ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

RECENT PROGRESS ON NONLINEAR BEAM MANIPULATIONS IN CIRCULAR ACCELERATORS

A. Bazzani, F. Capoani[1], Bologna University and INFN, Bologna, Italy
M. Giovannozzi*, CERN Beams Department, Geneva, Switzerland
[1]also at CERN Beams Department, Geneva, Switzerland

Abstract

In recent years, transverse beam splitting by crossing a stable resonance has become the operational means to perform MultiTurn Extraction (MTE) from the CERN PS to the SPS. This method delivers the high-intensity proton beams for fixed-target physics at the SPS. More recently, further novel manipulations have been studied, with the goal of devising new techniques to manipulate transverse beam properties. AC magnetic elements can allow beam splitting to be performed in one of the transverse degrees of freedom. Crossing 2D nonlinear resonances can be used to control the sharing of the transverse emittances. Furthermore, cooling the transverse emittance of an annular beam can be achieved through an AC dipole. These techniques will be presented and discussed in detail, considering future lines of research.

INTRODUCTION

Nonlinear effects introduce new phenomena in beam physics. In recent years, they have been used extensively to design novel beam manipulations in which the transverse beam distribution is modified in a controlled way for different purposes. This is the case for the beam splitting that is at the heart of the CERN Multiturn Extraction (MTE) [1–4].

The possibility of a controlled manipulation of the phase space by means of an adiabatic change of a parameter opened the road-map to new applications in accelerator and plasma physics [1, 5–9]. In particular, the adiabatic transport performed by means of nonlinear resonance trapping allows manipulation of a charged particle distribution, as to minimize the particle losses during the beam extraction process in a circular accelerator. Furthermore, the control of the beam emittance can be obtained by a similar approach [4, 10, 11]. The experimental procedures [4, 10, 11] require a very precise control of the efficiency of the adiabatic trapping into resonances [12–14], as well as of the phase-space change during the adiabatic transport, when a parametric modulation is introduced by means of an external perturbation. All these processes can be represented by multi-dimensional Hamiltonian systems or symplectic maps [15].

The adiabatic theory for Hamiltonian systems is a key breakthrough towards an understanding of the effects of slow parametric modulation on the dynamics. The concept of adiabatic invariant allows the long-term evolution of the system to be predicted and the fundamental properties of the action variables to be highlighted upon averaging over the fast variables [16, 17]. The theory has been well developed for systems with one degree of freedom [12, 18–21], but the extension of some analytical results to multi-dimensional systems or to symplectic maps [22] has to cope with the issues generated by the ubiquitous presence of resonances in phase space [23, 24]. For these reasons, such an extension is still an open problem.

The combination of nonlinear effects that do not preserve the linear Courant-Snyder invariant, and adiabatic variation of the system parameters that allow crossing separatrices, opens new regimes that can be used to propose novel beam manipulations, in which essential beam parameters, such as the emittances can be changed in a controlled way.

In this paper, three novel beam manipulations are reviewed, namely beam splitting by means of AC elements [25], sharing of transverse emittances by crossing a nonlinear 2D resonance [26], and cooling of an annular beam distribution [27, 28].

* massimo.giovannozzi@cern.ch

ADIABATIC THEORY OF SEPARATRIX CROSSING

Phenomena occurring when a Hamiltonian system is slowly modulated have been widely studied in the framework of adiabatic theory [18, 19]. As the modulation of the Hamiltonian changes the shape of the separatrices in phase space, the trajectories can cross separatrices and enter into different stable regions associated with nonlinear resonances. The separatrix crossing can be described in a probabilistic way due to the sensitive dependence on initial conditions, and the crossing probabilities can be computed in the adiabatic limit, like the change of adiabatic invariant due to the crossing [18, 19].

Let us consider a Hamiltonian $\mathcal{H}(p, q, \lambda = \epsilon t)$, $\epsilon \ll 1$, where the parameter λ is slowly modulated and whose phase space is sketched in Fig. 1. An initial condition in Region III has a probability to be trapped into Region I or II of phase space given by [18]

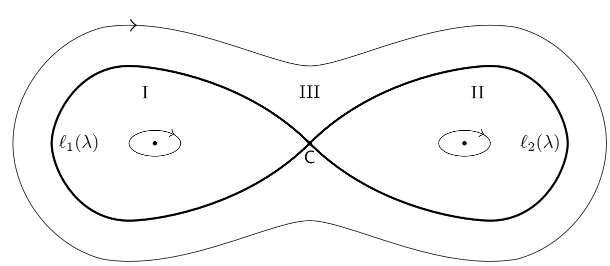

Figure 1: A generic phase-space portrait divided into three regions (I, II, III) by separatrices $\ell_1(\lambda)$ and $\ell_2(\lambda)$.

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

$$\mathcal{P}_{\text{III}\to\text{I}} = \frac{\Theta_\text{I}}{\Theta_\text{I}+\Theta_\text{II}} \qquad \mathcal{P}_{\text{III}\to\text{II}} = 1 - \mathcal{P}_{\text{III}\to\text{I}}\,, \tag{1}$$

where

$$\Theta_i = \left.\frac{\mathrm{d}A_i}{\mathrm{d}\lambda}\right|_{\tilde\lambda} = \oint_{\partial A_i} \mathrm{d}t \left.\frac{\partial \mathcal{H}}{\partial \lambda}\right|_{\tilde\lambda} \qquad i = \text{I, II}\,, \tag{2}$$

with A_i the area of region i, ∂A_i the boundary of region i, and $\tilde\lambda$ the value of λ when the separatrix is crossed. In case $\mathcal{P}_{\text{III}\to i} < 0$, then $\mathcal{P}_{\text{III}\to i} = 0$, whereas when $\mathcal{P}_{\text{III}\to i} > 1$ then $\mathcal{P}_{\text{III}\to i} = 1$.

When a separatrix is crossed, the adiabatic invariant J changes according to the area difference between the two regions at the crossing time, and just after the crossing into a region of area A, $J = A/(2\pi)$. This occurs only if the modulation is perfectly adiabatic, i.e. $\epsilon \ll 1$, but a correction to the value of the new action can be found following [19].

The adiabatic trapping into resonances has been studied in various works [18, 29] to show the possibility of transport in phase space when some system's parameters are slowly modulated. This phenomenon suggests possible applications in different fields and, in particular, in accelerator physics where MTE has been proposed [1] and successfully made into an operational beam manipulation at the CERN PS [2, 4]. In this case, an extension of the results of adiabatic theory to quasi-integrable area-preserving maps has been considered, and the probability to be captured in a resonance can be computed analogously to those in Eq. (1) [15], when the Poincaré–Birkhoff theorem [30] can be applied to prove the existence of stable islands in phase space. The properties of such resonance islands for polynomial Hénon-like maps [31] have been studied in [32] and the possibility of performing an adiabatic trapping into a resonance by modulating the linear frequency at the elliptic fixed point has been studied [15].

BEAM SPLITTING USING AC ELEMENTS

The Model

A new approach can be devised to perform beam splitting by considering a Hénon-like symplectic map of the form

$$\mathcal{M}_{\ell,m} : \begin{pmatrix} q_{n+1} \\ p_{n+1} \end{pmatrix} = R(\omega_0)\times \times \begin{pmatrix} q_n \\ p_n - \sum_{j>2} k_j q_n^{j-1} - q^{\ell-1} \varepsilon_\text{m} \cos\omega\, n \end{pmatrix}\,, \tag{3}$$

where $R(\omega_0)$ is a rotation matrix of an angle ω_0, n is the iteration number, $\ell \in \mathbb{N}$, and the dynamics is perturbed by a modulated kick of amplitude ε_m whose frequency ω is close to a resonance condition $\omega = m\,\omega_0 + \delta$, $\delta \ll 1$. When $\ell = 1$, the fixed point at the origin of the unperturbed system becomes an elliptic periodic orbit of period $2\pi/\omega$, and the linear frequencies depend on the perturbation strength, so that they are adiabatically modulated. This is not the case when $\ell \geq 2$, which is also interesting for applications.

The Birkhoff Normal Form theory allows a relationship between the map of Eq. (3) and the Hamiltonian [32]

$$\mathcal{H}_{\ell,m}(p,q,t) = \omega_0 \frac{q^2+p^2}{2} + \sum_{j>2} \hat{k}_j \frac{q^j}{j} + \varepsilon_\text{h} \frac{q^\ell}{\ell} \cos\omega\, t \tag{4}$$

to be established. Note that in [25] the adiabatic theory for the Hamiltonian (4) is used to analyze the results obtained with the map (3) and found in excellent agreement. This observation is essential as it demonstrates that a splitting protocol can be designed based on adiabatic theory for the Hamiltonian (4) and it will be valid, with minor adaptations, also for the map (3).

Splitting with AC Elements

To study the possibility of beam splitting by means of AC elements, the third-order resonance is selected, but the concepts used can be generalized to any resonance order.

When the system parameters are adiabatically modulated, the trapping of the orbits into the stable islands and the adiabatic transport are possible [18]. To optimize the trapping probability, we propose a protocol divided into two steps. In the first one, the perturbation frequency ω is kept constant at a value $\omega_\text{i} < m\,\omega_0$, near the mth-order resonance, while the exciter is slowly switched on, increasing its strength ε_h from 0 to the final value $\varepsilon_\text{h,f}$. In the second stage, the exciter strength is kept fixed at $\varepsilon_\text{h,f}$, and the frequency is modulated from ω_i to ω_f. Both modulations are performed by means of a linear variation in N time steps. It is essential to mention that, unlike MTE where ω_0 varies as to cross a resonance, with AC elements the resonance is created between ω_0 and ω, and this is an essential advantage in case the value of ω_0 is imposed by, e.g. space charge considerations.

An example of the behavior described above is shown in Fig. 2, where the evolution of a set of initial conditions, uniformly distributed on a disk of radius R, under the dynamics generated by $\mathcal{H}_{1,3}$, using the protocol for trapping and transport described above, is shown. The plots show the evolution of an ensemble of initial conditions under the same dynamics generated by $\mathcal{H}_{1,3}$ and the colors are used to indicate which region the initial conditions are trapped into. The trapping and transport phenomena are clearly visible, thus indicating that the proposed protocol works efficiently. It is worth stressing that no initial condition moves towards very large amplitudes and that there are no particles in between islands, which means that multi-turn extraction would be free of losses also for this type of splitting.

EMITTANCE SHARING BY CROSSING 2D NONLINEAR RESONANCES

We remark that this idea was inspired by [33], where the analysis of the crossing of a 2D nonlinear resonance was carried out with the goal of quantifying the emittance growth due to a fast resonance crossing. This process is sometimes unavoidable in many high-power accelerators,

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

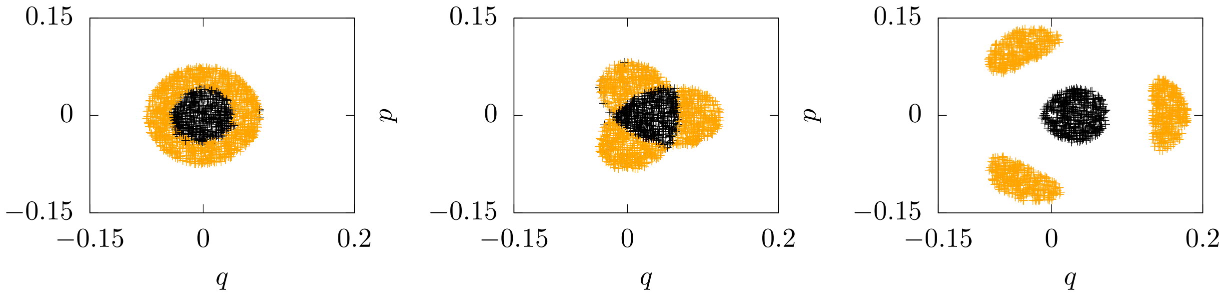

Figure 2: Evolution of an ensemble of particles in phase space with the colors used to identify in which region each initial condition has been trapped into (core, black, and islands, orange) for the Hamiltonian model (4) with $\ell = 1$, $m = 3$ at the beginning of the process (left column), at the end of the ε_{h} variation (mid column) and at the end of the frequency variation (right column). Parameters: $k_3 = 1$, $\omega_0/(2\pi) = 0.17133$, $\omega_{\mathrm{i}} = 2.995\,\omega_0$, $\omega_{\mathrm{f}} = 2.983\,\omega_0$, $\varepsilon_{\mathrm{h,f}} = 0.28$.

such as isochronous cyclotrons, non-scaling fixed field alternating gradients, and other low-energy accelerators.

The Model

Using the action-angle variables and averaging procedures, transverse motion in a circular accelerator close to a (m,n) resonance (with $m, n \in \mathbb{N}$) is described by the Hamiltonian

$$\begin{aligned}\mathcal{H}(\phi_x, J_x, \phi_y, J_y) &= \omega_x J_x + \omega_y J_y + \alpha_{xx} J_x^2 + \alpha_{yy} J_y^2 + \\ &+ 2\alpha_{xy} J_x J_y + G J_x^{m/2} J_y^{n/2} \cos(m\phi_x - n\phi_y),\end{aligned} \tag{5}$$

the resonance condition being $m\omega_x - n\omega_y \approx 0$, and α_{xx}, α_{xy}, α_{yy} are the amplitude-detuning parameters.

The canonical transformation (see [16, p. 410])

$$J_x = mJ_1 \qquad \phi_1 = m\phi_x - n\phi_y \tag{6}$$

$$J_y = J_2 - nJ_1 \qquad \phi_2 = \phi_y \tag{7}$$

transforms the Hamiltonian into

$$\begin{aligned}\mathcal{H}(\phi_1, J_1) &= \delta J_1 + \alpha_{11} J_1^2 + \alpha_{12} J_1 J_2 + G(mJ_1)^{\frac{m}{2}} \times \\ &\times (J_2 - nJ_1)^{\frac{n}{2}} \cos\phi_1 + \left(\omega_y J_2 + \alpha_{22} J_2^2\right)\end{aligned} \tag{8}$$

where $\delta = m\omega_x - n\omega_y$ is the resonance-distance parameter and $\alpha_{11}, \alpha_{12}, \alpha_{22}$ are functions of $\alpha_{xx}, \alpha_{xy}, \alpha_{yy}$. Note that the Hamiltonian (8) does not depend on ϕ_2, hence J_2 is a constant of motion and the last term in (8) can be neglected. Furthermore, J_2, which is a constant parameter, induces a shift of the resonance condition, which will not be met when $\delta = 0$, but rather when $\delta + \alpha_{12} J_2 = 0$.

The phase-space topology of the Hamiltonian (8) depends on m, n, but some elements are common. The condition $J_y > 0$ constrains the motion to $J_1 < J_2/n$, the *allowed disk*. When unstable fixed points lie on the border of this disk, it is possible to draw a separatrix that joins them, the *coupling arc*.

Emittance Sharing

Let us consider a process where a particle evolves under the Hamiltonian (5), while either ω_x or ω_y is changed with time to cross the (m, n) resonance. This means varying δ from a situation where $\delta + \alpha_{12} J_2 \gg 0$ to one where $\delta + \alpha_{12} J_2 \ll 0$. The variation of δ changes the position of a separatrix that then sweeps the allowed disk inside which particles are constrained to move.

A particle starts evolving, far from resonance, with an initial action $J_{1,\mathrm{i}} = J_{x,\mathrm{i}}/m$. Its orbit, being far from the resonance, will be close to a circle of area $2\pi J_1$. This area, being the adiabatic invariant, is conserved when δ is slowly varied. As δ is decreased, the separatrix reduces the region in which the particle is moving, dividing the allowed disk in two (the two regions will be equal on resonance). When the area of the initial region is equal to $2\pi J_1$, according to separatrix crossing theory [18], the particle crosses the coupling arc entering the other region of the allowed disk, with an action corresponding to 2π times the area of the arrival region at the jump time.

Since the allowed disk has an area $2\pi J_2/n$, the resulting action will be

$$J_{1,\mathrm{f}} = \frac{J_2}{n} - J_{1,\mathrm{i}} \tag{9}$$

and, going back to the x and y actions

$$J_{x,\mathrm{f}} = mJ_{1,\mathrm{f}} = m\left(\frac{J_{y,\mathrm{i}} + nJ_{x,\mathrm{i}}/m}{n} - \frac{J_{x,\mathrm{i}}}{m}\right) = \frac{m}{n} J_{y,\mathrm{i}} \tag{10}$$

and

$$J_{y,\mathrm{f}} = \frac{n}{m} J_{x,\mathrm{i}}\,. \tag{11}$$

As δ continues to decrease, the area where the particle orbits increases. At the end of the resonance crossing process, far from resonance, the particle will orbit on a circle around the origin at the new action.

As the x and y emittances are the averages of J_x and J_y, at the end of the process an *emittance sharing* between the two directions occurs, keeping the product $J_x J_y$ constant. An example for the $m = 1, n = 2$ case is given in Fig. 3.

It is worth stressing that the picture described here holds in all cases with no hyperbolic fixed points inside the allowed disk. Otherwise, the situation becomes much more involved, as separatrices, linked with the hyperbolic fixed points, appear, partitioning the phase space into more regions. A different analysis is required in that case [26].

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

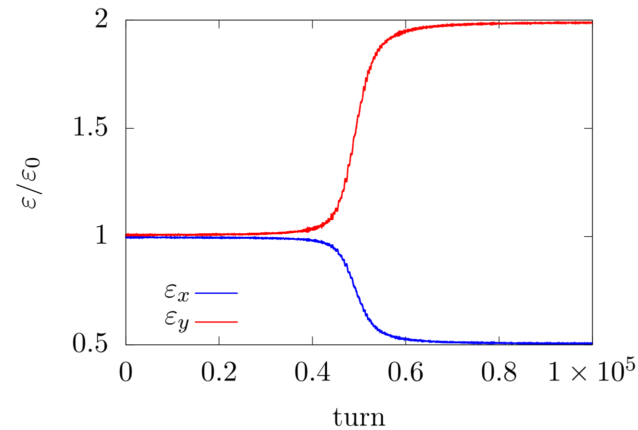

Figure 3: Example of the emittance evolution during the resonance-crossing process for the $m = 1, n = 2$ case.

COOLING OF AN ANNULAR BEAM

The Model

The generic Hamiltonian (4) can be recast by using the unperturbed ($\varepsilon = 0$) action-angle coordinates (ϕ, J) in the form

$$\mathcal{H}(\phi, J) = \omega_0 J + \frac{\Omega_2}{2} J^2 + \varepsilon\sqrt{2J}\cos\phi\cos\omega t\,, \tag{12}$$

where we introduce the detuning term $\Omega_2 = O(k_3^2)$.

Several transformations can then be applied [27, 28], which include moving to a rotating-frame reference, using the angle $\gamma = \phi - \omega t$, and averaging the perturbation term over the fast variable ωt, to obtain the Hamiltonian of the slow dynamics that, after re-scaling of the action, reads

$$\mathcal{H}(\gamma, J) = 4J^2 - 2\,\lambda\,J + \mu\sqrt{2J}\cos\gamma\,, \tag{13}$$

where the parameters λ, μ are defined as

$$\lambda = \frac{4}{\Omega_2}(\omega - \omega_0), \qquad \mu = \frac{4\varepsilon}{\Omega_2}\,, \tag{14}$$

and can be changed upon acting on ε and ω. The Hamiltonian (13) is well-known [18, 29] and can be written

$$\mathcal{H}(X, Y) = (X^2 + Y^2)^2 - \lambda(X^2 + Y^2) + \mu X\,, \tag{15}$$

upon using the co-ordinates $X = \sqrt{2J}\cos\gamma$, $Y = \sqrt{2J}\sin\gamma$.

Its phase space can be analyzed and the existence and position of the fixed points can be determined analytically [27, 28]. The separatrix divides the phase space in three regions, as shown in Fig. 4, whose areas A_i are computed analytically [27, 28], which is essential for designing cooling protocols.

Cooling Protocols

The idea at the heart of the cooling protocol is based on a careful control of the time variation of the size of the phase-space regions G_1 and G_2 so to trap particles in an annular beam distribution and then reduce the value of their action

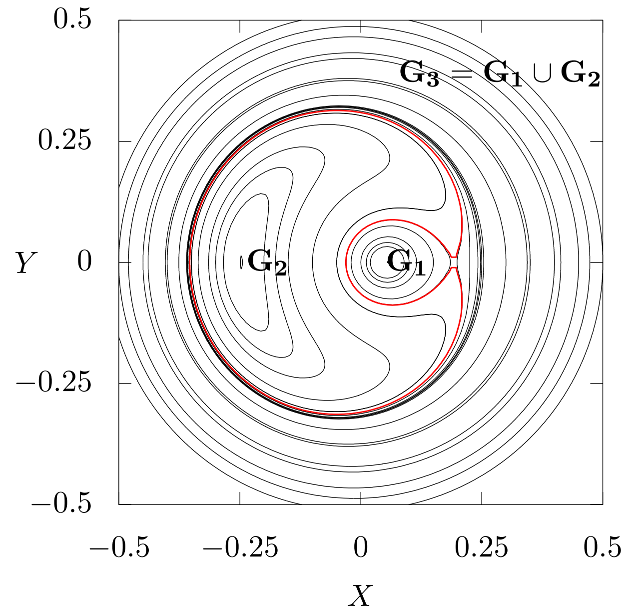

Figure 4: Phase-space portrait of the Hamiltonian (15) with $\lambda = 0.1$, $\mu = 0.01$. The red line represents the separatrix.

at the end of the trapping and transport processes. This can be carried out by using the theory outlined in the previous section, and the details can be found in [27, 28].

At first, an initial condition from an annular-shape distribution evolves in the outer region with an initial action J_0, and λ and μ are slowly varied. At time t^*, $\lambda = \lambda^*$, $\mu = \mu^*$, and $A_3 = 2\pi J_0$, and according to adiabatic separatrix-crossing theory [16, 18], having defined $\xi = \dfrac{\mathrm{d}A_i/\mathrm{d}t}{\mathrm{d}A_3/\mathrm{d}t}$ then

$$P_i = \xi \text{ if } \xi \in]0, 1[, \quad P_i = 0 \text{ if } \xi < 0, \quad P_i = 1 \text{ if } \xi > 1\,, \tag{16}$$

and the orbit is trapped in the region G_i ($i = 1, 2$) with probability P_i, and an action value after trapping of $A_i/2\pi$. Given a distribution of initial conditions with action $J \in [J_0 - \Delta, J_0 + \Delta]$, the expected value of their final action after trapping, if Δ is sufficiently small, is $\langle J\rangle_\mathrm{f} = (A_1P_1 + A_2P_2)/2\pi \leq J_0$, which means that the separatrix-crossing process reduces the emittance of the annular distribution.

To optimize the cooling process, two protocols have been considered: one consists in trapping all particles in G_1, the other in trapping all particles in G_2. For both processes, the trapping phase is followed by the adiabatic transport obtained by moving the resonance island toward the origin of the phase space.

The two approaches are presented and discussed in detail in [28]. Here we report an example of the evolution of the annular distribution in Fig. 5. The evolution of the distribution, the optimized variation of the parameters λ and μ, and the projected distribution of the actions are shown. The cooling of the initial action distribution is clearly visible, both in the phase-space plots and in the action projection. The color code used to identify the action values provides an indication of a certain level of mixing that occurs during

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

the trapping phase. Indeed, at the end of the cooling process, the ordering of the colors used to identify action values is only approximately respected and in the outer zone of the action distribution all colors are represented.

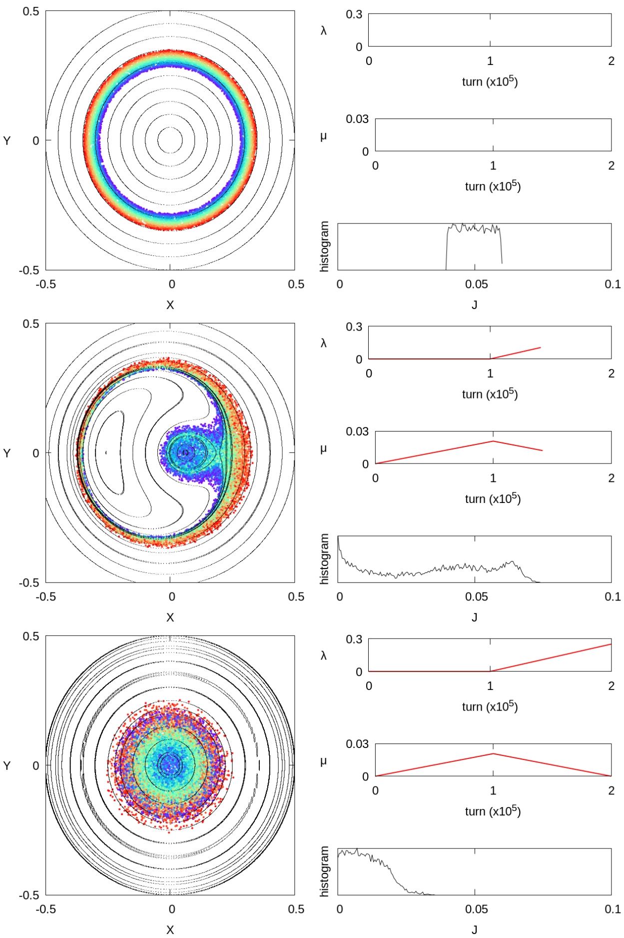

Figure 5: Example of the cooling protocol based on trapping in G_1, showing the evolution of the beam distribution, its projection, and the parameters λ, μ.

Finally, in Fig. 6, the performance of the proposed cooling approach is shown as a function of the special parameter values λ^* and μ^* that characterize the two types of protocols. The plots report the results of numerical simulations as well as those of theoretical estimates. The agreement is clearly visible and the possibility of achieving very high values of cooling is also evident. The disagreement is due to a lack of adiabaticity for large μ^* [28].

CONCLUSIONS AND OUTLOOK

The developments and recent results of novel beam manipulations based on nonlinear beam dynamics have been reviewed in this paper. The precursor has been beam splitting that is used to perform multiturn extraction in the CERN PS and it allows controlling the emittance in the horizontal plane and stretching the beam beyond the length of the ring circumference.

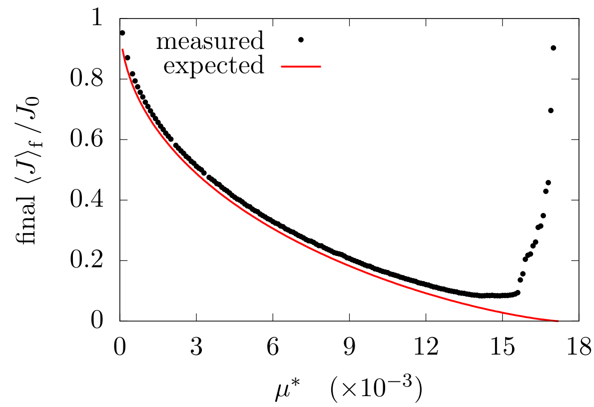

Figure 6: Expected and simulated cooling ratio for trapping in G_1 as a function of μ^*. Initial distribution is an annulus at $J_0 = 0.05$. The Hamiltonian of Eq. (4) has been used, with $\hat{k}_3 = 1$, $\omega_0 = 0.414 \times 2\pi$, $\Omega_2 = -0.3196$.

The first generalization of this technique consists in performing beam splitting by means of AC elements. The most natural approach is the use of an AC dipole, but high-order magnets could also be considered. This approach aims at providing the same type of manipulation as the standard beam splitting with, however, a major advantage: the resonance condition is created between the ring tune and the frequency of the AC element. Therefore, even if the tune would be constrained, e.g. by space charge considerations, thus preventing to cross a resonance, beam splitting could still be performed by setting the frequency of the AC element to the appropriate resonant value and then changing it to cross the resonance.

Extending the type of nonlinear manipulation to the crossing of 2D resonances allows entering a new regime, in which the emittance values in both transverse planes are affected and not only that in a single plane. This implies that the redistribution of the values of the transverse emittances is a feasible option.

Finally, the cooling of an annular beam distribution by means of an AC dipole has been successfully studied. Two protocols have been considered, both featuring excellent properties in terms of cooling performance, as well as in terms of the range of amplitudes that can be cooled. Note that the annular beam distribution considered in this study is an excellent model for the beam halo. Therefore, this could be the basis for future applications to halo manipulation, possibly including experimental tests at the LHC.

In the near future, it is planned to pursue these studies using realistic ring lattices in view of experimental tests of the proposed nonlinear manipulations.

REFERENCES

[1] R. Cappi and M. Giovannozzi, "Novel method for multiturn extraction: Trapping charged particles in islands of phase space," *Phys. Rev. Lett.*, vol. 88, p. 104801, 2002. doi:10.1103/PhysRevLett.88.104801

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

[2] J. Borburgh *et al.*,"First implementation of transversely split proton beams in the CERN Proton Synchrotron for the fixed-target physics programme," *EPL*, vol. 113, no. 3, p. 34001, 2016. doi:10.1209/0295-5075/113/34001

[3] S. Abernethy *et al.*, "Operational performance of the CERN injector complex with transversely split beams," *Phys. Rev. Accel. Beams*, vol. 20, p. 014001, 2017. doi:10.1103/PhysRevAccelBeams.20.014001

[4] A. Huschauer *et al.*, "Transverse beam splitting made operational: Key features of the multiturn extraction at the CERN Proton Synchrotron," *Phys. Rev. Accel. Beams*, vol. 20, p. 061001, 2017. doi:10.1103/PhysRevAccelBeams.20.061001

[5] A. Neishtadt, "Jumps in the adiabatic invariant on crossing the separatrix and the origin of the 3:1 Kirkwood gap," *Sov. Phys. Dokl.*, vol. 32, p. 571, 1987.

[6] S. Sridhart and J. Touma, "Adiabatic evolution and capture into resonance: vertical heating of a growing stellar disc," *Mon. Not. R Astron. Soc.*, vol. 279, p. 1263, 1996.

[7] V. Chirikov, "Particle confinement and adiabatic invariance," *Proc. Royal Soc. London A*, vol. 413, p. 145, 1987.

[8] D. Escande, "Contributions of plasma physics to chaos and nonlinear dynamics," *Plasma Physics and Controlled Fusion*, vol. 58, no. 11, p. 113001, 2016. doi:10.1088/0741-3335/58/11/113001

[9] R. Cappi and M. Giovannozzi, "Multiturn extraction and injection by means of adiabatic capture in stable islands of phase space," *Phys. Rev. ST Accel. Beams*, vol. 7, p. 024001, 2004. doi:10.1103/PhysRevSTAB.7.024001

[10] S. Gilardoni *et al.,* "Experimental evidence of adiabatic splitting of charged particle beams using stable islands of transverse phase space," *Phys. Rev. ST Accel. Beams*, vol. 9, p. 104001, 2006. doi:10.1103/PhysRevSTAB.9.104001

[11] A. Franchi, S. Gilardoni, and M. Giovannozzi, "Progresses in the studies of adiabatic splitting of charged particle beams by crossing nonlinear resonances," *Phys. Rev. ST Accel. Beams*, vol. 12, p. 014001, 2009. doi:10.1103/PhysRevSTAB.12.014001

[12] A. Neishtadt, "Capture into resonance and scattering on resonances in two-frequency systems," *Proceedings of the Steklov Institute of Mathematics*, vol. 250, p. 183, 2005.

[13] A. Neishtadt, A. A. Vasiliev, and A. Itin, "Captures into resonance and scattering on resonance in dynamics of a charged relativistic particle in magnetic field and electrostatic wave," *Physica D*, vol. 141, p. 281, 2000.

[14] A. N. Vasil'ev and M. A. Guzev, "Particle capture by a slowly varying potential," *Theor. Math. Phys.*, vol. 68, p. 907, 1986.

[15] A. Bazzani, C. Frye, M. Giovannozzi, and C. Hernalsteens, "Analysis of adiabatic trapping for quasi-integrable area-preserving maps," *Phys. Rev. E*, vol. 89, p. 042915, 2014. doi:10.1103/PhysRevE.89.042915

[16] V. Arnol'd, V. Kozlov, and A. Neishtadt, *Mathematical aspects of classical and celestial mechanics. Dynamical systems III; 3rd rev. version*, ser. Encyclopaedia of mathematical sciences, Heidelberg: Springer, 2006.

[17] B. V. Chirikov and V. V. Vecheslavov, "Adiabatic Invariance and Separatrix: Single Separatrix Crossing," *Journal of Experimental and Theoretical Physics*, vol. 90-3, p. 562, 2000.

[18] A. Neishtadt, "Passage through a separatrix in a resonance problem with a slowly-varying parameter," *Journal of Applied Mathematics and Mechanics*, vol. 39, no. 4, pp. 594 — 605, 1975. doi:10.1016/0021-8928(75)90060-X

[19] A. Neishtadt, "Change of an adiabatic invariant at a separatrix," *Fiz. Plasmy*, vol. 12, p. 992, 1986.

[20] A. Neishtadt, "Scattering by resonances," *Celestial Mechanics and Dynamical Astronomy*, vol. 65, p. 1, 1997.

[21] D. Vainchtein and I. Mezić, "Capture into Resonance: A Method for Efficient Control," *Phys. Rev. Lett.*, vol. 93, p. 084301, 2004.

[22] A. Bazzani, F. Brini, and G. Turchetti, "Diffusion of the Adiabatic Invariant for Modulated Symplectic Maps," *AIP Conf. Proc.*, vol. 395, p. 129, 1997. doi:10.1063/1.52959

[23] V. I. Arnol'd, "On the behavior of adiabatic invariants under a slow periodic change of the Hamiltonian function," *Doklady*, vol. 142, p. 758, 1962.

[24] V. I. Arnol'd, "Conditions of the applicability and an estimate of the mistake of the averaging method for systems, which goes through the resonances during the evolution process," *Doklady*, vol. 161, p. 9, 1965.

[25] A. Bazzani, F. Capoani, and M. Giovannozzi, "Analysis of adiabatic trapping phenomena for quasi-integrable area-preserving maps in the presence of time-dependent exciters," in preparation.

[26] A. Bazzani, F. Capoani, and M. Giovannozzi, "Manipulations of transverse invariants in 2D Hamiltonian systems," in preparation.

[27] F. Capoani, A. Bazzani, M. Giovannozzi, and R. Tomás García, "Cooling of an Annular Beam by Using Nonlinear Effects", in *Proc. IPAC'21*, Campinas, Brazil, May 2021, pp. 1968–1971. doi:10.18429/JACoW-IPAC2021-TUPAB231

[28] A. Bazzani, F. Capoani, M. Giovannozzi, and R. Tomás, "Nonlinear cooling of an annular beam distribution," in preparation.

[29] A. Neishtadt, A. Vasil'ev, and A. Artem'ev, "Capture into resonance and escape from it in a forced nonlinear

ISBN: 978-3-95450-225-7 ISSN: 2673-5571 doi:10.18429/JACoW-HB2021-MOP08

pendulum," *Regular and Chaotic Dynamics*, vol. 18, no. 6, pp. 686–696, 2013.

[30] V. I. Arnold and A. Avez, *Problèmes Ergodiques de la Mécanique Classique*, Paris: Gauthier-Villars, 1967.

[31] M. Hénon, "Numerical study of quadratic area-preserving mappings," *Q. Appl. Math.*, vol. 27, p. 291, 1969.

[32] A. Bazzani, G. Servizi, E. Todesco, and G. Turchetti, *A normal form approach to the theory of nonlinear betatronic motion*, ser. CERN Yellow Reports: Monographs, CERN, Geneva, Switzerland, 1994. doi:10.5170/CERN-1994-002

[33] S. Y. Lee, K. Y. Ng, H. Liu, and H. C. Chao, "Evolution of beam distribution in crossing a Walkinshaw resonance," *Phys. Rev. Lett.*, vol. 110, p. 094801, 2013. doi:10.1103/PhysRevLett.110.094801