattern and no massless fermions, and option 1, with the full chiral symmetry broken to $SU(N_f) \times U(1)_B \times U(1)_R$ and a multiplet of massless fermions necessary to satisfy the 't Hooft anomaly conditions for the unbroken $U(1)_R$. The required composite fermions can be constructed from massless quarks, antiquarks, and gauginos. We have not found any argument based on QCD inequalities to rule out this possibility. However, only in the the baryon-number-conserving vacuum for $N_f = N_c$ in the R case did this symmetry-breaking pattern arise naturally. In all other cases, this pattern still requires a second-order phase transition from the vacuum which is preferred at small m_Q.

On top of the exotic behavior discussed above, there are further obvious differences between the infrared domain of the supersymmetric gauge theories and their decoupling limits. Here are several examples: (1) In the supersymmetric case the order parameters associated with the chiral symmetry breaking are expectation values of squark bilinear operators, whereas in QCD quark bilinears play this role. (2) Supersymmetric fermionic baryons are composites of $N_c - 1$ squarks and one quarks. (3) Only totally antisymmetric flavor representations are relevant for the SQCD baryons. In the cases in which the vacuum at small m_Q can go continuously into a vacuum of the decoupling limit, we have found that the order parameter is in fact a mixture of the condensates of both bilinears, and that it shows level-crossing behavior. Close to the supersymmetric limit, the dominant component is the squark-squark condensate. As we go to the QCD limit, this contribution becomes negligible and the quark-quark condensate takes over. If option 2 for the R case, as described above, is realized, there is a related level-crossing phenomenon, in which squark building blocks of composite fermions in the supersymmetric limit are replaced in the decoupling limit by a quark-gluino combination that has identical quantum numbers.

(iii) *The exotic behavior of the region close to the supersymmetric limit should be detectable in lattice simulations.*

Simulations of softly broken SQCD should be easier to perform than direct simulations of SQCD, since it is difficult to maintain supersymmetry on the lattice. It may still be nontrivial to locate the region of the lattice coupling constants which reflects softly broken SQCD, because this theory still has specific relations among its renormalizable couplings. However, we have argued that this region can be found without unusual fine-tuning. In particular, we have discussed the issue of possible first-order phase transitions in lattice gauge theories with scalars and indicated how to avoid them. With this barrier removed, we expect the lattice simulations to reach

the continuum limit and reveal the rich structure of the exotic phenomena described in this paper.

Finally, we list some additional issues which we have not resolved, and which remain problems for future work.

The major difficulty encountered in passing from the supersymmetric gauge models to QCD is the identification of the SUSY-breaking operators. It is usually not easy to identify the relevant SUSY-breaking operators in the low-energy effective potential description. As we explained in Sec. II, in softly broken supersymmetric theories we do have some control over this problem. Following [14], we can show that our choice of the SUSY-breaking operators corresponds to those obtained from a supersymmetric theory which includes some additional chiral superfields via spontaneous supersymmetry breaking. In fact, with this approach one can relate the resulting squark mass term to the Kähler kinetic term in the underlying original SUSY theory. Thus, our lack of control over the soft breaking terms is related to our lack of control over the Kähler term. Clearly this question deserves further study. We have also noted that other terms which may appear in the operator identifications (such as, e.g., $\mathrm{tr}\left[(T^\dagger T)^n\right]$ for $n > 1$) are typically suppressed by powers of Λ. Hence, as long as we are considering vacua close to the origin, we are justified in retaining only the lower terms we worked with. In these cases we have more confidence in our results and can rely even on their quantitative aspects. This is typically the situation for $N_f > N_c$. However, when expectation values at the vacuum we are considering are of order Λ and higher, our neglect of the other higher terms is not justified. This is the case for $N_f = N_c$, when the expectation values are of order Λ, and for $N_f < N_c$ when the vacuum of the theory runs to infinity in the supersymmetric limit. We believe that the qualitative features of our results still hold in these cases, but we certainly cannot trust the quantitative aspects. This is the reason that in the $N_f = N_c$ case we could not decide which of the two possible vacuum states is preferred.

In addition, we have assumed throughout this work that the overall coefficients B_T and B_B of the SUSY-breaking contributions to the effective Lagrangian are positive. This is not an innocent assumption, since, had they been negative, we would have found chiral symmetry breaking instead of several of the exotic features we encountered in Secs. IV-VII. At present, we do not have a proof of the positivity of these coefficients, though the picture we have derived assuming this positivity is a coherent one.

Another avenue of possible future research is the analysis of softly broken supersymmetric gauge theories of other types, in particular, chiral models, which also admit dual representations. Recently, a number of generalizations of Seiberg's original proposal have been presented [27-34]. We expect the behavior of these theories upon adding soft supersymmetry-breaking terms to be similar to the behavior we found above for the $SU(N_c)$ case. Perhaps the study of these theories will open even wider the unusual possibilities for nonperturbative gauge dynamics.

ACKNOWLEDGMENTS

APPENDIX: QCD INEQUALITY

In this appendix we demonstrate an inequality which is useful in understanding the limit of supersymmetric QCD in which the squark mass is taken to infinity. This limit is a vectorlike gauge theory of quarks and the gluino, with no scalar fields. We will show that, in this limit, a flavor nonsinglet composite hadron cannot be massless if the pion is massive. Our argument is a straightforward generalization of arguments used to analyze QCD by Weingarten [21].

To prove our claim, we follow Weingarten's proof that the baryon is heavier than the pion. Though Weingarten's original argument was given on the lattice (and therefore was completely rigorous at the price of some complication), we will apply a continuum version of the argument. The crucial observation is that, in vectorlike gauge theories, the measure of integration over gauge fields, which includes the determinants from the integration over the fermions, is non-negative. This can be seen simply in the following way: For fermions of mass m, the fermion determinant is $\det(\slashed{D}_A + m)$ where $\slashed{D}_A$ is the covariant derivative with gauge field A. In vectorlike theories this is always positive, since the eigenvalues of $\slashed{D}_A$ are imaginary, and for every eigenvalue ia with eigenvector ψ_1, $\gamma_5\psi_1$ is an eigenvector with eigenvalue $-ia$, and the product of the contributions of both eigenvalues to the determinant is always positive. In the limit $m \to 0$ one could have zero modes in gauge sectors of nontrivial Pontryagin number. However, these sectors do not contribute to any correlation function we will consider.

In the analysis of Sec. IV D, we are most concerned with the possibility of a quark-antiquark-gluino bound state $\bar{\psi}_a^i \lambda_b^a \psi_j^b$, and so let us begin by considering this state. Its propagator from x to y is given by

$$\int d\mu \sum_{a,a',b,b'} S_{\psi_{a,a'}} \delta^{ii'} S_{\bar{\psi}_{b,b'}} \delta_{jj'} S_{\lambda_{a,a',b,b'}}, \tag{A1}$$

where S_ψ, $S_{\bar\psi}$, and S_λ are the quark, antiquark, and gluino propagators in the presence of fixed background gauge fields (we do not write the space-time indices explicitly), and $d\mu$ is the measure of integration over the gauge fields (including the fermion determinants). Since the integration measure is positive, this is smaller than

$$\int d\mu \left(\sum_{a,a'}|S_{\psi_{a,a'}}|^2\right)^{\frac12} \left(\sum_{a,a'}|S_{\bar\psi_{a,a'}}|^2\right)^{\frac12}$$
$$\times \left(\sum_{a,a',b,b'}|S_{\lambda_{a,a',b,b'}}|^2\right)^{\frac12}. \tag{A2}$$

Next, we use the Hölder inequality, which says that, for any positive measure,

$$\left|\int d\mu(fg)\right| \le \left(\int d\mu|f|^2\right)^{\frac12}\left(\int d\mu|g|^2\right)^{\frac12}, \tag{A3}$$

to bound the propagator from above by

$$\left(\int d\mu \sum_{a,a'}|S_{\psi_{a,a'}}|^2\right)^{\frac12}\left[\int d\mu\left(\sum_{a,a',b,b'}|S_{\lambda_{a,a',b,b'}}|^2\right)\right.$$
$$\left.\times\left(\sum_{a,a'}|S_{\psi_{a,a'}}|^2\right)\right]^{\frac12}. \tag{A4}$$

The next stage is to interpret each of the integrals in (A4) as some correlation function. The first integral is proportional to the propagator of the pion, $\psi^i \bar\psi_j$ directly. In general, a propagator falls asymptotically as $e^{-m|x-y|}$, where m is the lowest mass possible in the intermediate state. For the first integral m is the pion mass. The second integral can, at worst, approach a constant asymptotically. Thus, the correlation function of $\bar\psi_a^i \lambda_b^a \psi_j^b$ is bounded above by a constant times $\exp(-m_\pi|x-y|)$, where m_π is the pion mass. Then the mass of the quark-antiquark-gluino bound state must be greater than m_π. This argument goes through in the same way for any flavor-nonsinglet bound state, which necessarily contains at least one quark and one antiquark or at least N_c quarks.

Since the gluino-ball is a flavor singlet, there is no QCD inequality relating its mass to that of flavor-nonsinglet bound states. This leaves an ambiguity that we are not able to resolve. It is this ambiguity that leads to the presence of option 1 [unbroken $U(1)_R$] in the cases $N_f \ge N_c$.

[1] G. 't Hooft, in *Recent Development in Gauge Theories*, Proceedings of the Cargese Summer Institute, Cargese, France, 1979, edited by G. 't Hooft *et al.*, NATO Advanced Study Institute Series B: Physics Vol. 59 (Plenum Press, New York, 1980).

[2] G. Veneziano and S. Yankielowicz, Phys. Lett. **113B**, 321 (1982).

[3] T. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. **B218**, 493 (1983).

[4] I. Affleck, M. Dine, and N. Seiberg, Phys. Rev. Lett. **51**, 1026 (1983).

[5] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B241**, 493 (1984).

[6] N. Seiberg, Phys. Rev. D **49**, 6857 (1994).

[7] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).

[8] A. Masiero and G. Veneziano, Nucl. Phys. **B249**, 593 (1985).

[9] A. Masiero, R. Pettorino, M. Roncadelli, and G. Veneziano, Nucl. Phys. **B261**, 633 (1985).

[10] S. Elitzur, E. Rabinovici, and A. Schwimmer, Phys. Lett. B **199**, 165 (1982); S. Elitzur and A. Schwimmer, Nucl. Phys. **B226**, 109 (1983).

[11] J. Preskill and S. Weinberg, Phys. Rev. D **24**, 1059 (1981).

[12] S. Dimopoulos and J. Preskill, Nucl. Phys. **B199**, 206 (1982).

[13] I. Montvay, Phys. Lett. B **344**, 176 (1995); Nucl. Phys. **B445**, 399 (1995).

[14] N. Evans, S. D. H. Hsu, and M. Schwetz, "Exact Results in Softly Broken Supersymmetric Models," Report No. YCTP-P8-95, hep-th/9503186 (unpublished).

[15] K. Intriligator, R. Leigh, and N. Seiberg, Phys. Rev. D **50**, 1092 (1994).

[16] S. Coleman and E. Witten, Phys. Rev. Lett. **45**, 100 (1980).

[17] T. Banks, I. Frishman, A. Schwimmer, and S. Yankielowicz, Nucl. Phys. **B177**, 157 (1981).

[18] Y. Meurice and G. Veneziano, Phys. Lett. **141B**, 69 (1984).

[19] W. Marciano, Phys. Rev. D **21**, 2425 (1980).

[20] J. Kogut, M. Stone, H. W. Wyld, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Phys. Rev. Lett. **48**, 1140 (1982).

[21] D. Weingarten, Phys. Rev. D **51**, 1830 (1983).

[22] C. Vafa and E. Witten, Nucl. Phys. **B234**, 173 (1984).

[23] K. Intriligator and N. Seiberg, Nucl. Phys. **B444**, 125 (1994).

[24] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).

[25] E. Fradkin and S. Shenker, Phys. Rev. D **19**, 3682 (1979).

[26] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[27] D. Kutasov, Phys. Lett. B **351**, 230 (1995).

[28] O. Aharony, J. Sonnenschein, and S. Yankielowicz, "Flows and Duality in $N = 1$ Supersymmetric Gauge Theories," Report No. TAUP-2246-95, hep-th/9504113 (unpublished).

[29] D. Kutasov and A. Schwimmer, Phys. Lett. B **354**, 315 (1995).

[30] K. Intriligator and P. Pouliot, Phys. Lett. B **353**, 471 (1995).

[31] K. Intriligator, Nucl. Phys. **B448**, 187 (1995).

[32] M. Berkooz, "The Dual of Supersymmetric $SU(2k)$ with an Antisymmetric Tensor and Composite Dualities," Report No. RU-95-29, hep-th/9505067 (unpublished).

[33] R. G. Leigh and M. J. Strassler, Phys. Lett. B **356**, 492 (1995).

[34] K. Intriligator, R. G. Leigh, and M. J. Strassler, "New Examples of Duality in Chiral and Nonchiral Supersymmetric Gauge Theories," Report No. RU-95-38, hep-th/9506148 (unpublished).