

## EXTRACTING THE STRANGE DENSITY FROM $xF_3$

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### Abstract

We present a QCD analysis of the strange and charm contributions to the neutrino deep inelastic structure function  $xF_3$ . We show that next-to-leading order effects, which are relatively important for  $F_2$ , play a lesser role in the case of  $xF_3$ . The neutrino-antineutrino difference  $xF_3^\nu - xF_3^{\bar{\nu}}$  provides a new determination of the strange density, which exhibits some advantages with respect to other traditional methods.

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Two methods are traditionally used to extract the strange-sea density from deep inelastic scattering (DIS) data: the first consists in studying charm production in charged-current neutrino DIS (the characteristic signature of this process being the presence of dimuons in the final state); the second is to subtract the  $F_2$  structure functions measured in neutrino and muon DIS, thus selecting the strange contribution.

Until last year, these two determinations, based on the NMC  $\mu$ DIS data [1] and on the CCFR  $\nu$ DIS data [2, 3] available at that time, seemed to yield contradictory results for  $s(x)$ . This discrepancy, which has strongly challenged the attempts at global parton fits [4, 5], had been actually predicted [6, 7], and is simply explained [8, 9] by the observation that the dimuon and  $\nu - \mu$  determinations of  $s(x)$  actually measure different quantities, related to – but not coincident with – the strange density. This is due to the relevance of quark-mass effects <sup>2</sup> and longitudinal contributions, in the region of small and moderate  $Q^2$  values (of order of 10-30 GeV<sup>2</sup>) <sup>3</sup>.

A recent QCD analysis [10] of the new CCFR dimuon data [11] has shown that, when all important physical effects are taken into account, the different measurements converge – as they should – towards a unique result for  $s(x)$ . Incidentally, the strange-quark distribution emerging from all data is well reproduced by a “traditional” fit, such as, for instance, MRS(A) [12], and does not support a nearly  $SU(3)$  symmetric fit, such as CTEQ1 [5].

Both the dimuon and the  $\nu$ - $\mu$  extractions of  $s(x)$  present problems and subtleties (for a detailed discussion see [9, 10]). In particular, the dimuon determination implies, for experimental reasons, an acceptance-dependent separation of the  $t$ - and  $u$ -channel diagrams that constitute the  $W$ -gluon fusion QCD process. On the other hand, the  $\nu - \mu$  result is affected by large uncertainties due to the very unsafe procedure of

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<sup>2</sup>Remember that in charged-current neutrino DIS strange and charm excitations are inseparable.

<sup>3</sup>For instance, the average  $Q^2$  value of the CCFR data is  $\langle Q^2 \rangle \simeq 22 \text{ GeV}^2$ .

subtracting data from two different experiments.

However, the idea of obtaining  $s(x)$  by an appropriate combination of structure functions can be further exploited. There is in fact another way to isolate  $s(x)$  from DIS structure functions, which makes use of the third  $\nu$ DIS structure function,  $F_3$ . In the parton model these are [13]

$$xF_3^{\nu N}(x) = xV(x) - 2x\bar{c}(x) + 2xs(x), \quad (1)$$

$$xF_3^{\bar{\nu}N}(x) = xV(x) + 2xc(x) - 2x\bar{s}(x), \quad (2)$$

where  $V(x)$  is the valence distribution and  $N$  denotes an isoscalar nucleon (hereafter we shall drop the suffix  $N$  from our formulas). Therefore, the  $\nu - \bar{\nu}$  difference effectively measures the strange density, since the charm contribution is very small, at least in the kinematical region investigated by the present experiments (we assume  $s = \bar{s}$  and  $c = \bar{c}$ ):

$$xF_3^{\nu N}(x) - xF_3^{\bar{\nu}N}(x) = 4x[s(x) - c(x)]. \quad (3)$$

Needless to say, moving from the parton model to leading-order QCD, the quark distributions acquire a  $Q^2$  dependence governed by the Altarelli–Parisi equations.

The use of eq. (3) to extract  $xs(x)$  has the immediate advantage, over the  $\nu - \bar{\nu}$  method, of combining data from the same experiment, thus with no relative-normalization problems. However, in practice there are at least two shortcomings. First of all,  $xF_3$  is a small, valence-dominated, quantity, more difficult to measure than  $F_2$ . Secondly, the statistics for  $xF_3^\nu - xF_3^{\bar{\nu}}$  is much lower than that for  $F_2$ : the measurement of  $xF_3^\nu - xF_3^{\bar{\nu}}$  requires neutrino and antineutrino data separately, whereas they can be combined for  $F_2$ , which is the same in  $\nu N$  and  $\bar{\nu}N$  deep inelastic scattering. In general, the number of  $\bar{\nu}$ -induced events is considerably smaller than that of  $\nu$ -induced events: for instance, the ratio is about 1 to 5 in the CCFR experiment [11].

The CCFR/NuTeV Collaboration is at present working on the extraction of  $xs(x)$  from  $x F_3^{\nu N} - x F_3^{\bar{\nu} N}$ , and data will be available in the near future [14].

Previous studies on the determination of the strange density at moderate  $Q^2$  from neutrino DIS have taught us the importance of quark-mass corrections and current non-conservation effects, which manifest themselves through the order- $\alpha_s$  vector-boson-gluon fusion diagrams. These represent the dominant contribution near the heavy-quark threshold, in particular at small  $x$ . It is then natural to go beyond the leading order also in the analysis of the extraction of  $xs(x)$  from  $x F_3$ . The Next-to-Leading Order corrections are known to affect mostly the longitudinal component of structure functions [7]: thus we expect the NLO effects to be smaller in  $x F_3$  than in  $F_2$ , because  $x F_3$  is a purely transverse structure function. However, only an explicit computation can give us precise information on the charm-strange content of  $x F_3$ .

In the following we shall present a QCD calculation of  $x F_3^\nu - x F_3^{\bar{\nu}}$  at order  $\alpha_s$ , taking into account the contribution of the gluon-fusion diagrams. The possibility of a safe, unambiguous, extraction of the strange density from  $x F_3^{\nu N} - x F_3^{\bar{\nu} N}$  will be explored and discussed.

At order  $\alpha_s$  the main contribution to the  $cs$  component of  $F_3^\nu$  is given by the  $W$ -gluon fusion (GF) term <sup>4</sup>, which, for the strange-charm sector, reads

$$F_{3,GF}^{\nu,cs}(x, Q^2) = \left(\frac{\alpha_s}{2\pi}\right) \int_{ax}^1 \frac{dz}{z} g(z, \mu^2) C_3\left(\frac{x}{z}, Q^2\right). \quad (4)$$

Here  $a = 1 + (m^2 + m'^2)/Q^2$  and, for neutrino scattering,  $m$  is the mass of the charmed quark,  $m'$  is the mass of the strange antiquark. The Wilson coefficient  $C_3$  represents the  $W^+g \rightarrow c\bar{s}$  cross section difference  $\sigma_L - \sigma_R$  ( $L$  and  $R$  standing for left- and right-transverse  $W$ , respectively); it has the explicit form [15, 16]:

$$C_3(z, Q^2) = 2 \left\{ \beta_+ \beta_- \frac{m^2 - m'^2}{Q^2} 2z(1-z) \right.$$

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<sup>4</sup>The  $O(\alpha_s)$   $W$ -quark fusion diagrams are a negligible correction.

$$\begin{aligned}
& - \left[ \frac{1}{2} - z(1-z) + \frac{m^2 - m'^2}{Q^2} z(1-2z) - \frac{m^4 - m'^4}{Q^4} z^2 \right] L(m, m') \\
& + [m \leftrightarrow m'] L(m', m) \} , \tag{5}
\end{aligned}$$

where

$$\beta_{\pm}^2 = 1 - \frac{(m \pm m')^2}{Q^2} \frac{z}{1-z} \tag{6}$$

and

$$L(m, m') = \log \frac{1 + \frac{m^2 - m'^2}{Q^2} \frac{z}{1-z} + \beta_+ \beta_-}{1 + \frac{m^2 - m'^2}{Q^2} \frac{z}{1-z} - \beta_+ \beta_-} . \tag{7}$$

If the two quark masses are non-zero, the Wilson coefficient  $C_3$  is free from singularities. In the limit  $m' \rightarrow 0$ , *i.e.* treating the strange quark as massless,  $L(m, m')$  and  $L(m', m)$  behave as

$$L(m, m') \rightarrow \log \frac{\hat{s}}{m^2} \tag{8}$$

$$L(m', m) \rightarrow \log \frac{(\hat{s} - m^2)^2}{\hat{s} m'^2} \tag{9}$$

where  $\hat{s} = Q^2(1-z)/z$ . In the same limit, the factor multiplying  $L(m', m)$  becomes  $\mathcal{P}_g(\xi)$ , namely the usual  $g \rightarrow q\bar{q}$  splitting function  $\mathcal{P}_g$  expressed in the rescaled variable  $\xi = x(1 + m^2/Q^2)$ . While  $L(m, m')$  is regular,  $L(m', m)$  has a collinear singularity. This is subtracted out by setting [17]

$$L(m', m) = \log \frac{(\hat{s} - m^2)^2}{\hat{s} \mu^2} , \tag{10}$$

where the scale  $\mu^2$  is customarily taken to be equal to the factorization scale (*i.e.* to the scale that separates the perturbative part from the non-perturbative one in the DIS QCD diagrams).

Let us now come to the total  $cs$  contribution to  $F_3$ . We start from the QCD factorization formula, which formally reads ( $\otimes$  means convolution)

$$F(x, Q^2) = \sum_i f_i(z, \mu^2) \otimes C\left(\frac{x}{z}, \mu^2, Q^2\right) , \tag{11}$$

where the sum is made over all parton species.

At leading order only quark and antiquark contribute and the corresponding Wilson coefficients are delta functions of the slow-rescaling variable  $\xi = x(1+m^2/Q^2)$ . At next-to-leading order the contribution is given by eq. (4) with the above-defined subtraction. Thus we get

$$\begin{aligned} F_3^{\nu,cs}(x, Q^2) &\equiv F_{3,QE}^{\nu,cs}(x, \mu^2) + \tilde{F}_{3,GF}^{\nu,cs}(x, Q^2) \\ &= 2 \left[ \bar{s}(\xi, \mu^2) - c(x, \mu^2) \right] + \left( \frac{\alpha_s}{\pi} \right) \int_{ax}^1 \frac{dz}{z} g(z, \mu^2) \tilde{C}_3 \left( \frac{x}{z}, Q^2 \right). \end{aligned} \quad (12)$$

Here  $\tilde{C}_3$  stands for the subtracted Wilson coefficient, *i.e.* for  $C_3$  with the replacement (10). Notice that the quark excitation (QE) term,  $\bar{s} - c$ , is taken at the factorization scale  $\mu^2$ . The  $Q^2$  evolution (at least the dominant part of it, due to  $g \rightarrow q\bar{q}$  splitting) is already contained in the gluon-fusion Wilson coefficient. Both taking the quark densities in eq. (12) at the physical scale  $Q^2$  and using  $C_3$  instead of  $\tilde{C}_3$  in eq. (12) would represent a double counting. A most often used approximation consists in setting

$$F_3^{\nu,cs}(x, Q^2) = 2 \left[ \bar{s}(\xi, Q^2) - c(x, Q^2) \right], \quad (13)$$

which means combining massless QCD, for the  $Q^2$  evolution, with slow rescaling, to account for quark mass effects. In the following we shall check the goodness of this slow rescaling procedure for  $F_3$ .

At large momentum transfer ( $Q^2 \gg m_c^2$ , say  $Q^2 \gtrsim 30 \text{ GeV}^2$ ) the charmed quark should also be treated as a massless parton. This means that  $L(m, m')$  too becomes large and should undergo a subtraction similar to that performed by eq. (10). Asymptotically, the massless QCD formulas are of course regained: in the limit  $m, m' \rightarrow 0$  the contribution of the gluon-fusion diagram to  $F_3$  vanishes, *i.e.*  $\tilde{F}_{3,GF}^{\nu,cs} \rightarrow 0$ .

So much for the neutrino DIS. In the case of antineutrino scattering the formulas written above must be modified by exchanging  $m$  and  $m'$ :  $m$  becomes the strange mass,

$m'$  the charm mass. The GF contribution changes sign and so does the QE term. The difference of the whole structure functions,  $F_3^\nu - F_3^{\bar{\nu}}$ , is simply twice the  $cs$  component for neutrino,  $2F_3^{\nu,cs}$ , since the valence part cancels out (notice that the  $cs$  component disappears in the sum  $F_3^\nu + F_3^{\bar{\nu}}$  and hence is irrelevant for the Gross–Llewellyn Smith sum rule).

Let us now present the results of our calculations. The  $\nu - \bar{\nu}$  difference of  $x F_3$  structure functions,  $x F_3^\nu - x F_3^{\bar{\nu}}$ , has been evaluated by using eq. (12) and the MRS(A) fit [12] for the strange, charm and gluon densities at the factorization scale  $\mu^2$ . The choice of  $\mu^2$  is a delicate issue. As we shall see, one of the advantages of working with  $x F_3^{cs}$  is that its dependence on  $\mu^2$  turns out to be quite small (smaller than the  $\mu^2$ -dependence of  $F_2^{cs}$ ). In any case,  $\mu^2 = m_c^2$ , where  $m_c$  is the charm mass, seems to be a natural choice near (or not much above) threshold. At large  $Q^2$  it is reasonable to take a factorization scale of order  $Q^2$ , rather than  $m_c^2$ . The prescription of Ref. [17], for instance, gives  $\mu^2 \simeq m_c^2$  at small  $Q^2$  and  $\mu^2 \simeq Q^2/2$  at  $Q^2 \gg m_c^2$ .

In Fig. 1 we present our results for  $x F_3^\nu - x F_3^{\bar{\nu}}$  just above threshold ( $Q^2 = 10 \text{ GeV}^2$  and  $25 \text{ GeV}^2$ ), with  $\mu^2 = m_c^2$  (we use  $m_c^2 = 2.7 \text{ GeV}^2$ ). In this case  $c(x, \mu^2)$  obviously vanishes. The total result, eq. (12), the quark excitation term, the unsubtracted gluon fusion contribution, eq. (4), and the slow-rescaling prediction, eq. (13), are plotted in the figure. An interesting feature is clearly visible: the complete NLO result for  $x F_3^\nu - x F_3^{\bar{\nu}}$  nearly coincides with the slow-rescaling expectation, whereas it differs sensibly from the unsubtracted GF result, especially for  $x \gtrsim 0.01$ . Thus, the slow rescaling mechanism, whose application to the longitudinal+transverse structure function  $F_2$  at small  $Q^2$  is rather unsafe (see [10, 18]), turns out to be an excellent approximation when dealing with the charm–strange contribution to  $F_3$ . This is due to the fact that the main next-to-leading order effects that slow rescaling mimicks too crudely are related to the longitudinal component of structure functions, which is absent in  $F_3$ . Notice

also that the QE component, which is the only term containing the strange density that experiments aim to extract, is comparable in magnitude to – and actually not much different from – the complete result.

We checked the dependence of the results on the factorization scale  $\mu^2$ . This dependence is shown in Fig. 2 for two values of  $x$  (0.01 and 0.1) and for  $Q^2 = 25 \text{ GeV}^2$  (close to the average  $Q^2$  value of the CCFR experiment). It is reassuring to see that, not only the total result for  $x F_3^\nu - x F_3^{\bar{\nu}}$ , but each single term (and in particular the QE term) has a very mild dependence on  $\mu^2$ . Moreover, the (small)  $\mu^2$ -dependence of the QE term is approximately the same as that of the full  $\nu - \bar{\nu}$  difference (the solid and dotted curves in the left part of Fig. 2 are parallel). For comparison, in the right part of Fig. 2 we present also the scale dependence of  $F_2^{\nu,cs}$ , which appears to be more dramatic, in particular in the QE component (for more details we refer the reader to a forthcoming paper [18]). Since it is the latter component that contains the strange and charm densities, it is evident that the extraction of  $xs(x)$  from  $x F_3^\nu - x F_3^{\bar{\nu}}$  is affected by a factorization scale uncertainty much smaller than that occurring in a measurement based on  $F_2^{\nu,cs}$ , such as the dimuon measurement.

At large  $Q^2$  both strange and charm can be considered as massless partons and one expects to regain the results of massless QCD: in particular, the subtracted GF term should vanish. This is clearly visible in Fig. 3, where one can see that the full result coincides asymptotically both with the QE term and with the slow-rescaling prediction (this means that  $F_{3,GF}^{\nu,cs}$  – represented by the dashed curve – is exactly cancelled by the subtraction term, so that  $\tilde{F}_{3,GF}^{\nu,cs} = 0$ ).

Leaving aside the lack of statistics that may make the determination of the strange density from  $x F_3^\nu - x F_3^{\bar{\nu}}$  difficult in practice, it is clear that this method has some indisputable advantages: *i*) it is not plagued by relative-normalization errors; *ii*) it is not affected by the ambiguities inherent in other methods, such as the spurious



separation of  $t$ - and  $u$ -channel diagrams that occurs in the dimuon separation (see [9, 10]); *iii*) large longitudinal contributions arising from the non-conservation of weak currents are obviously absent; *iv*) charm mass effects are very well accounted for by the slow-rescaling prescription, making the analysis of data and the extraction of the strange density particularly simple (all the next-to-leading order QCD machinery can be safely avoided); *v*) the dependence on the factorization scale of the full  $O(\alpha_s)$  result and of the quark excitation term, which contains the strange density to be extracted, is rather mild and does not represent a worrisome source of uncertainty.

The main conclusion of our study is that a precision measurement of  $x F_3^\nu - x F_3^{\bar{\nu}}$  may well represent a new, interesting source of information on the strange quark distribution: being cleaner and more direct than other determinations, it is certainly worth exploiting.

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## Figure Captions

- Fig. 1 The difference  $xF_3^\nu - xF_3^{\bar{\nu}}$  at  $Q^2 = 10 \text{ GeV}^2$  and  $25 \text{ GeV}^2$ . The solid curve is the complete result, eq. (12). The dotted and dashed curves are the quark excitation (QE) and the unsubtracted gluon fusion (GF) contributions, respectively. The dot-dashed curve is the slow-rescaling expectation, eq. (13). The factorization scale is  $\mu^2 = m_c^2$ .
- Fig. 2 The dependence of  $xF_3^\nu - xF_3^{\bar{\nu}}$  and of  $F_2^{\nu,cs}$  on the factorization scale  $\mu^2$ . The meaning of the curves is the same as in Fig. 1.
- Fig. 3 Same as Fig. 1 at  $Q^2 = 100 \text{ GeV}^2$  and  $1000 \text{ GeV}^2$ , with  $\mu^2 = Q^2/2$ . Note that in the bottom window the solid, dotted and dot-dashed curves coincide.