Improved Hot Dark Matter Bound on the QCD Axion

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We obtain a reliable cosmological bound on the axion mass m_a by (1) deriving the production rate directly from pion-pion scattering data, which overcomes the breakdown of chiral perturbation theory and results in ∼30% differences from previous estimates; (2) including momentum dependence in the Boltzmann equations for axion-pion scatterings, which enhances the relic abundance by ∼40%. Using present cosmological datasets we obtain $m_a \leq 0.24$ eV, at 95% C.L. We also constrain the sum of neutrino masses, $\sum m_\nu \leq 0.14$ eV at 95% C.L., in the presence of relic axions and neutrinos. Finally, we show that reliable nonperturbative calculations above the QCD crossover are needed to exploit the reach of upcoming cosmological surveys for axion detection.

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Introduction.—The absence of *CP* violation in QCD is one of the long-standing motivations for physics beyond the standard model (SM). Its simplest explanation, the Peccei-Quinn (PQ) mechanism [1,2], predicts the existence of a light (pseudo)scalar particle [3,4], the axion a, which can, moreover, play the role of the dark matter and whose detection is among the most important endeavours in particle physics of our times. As such, it is currently being tackled via a diverse array of experimental, astrophysical, and cosmological strategies.

This effort is especially challenging, in that first, axion interactions must be very suppressed in order not to affect stellar evolution (see Ref. [5]); second, relic cold axions produced via the misalignment mechanism [6–8] and the decay of topological defects [9–11] may only make a small fraction of the observed dark matter abundance, depending on the axion mass m_a, and this would then hinder any detection strategy which relies on identifying the axion with the dark matter. Astrophysical searches are a promising alternative, but are affected by larger uncertainties (see, e.g., recent conflicting reassessments [12–14] of the SN1987A constraint, for bounds from other sources, see Refs. [15–18]).

In this Letter, we focus on a different route to axion detection, which partially escapes the shortcomings above, while relying only on the unavoidable ingredient of the PQ mechanism: the axion coupling to QCD. The resulting scattering processes (most importantly with pions) can be effective in the early Universe at temperatures $T \sim$ 100 MeV and above, thereby generating a population of relativistic axions [19–24]. Very much like neutrinos, these behave as "hot," rather than cold, dark matter (HDM) components, and can thus be searched for, or constrained, using observations of the cosmic microwave background (CMB) [25] and of cosmic large scale structure (LSS). Given the recently attained precision of such datasets, as well as the important sensitivity improvements of current [26–28] and upcoming [29–32] cosmological surveys, the crucial theoretical task is to reliably predict the axion HDM abundance. This sets the goal of this Letter.

Long-employed estimates of axion production via scatterings with pions, based on computations at leading order (LO) in chiral perturbation theory (χPT) [22] (see Refs. [33–37] for corresponding HDM bounds on m_a from CMB data), have been recently shown to be unreliable [38], since the rate receives large one-loop corrections at $T \gtrsim 70$ MeV. This significantly reduces the range of temperatures up to which theoretical control of uncertainties can be maintained. Consequently, the corresponding cosmological constraints [39] are weaker than those set by solar axion experiments [40] (although these rely on the model-dependent axion coupling to photons).

We overcome this obstacle using a novel approach: we employ experimental data on $\pi\pi \leftrightarrow \pi\pi$ scattering, which indeed extend to regions where LO χPT fails. We then point out that the $a\pi$ cross sections can be obtained by a simple rescaling, owing to the well-known fact that a π^0 field contains an a component (see [41] for a similar strategy and [42] for production in stars). This allows us to obtain a reliable axion production rate up to the QCD confinement temperature $T_c \lesssim 150$ MeV. In particular, unlike [43–45], conservatively we do not rely on interpolations between LO χPT and perturbative QCD rates.

We then present an equally important novelty for the axion HDM abundance calculation. At the temperatures of interest, the evolution of the thermal plasma in the Universe is strongly affected by the QCD crossover, whereby heavy hadrons form and rapidly transfer significant entropy to the remaining light degrees of freedom. This invalidates the previously employed equilibrium assumption for the axion phase space distribution, which we indeed find to be significantly distorted, by solving momentum-dependent Boltzmann equations. This leads to a previously overlooked enhancement of the axion HDM abundance, which is much more significant than in the familiar case of neutrino decoupling during e^+e^- annihilation [46,47] (see Ref. [48] for related work focused on heavy axions).

The two advances above allow us to set a reliable upper bound on m_a from cosmology, which importantly extends to the region targeted by future solar axion experiments [49,50]. Furthermore, our work additionally reveals previously neglected nonperturbative QCD contributions to axion production at T_c and above, motivating dedicated studies to correctly interpret the implications of forthcoming experiments for axion detection.

Boltzmann equations.—We focus on the minimal model-independent axion interaction, $\mathcal{L}_{\rm int} = \alpha_s a G\tilde{G}/(8\pi f_a)$, with G the gluon field strength, $\tilde{G}$ its dual, α_s the strong coupling constant, and $f_a = 10^7\ \mathrm{GeV}(0.57\ \mathrm{eV}/m_a)$ [51] the axion decay constant.

We use the momentum-dependent Boltzmann equations to compute the actual spectrum of axions produced via scatterings. We find this to be necessary, for two reasons. First, the initial axion abundance may be negligible and production may not be efficient enough, so that the axion spectrum never reaches the equilibrium distribution. Second, even when reaching equilibrium, interaction rates depend on the axion momentum, so that high momenta decouple later than low momenta. Since the number of relativistic degrees of freedom $g_{*,S}$ decreases quickly and substantially with temperature around T_c, high momenta are less diluted with respect to photons than low momenta.

The Boltzmann equations for the axion distribution function $f_{\mathbf{p}}$, with comoving 3-momenta $\mathbf{p}$, reads [20,52]

$$\frac{df_{\mathbf{p}}}{dt} = (1+f_{\mathbf{p}})\Gamma^{<} - f_{\mathbf{p}}\Gamma^{>}, \tag{1}$$

where the axion, with negligible mass at the time of production, has physical energy $E = |\mathbf{p}|/R$ and four-momentum $k^\mu = (E, \mathbf{k})$, R is the scale factor, t is cosmic time, and the energy dependent rates $\Gamma^{<}$ and $\Gamma^{>}$ describe creation and destruction of an axion. Thermal equilibrium of the QCD bath implies $\Gamma^{>} = e^{E/T}\Gamma^{<}$. At weak coupling the rates are dominated by $2 \leftrightarrow 2$ scatterings, with amplitude $\mathcal{M}$, leading to [52]:

$$\Gamma^{<} = \frac{(2\pi)^4}{2E}\int\left(\prod_{i=1}^{3}\frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\rm eq} f_2^{\rm eq}(1+f_3^{\rm eq}) \times \delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2, \tag{2}$$

where $f_i^{\rm eq} \equiv (e^{E_i/T}-1)^{-1}$. For $\pi\pi \leftrightarrow \pi a$ scatterings, the physical four-momenta are k_1 and k_2 for the incoming pions, k_3 for the emitted pion.

The axion relic abundance is then commonly expressed in terms of the effective number of (massless) neutrino species beyond the SM neutrinos $N_\nu \approx 3.044$ [53–55]: $\Delta N_{\rm eff} \equiv N_{\rm eff} - N_\nu = (8/7)(11/4)^{4/3}(\tilde{\rho}_a/\rho_\gamma)_{\rm rec}$, where $\tilde{\rho}_a, \rho_\gamma$ are the energy densities of axions for $m_a = 0$ and photons respectively, evaluated at recombination. While in our region of interest m_a is actually close to the recombination temperature ($\lesssim$eV), we still use $\Delta N_{\rm eff}$ as a conventional parametrization of the axion abundance.

We make contact with previous momentum-independent treatments in the literature [56] through the averaged rate

$$\bar{\Gamma} \equiv \frac{1}{n^{\rm eq}}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Gamma^{<}, \tag{3}$$

with $n^{\rm eq} \equiv \int d^3\mathbf{k}/(2\pi)^3 f_{\mathbf{p}}^{\rm eq}$. Furthermore, under the assumption of instantaneous decoupling of an initial equilibrium axion abundance, the approximation $\Delta N_{\rm eff} \simeq 0.027[106.75/g_{*,S}(T_{\rm d})]^{4/3}$ is commonly used, with T_d defined by $\bar{\Gamma} = H|_{T=T_d}$ and $H \equiv \dot{R}/R$. As mentioned above, neither of these two approaches is justified in our case.

Axion rate below T_c.—At $T \lesssim T_c$ the QCD thermal bath is dominated by pions. As pointed out in Ref. [38], next-to-leading-order (NLO) corrections invalidate the LO χPT computation already at $T \gtrsim 60$ MeV. In retrospect, this is not surprising given that (1) the typical center of mass energy $\sqrt{s}$ for two pions at such temperatures is already above 0.4 GeV and (2) the scattering amplitudes grow with energy, thereby more energetic pions, for which χPT is even more unreliable, are weighted more in the integral (3).

In fact, the same problem was tackled long ago for pion damping rates [76–78] by using experimental $\pi-\pi$ scattering data directly. With this strategy, the pion rate was computed up to $T = T_c$ (beyond which the rate becomes rapidly comparable with the mass and pions cannot be considered as elementary particles anymore). In an attempt to rescue the χPT computation, Ref. [78] proposed also a unitarization approach, where the χPT expansion is applied to (functions of) the scattering phase shifts rather than to the amplitude $\mathcal{M}$. Unitarity is then respected, taming the growth at high energies. The unitarization procedure is, however, not unique. While some choices agree well with experimental data, without the latter it would be hard to defend the use of one particular prescription, or why higher-order corrections can be neglected.

Unfortunately, for axions there are no experimental data and at a first glance it seems that only unitarization could improve on the fixed order χPT computation. However, in the PQ mechanism the neutral pion mixes with the axion [3], such that a-π and π^0-π amplitudes are related by the simple rescaling:

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \frac{\epsilon f_\pi}{2f_a}\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right), \tag{4}$$

with $f_\pi = 92.3$ MeV, $m_\pi = 138$ MeV (we use the average pion mass), $\epsilon \equiv (m_d - m_u)/(m_d + m_u)$, where m_u and m_d are up and down quark masses, $m_u/m_d \simeq 0.47$ [79]. That this relation holds at LO and NLO order can be checked by a direct comparison of the amplitudes for π-π [80] and a-π [22,38] scattering. In fact, it remains valid at all orders in χPT (see [56] for details). The $\mathcal{O}(m_\pi^2/s)$ corrections near the two pions threshold can be computed directly at LO and they are at most $\mathcal{O}(10\%)$ and rapidly decrease at higher energies. While here we focused on the model-independent coupling of the QCD axion, Eq. (4) can easily accommodate general axion couplings, by replacing ϵ with the corresponding mixing [56].

Thanks to Eq. (4) phenomenological fits of π-π scattering data can be used to reconstruct a-π scattering amplitudes with a few percent precision up to T_c. To compute $\Gamma^>$ we applied Eq. (4) to the phenomenological π-π partial wave amplitudes provided in Refs. [81,82] (specifically the S0, S2, and P waves, valid up to the two-kaon threshold $\sqrt{s} = 2m_K \simeq 1$ GeV, while we checked that higher partial waves contribute negligibly). The result is presented in Fig. 1 (solid blue curve, upper panel), for a reference temperature $T = 120$ MeV. As expected, it decreases sharply at large momenta, in contrast to the LO a-π rate (dashed). The corresponding averaged rates are shown in Fig. 1 (lower panel) and differ by $\sim$30%.

The decrease of the a-π rate at high temperatures is in part due to the opening of new scattering channels (see [77] for an analogous discussion for pions). Using the LO χPT Lagrangian, we checked, for instance, that scatterings with kaons $\pi K \to aK$ are subleading below T_c, but would dominate above $T \simeq 200$ MeV [56] (similar considerations may apply to scattering off nucleons). While in general a χPT calculation for these processes is less reliable, such an estimate shows that the phenomenological a-π rate dominates the axion thermalization rate at low temperatures, representing a reliable lower bound at $T \lesssim T_c$.

Integrating numerically Eqs. (1) [56] and assuming conservatively no extra production from $T > T_c$, we get the lower bound on $\Delta N_{\rm eff}$ (boundary of the solid blue region) in Fig. 2. Because of the rapid change in $g_{*,S}$ axions decouple with a distorted spectrum. The main consequence is a large enhancement of $\Delta N_{\rm eff}$, by $\sim$40% (compared to

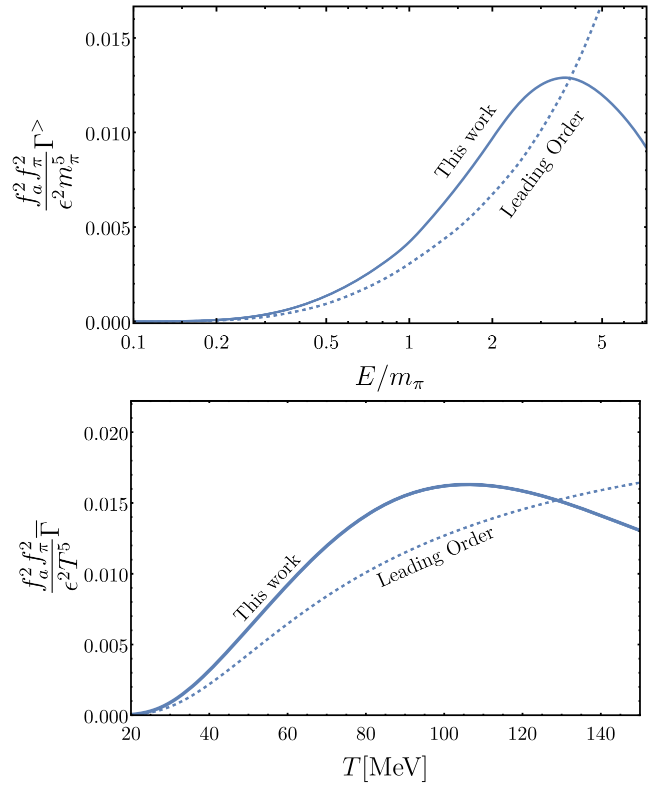

FIG. 1. Scattering rates for $a\pi \leftrightarrow \pi\pi$. Top: momentum dependent axion destruction rates at $T = 120$ MeV. Bottom: averaged rates.

the momentum-independent result, dotted black curve in Fig. 2), while the residual distortion of the shape only has minor effects on the cosmological analysis presented below (see also [56] and [83,84] for related discussion on neutrinos). A slightly less conservative assumption is to consider the axion in thermal equilibrium at some temperature $T > 1$ TeV (with $g_{*,S}$ saturated by the standard model), see Fig. 2 (boundary of the light blue region).

Estimates above T_c.—Above T_c the QCD bath has a smooth crossover from the hadronic phase to a quark-gluon plasma. The crossover is fully nonperturbative and the rates (2) should be replaced with $\Gamma^> = \Gamma^>_{\rm top}/(2Ef_a^2)$, with the topological rate

$$\Gamma^>_{\rm top} \equiv \int d^4x e^{ik^\mu x_\mu}\left\langle \frac{\alpha_s}{8\pi}G\tilde{G}(x^\mu)\frac{\alpha_s}{8\pi}G\tilde{G}(0)\right\rangle, \tag{5}$$

where $\langle\cdots\rangle$ stands for thermal average. At present, we are not aware of any computation of Eq. (5) in this regime, a challenging task beyond the aim of this work. All we can attempt is a very rough estimate of $\bar{\Gamma}$ during the crossover, say for $T_c \lesssim T \lesssim 2$ GeV, using dimensional analysis, i.e., $\bar{\Gamma} \sim \kappa T^3/f_a^2$, where κ is an unknown coefficient. For illustrative purposes we choose two reference values $\kappa = 0.01$ and $\kappa = 0.1$ in Fig. 2, justified as follows.

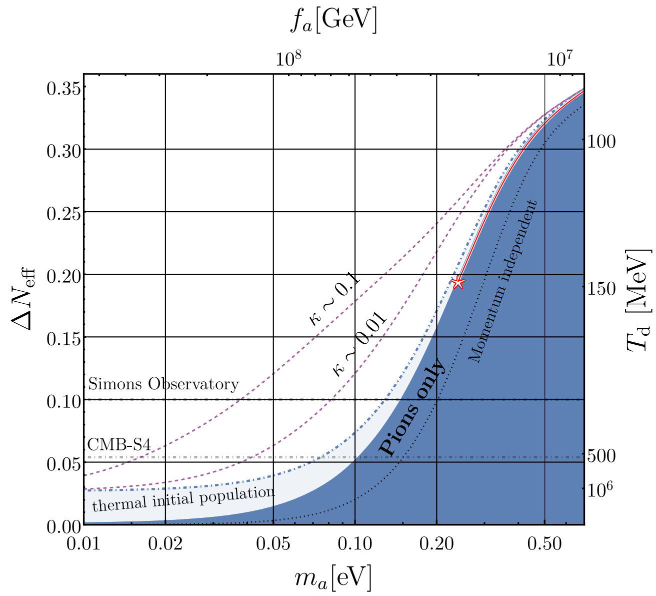

FIG. 2. Relic abundance for an axion minimally coupled to QCD. The 95% C.L. expected sensitivities of upcoming CMB surveys for massless species at recombination are shown by the dot-dashed gray lines, see text for details. The red curve above the star is excluded at 95% C.L. by our new analysis, and corresponds to a would-be decoupling temperature $T_d \lesssim 150$ MeV [T_d here is just a reparametrization of $\Delta N_{\rm eff}$ as defined below Eq. (3)]. The dot-dashed blue and dashed purple curves include an initial thermal population at $T = 1$ TeV. The purple curves also include tentative estimates of nonperturbative production above T_c.

First, at $T \gtrsim f_\pi$ the a-π rate just below T_c is $\sim 0.04\epsilon^2 T^3/f_a^2$. The isospin breaking suppression ϵ is present only in the coupling to pions but not to other hadrons [56]. Factoring that away and considering the contributions from other states above T_c, it is reasonable to expect $\kappa \sim \mathcal{O}(0.1)$ at around T_c and slightly above.

Second, recent attempts in pure SU(3) lattice gauge theory estimate $\Gamma_{\rm sph} \equiv \Gamma^>_{\rm top}(k^\mu = 0) \simeq \kappa_{\rm latt} T^4$ for the zero mode, with $\kappa_{\rm latt}$ ranging from $\mathcal{O}(0.1)$ to $\mathcal{O}(0.01)$ as T increases from T_c to 1 GeV [85,86].

We stress that our dimensional analysis estimate for the nonperturbative contribution is provided only to highlight its potential importance, especially for upcoming experiments.

Note also that in the same range of temperatures (150 MeV $\lesssim T \lesssim$ 500 MeV), the most interesting ones for the upcoming CMB experiments, entropy and energy densities have the largest relative variation [87]. Thus the spectral distortion discussed before is expected to be even more relevant in this case; it becomes therefore mandatory to study the full energy dependence in Eq. (5) with nonperturbative methods, in order to correctly assess the implications of forthcoming experiments for the axion parameter space.

At sufficiently large temperatures QCD becomes weakly coupled and the axion rates are dominated by the perturbative scattering with gluons and quarks [24,88,89]. In principle, one could interpolate these large temperature asymptotic computations with the low temperature ones from pion scattering discussed before to estimate the rates in the strongly coupled intermediate region, as in Refs. [43,44]. However, such strategy has two potential problems. First, perturbative computations at finite temperature are affected by infrared divergences of two kinds, one related to forward scattering [24,88] and the other to collective effects [90,91], which are intrinsically nonperturbative. The former have been improved in [89] but a full next-to-leading order computation is still missing and the minimum temperature at which such computation is reliable is unknown. The latter are connected to the strong sphaleron rates $\Gamma_{\rm sph}$ introduced above and a semiclassical estimate (see [56] for more details) seems to suggest that they dominate over the perturbative contribution well above T_c. The second potential problem is that many quantities, such as the free energy, present sudden changes in the crossover region due to the large number of degrees of freedom that freeze out. In particular, for the axion rates we already know that below T_c the only non-Boltzmann suppressed contribution comes from the scattering with pions, whose coupling is, however, isospin suppressed. Above T_c many more degrees of freedom will start contributing without isospin or Boltzmann suppressions. This suggests that the axion rate might not have a completely smooth interpolation between the low and high temperature regions, which raises doubts on the reliability of the rates used in [43–45].

Current bound and outlook.—Using the "pions only" curve in Fig. 2 we set a conservative upper bound on m_a from Planck 2018 CMB data [92], baryon acoustic oscillations [93–96] and pantheon supernovae [97] (see [56]). We modified the Boltzmann solver CLASS [98,99] to include the axion with its actual distribution function [56]. We ran a Markov chain Monte Carlo (MCMC) analysis of the ΛCDM $+ \sum m_\nu + m_a$ cosmological model, where $\sum m_\nu \geq 0.06$ eV [79] is the sum of the neutrino masses, using the MontePython sampler [100,101]. We find $m_a \leq 0.24$ eV, corresponding to $\Delta N_{\rm eff} \lesssim 0.19$ (red line with star in Fig. 2), and $\sum m_\nu \leq 0.14$, both at 95% C.L. (statistical error only); the 2D posterior distributions of m_a and $\sum m_\nu$ are reported in Fig. 3 [56]. Despite the large enhancement from momentum dependence, our bound on m_a is similar to the previous result based on the LO calculation until T_c [102], because our new rate is smaller at $T \lesssim T_c$.

Importantly, our new bound does not rely on interpolated rates above T_c, as in [45], which as we have argued are not under control, see also [56].

Our conservative bound, which uses only the axion coupling required to solve the strong CP problem, can be

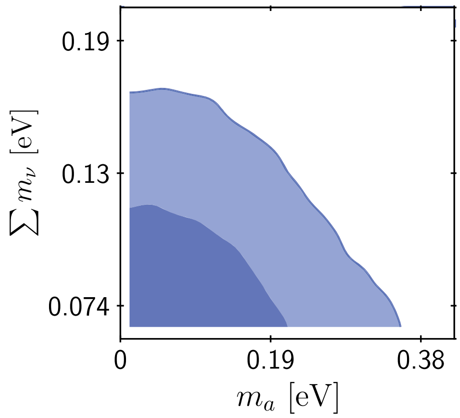

FIG. 3. 1D and 2D posterior distributions for $\sum m_\nu$ and m_a. The darker and lighter shades correspond to 1σ and 2σ confidence level regions.

either weaker or stronger than the astrophysical constraints from globular clusters [16] (see [17] for a recent more constraining update) and than solar axion searches [40], depending on the UV structure of the axion model. It is significantly weaker than the constraints of Refs. [14,18], which are, however, subject to astrophysical uncertainties. In contrast, our bound only relies on standard cosmology below T_c.

While we did not include nonperturbative production in our MCMC search in the conservative spirit of our work, Fig. 2 shows that it could significantly strengthen the current bound. Even more importantly, as shown on the right vertical axis of Fig. 2, the upcoming Simons Observatory [29] and CMB-S4 [30] will probe axion production during the QCD crossover. In particular, they could reach $m_a \sim 0.01$ eV, and possibly even below, close to the region where cold axions can be the dark matter (note that the dashed purple curves in Fig. 2 do not include production above $T = 2$ GeV nor the enhancement due to the rapid variation of $g_{*,S}$ above T_c).

Such exciting possibilities motivate a dedicated study of axion production rates by nonperturbative methods, for arbitrary axion momenta, beyond the attempts made so far only for the zero mode.

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Note added.—Recently, another preprint [103] appeared discussing axion-pion rates.

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