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ATTEMPT TO PROMOTE COLLISION AND COALESCING OF H=13 HOLES BY IMPOSED ENERGY MODULATION AT H=1 FOR PROTON BEAM IN PS BOOSTER

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The purpose of the 16th November 1999 MD was to investigate the behaviour of longitudinal holes in a coasting beam generated by depositing high harmonic empty buckets. Empirical evidence suggests that these holes are stabilized by space-charge forces, and that holes may collide as a result of momentum differences. Collision of holes was hastened by imposing an energy modulation on the beam using the fundamental RF system. The lower limit of the CO2 RF system is 0.1 kV. Unfortunately, it is too easy to make the modulation so large that although holes collide in real space (i.e. longitudinal position) they fail to collide in phase space and do not coalesce. Nevertheless, the principle of hastened collision was demonstrated using CO2 pulses of short duration ($\leq$ 1 ms).

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1 Introduction

Empty bucket deposition into a coasting beam produces a longitudinal phase space with periodic holes. Below transition energy these holes are stabilised (longitudinally) by space-charge forces. If the space-charge impulses are sufficient to reverse the direction of motion of particles with the maximal momentum inside the hole, then the hole is stable[1]. However, if the momenta are too large, the hole will shear in longitudinal phase space, the space-charge force will weaken and the hole will filament. It is not hard to derive an equilibrium condition for the case of rectangular holes (as the limiting case of bi-trapezoidal holes) which shows the connection between momenta and turn-average beam current.

1.1 Collision of holes

Consider now a longitudinal array of stable holes. Because the space-charge force is proportional to the derivative of the line current density, there is no force of attraction or repulsion between them until they contact. However, if there are small differences in their central momenta, then holes will move relative to one another. If the energy differences are quasi-random, then some holes will move closer together while others will move farther apart. Typically, this is a slow process because the energy differences are small. But eventually, two or more holes will collide. There is a clear mechanism by which two colliding holes will collide. Let us be clear, by collision we mean collide in "phase space"; collision merely in real space (i.e. longitudinal coordinate or RF-phase) is not sufficient condition for coalescence. Holes will collide if their central momenta differ by less than the their momentum width; if their relative momentum separation exceeds the momentum width, the holes will pass one another like "ships in the night" (albeit with a small filamentation due to the space-charge shock wave).

1.2 Coherent modulation

As a test of these ideas, one can hope to impress a long wavelength coherent energy modulation on the hole centres so that the collision process is speeded up and not governed by randomness. Hence one is led to making holes with a substantial high harmonic voltage, and then modulating them at the fundamental with a small voltage of short duration.

For demonstration, we take ring 2 with an MEFLAT cycle with C04 turned off, and the C02 and C16 cavities turned on. In every case, the C16 cavity followed the same frequency and voltage laws: a cleaning sweep followed by a deposition. The C16 GFA = 3V giving a gap voltage of 3.2 kV during 4 ms after injection; after which time the cavity voltage is held at zero. The radial position GFA GSRPOS starts –4 volt at injection, sweeps up to +4 volt in 2 ms, and then sweeps down to zero in 1 ms; and is zero thereafter. Thus the frequency law is completed 1 ms before the C16 voltage turns off.
At h=1, there is a frequency offset of 0.6933 kHz/volt of the GFA, and so 4 volts at h=13 gives a sweep of 36 kHz. In the absence of space-charge, the high harmonic bucket parameters are as follows: synchronous phase 17°, bucket height 198 keV and bucket length

[1] But one must not forget the possibility of negative mass instability: the hole shrinks in phase, the space-charge forces get stronger, which in turn further shrinks the hole, and so on.

238°. For comparison, the beam energy full width (not measured) is approximately 550 keV before the empty bucket deposition; this spread corresponds to a frequency width of 2.86 kHz at h=1, or 37 kHz at h=13. At the extrema of the C16 frequency program, the frequency separation of the beam from the empty buckets is 10.8 kHz at h=13; and so the buckets start outside of the beam.

Injection occurs at 275 ms of the C-train. 3.5 turns were injected giving 2.4E12 ppp. With hindsight one understands that it would have been much better to inject an exact integer number of turns. An integer number of turns would have avoided the large h=1 density modulation that slightly confuses the waterfall plots.

1.3 "Tomoscope"

The waterfall display of beam current within a single turn versus turn number, known at CERN PS as the "tomoscope", displays "phase at arrival" (at the pick-up) information about local beam intensity features. The relative intensity is represented by a grey scale in which black and white correspond to the highest and lowest values, respectively.

2 C02 cavity off

First let us show how the deposited holes behave when no modulation is applied. In the first two figures (1 & 2), we show behaviour (for a single intensity) over two time spans; while in the following figures (3 through 5) a single time span is studied for various injected intensity. The holes appear as the lighter traces. The C16 cavity harmonic number is 13, but because the horizontal span is 1.2 turns there appear to be 16 holes. The most significant feature of all these figures is that the holes survive at all; based upon their momentum spread, they should all debunch and disappear in 1-to-2 ms. The strong initial h=13 modulation in the plots arises from the fact that the plots start as the C16 voltage is turning off.

The rate of phase advance (radian/sec) is given by $d\varphi/dt=2\pi\times\nu_{rf}\times\eta\times\gamma/(\gamma+1)\Delta T/T$ where the symbols are defined below. In the absence of focusing forces, the holes should shear in longitudinal phase space at a rate given by this formula. ΔT, the kinetic energy difference, is equal to 198 keV and T = 50MeV. During an interval of 8.3 ms, the top and bottom of the empty RF buckets should shear by $\approx$3000 degrees (or 8.3 periods) of RF phase at h=1 (ν_{rf}=0.6MHz).

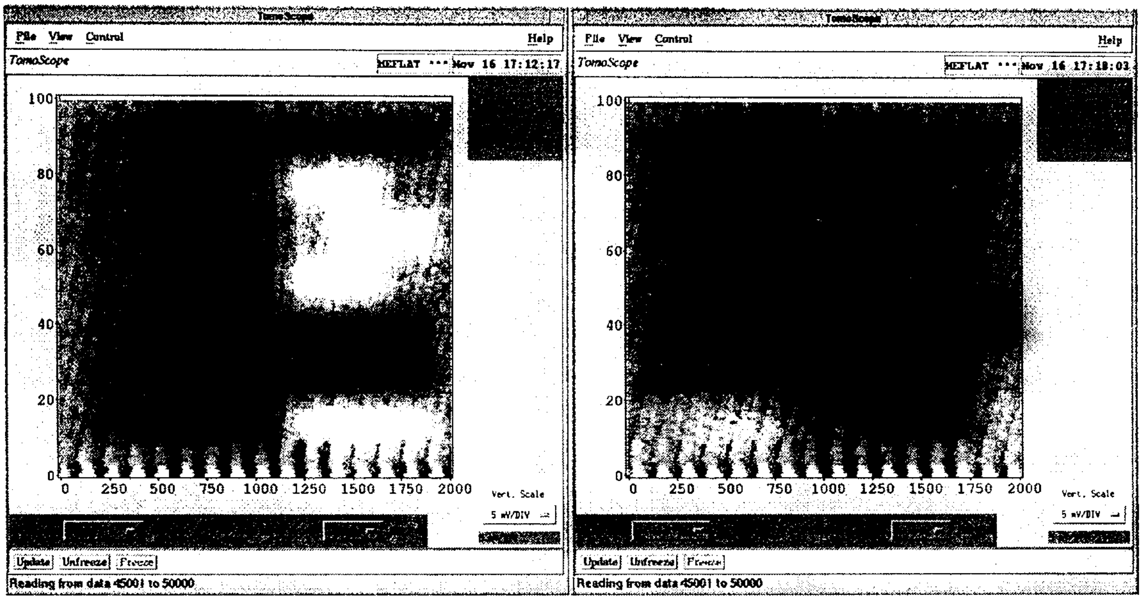

Figure 1: C02 off, 3.5 turns injected, 20 turns/trace, 100 profiles, span 3.3 ms, start @ injection +4ms .

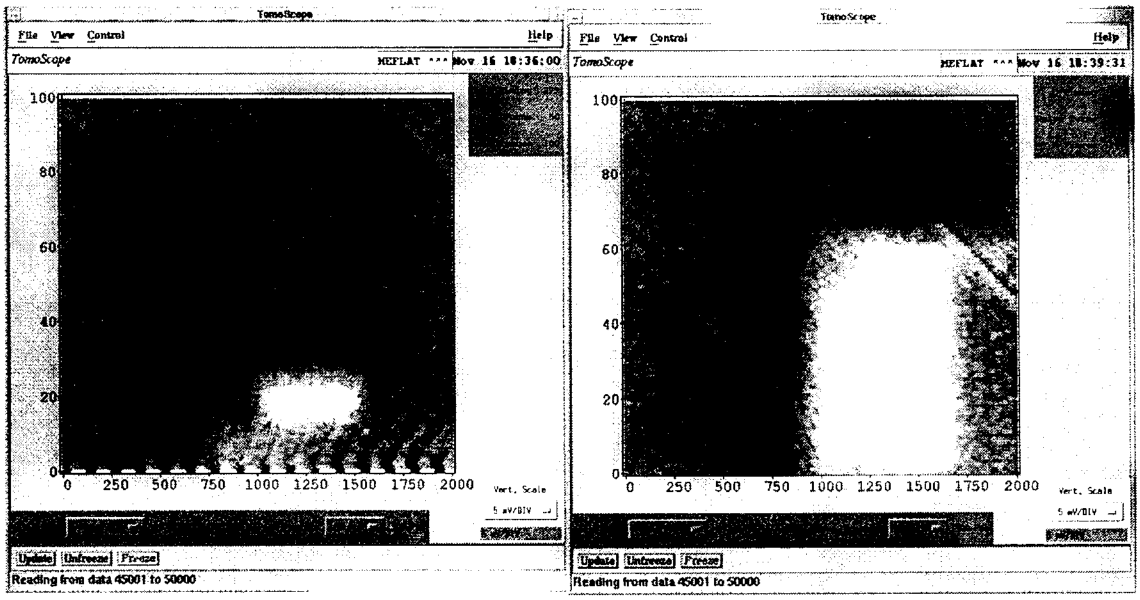

Figure 2: C02 off, 3.5 turns injected, 50 turns/trace, 100 profiles, span 8.3 ms; left => start @ inject+4ms, right => start @ inject+7ms .

The turn number and acquisition triggering of the tomoscope is taken from the fundamental frequency which may well not be the beam revolution frequency. A constant offset between these two frequencies produces a linear movement versus turns of any "phase-feature" in the beam. A ramped offset between beam and reference frequencies implies that phase accumulates quadratically with time. This is probably the explanation for the parabolic movement versus time of any beam intensity features over the longer time span (8.3ms) as we inherited an RF program (from a beam cycle intended for another purpose) that is a sequence of ramps. Other possibilities are beam energy loss or coherent instability.

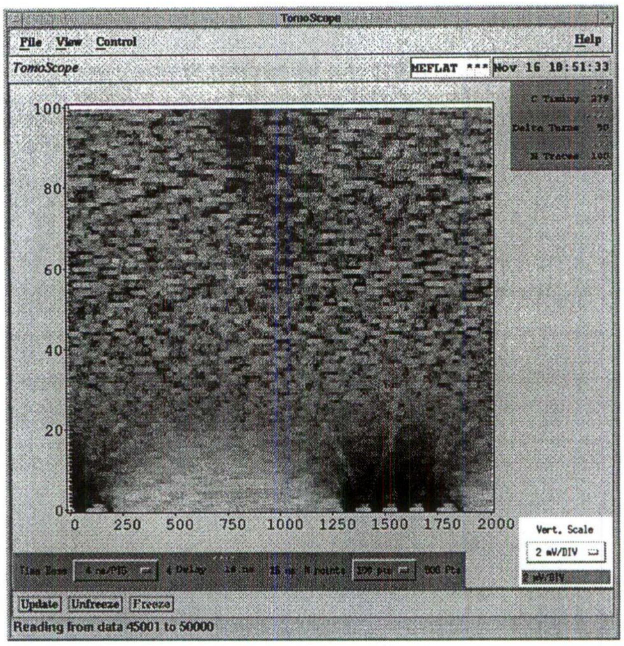

Figure 3:C02 off, 0.4 injected turns, 50 turns/trace, 100 profiles, span 8.3 ms, start @ inject+4ms .

In Figure 3, the four holes appear not survive the debunching of the initial partial turn of injected beam. However, this might be due to lack of resolution in the grey scale; ideally the first few profiles should be omitted to increase the dynamic range in the remainder of the waterfall plot.

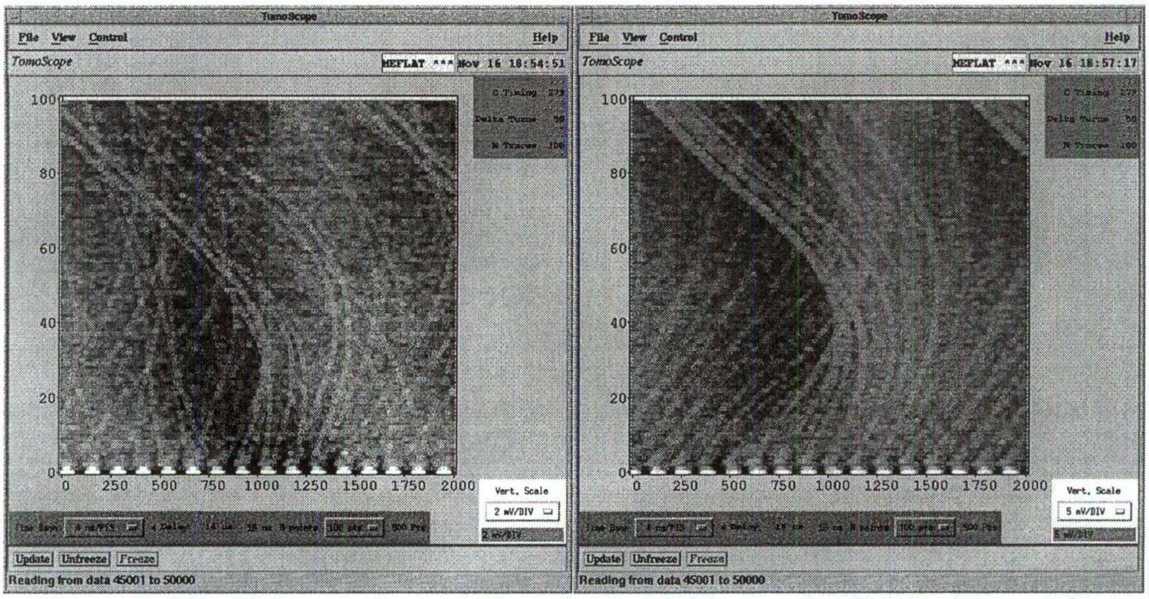

Figure 4: C02 off, 50 turns/trace, 100 profiles, span 8.3 ms, start @ inject+4ms; left => 1.4 turns & right => 3.1 turns injected .

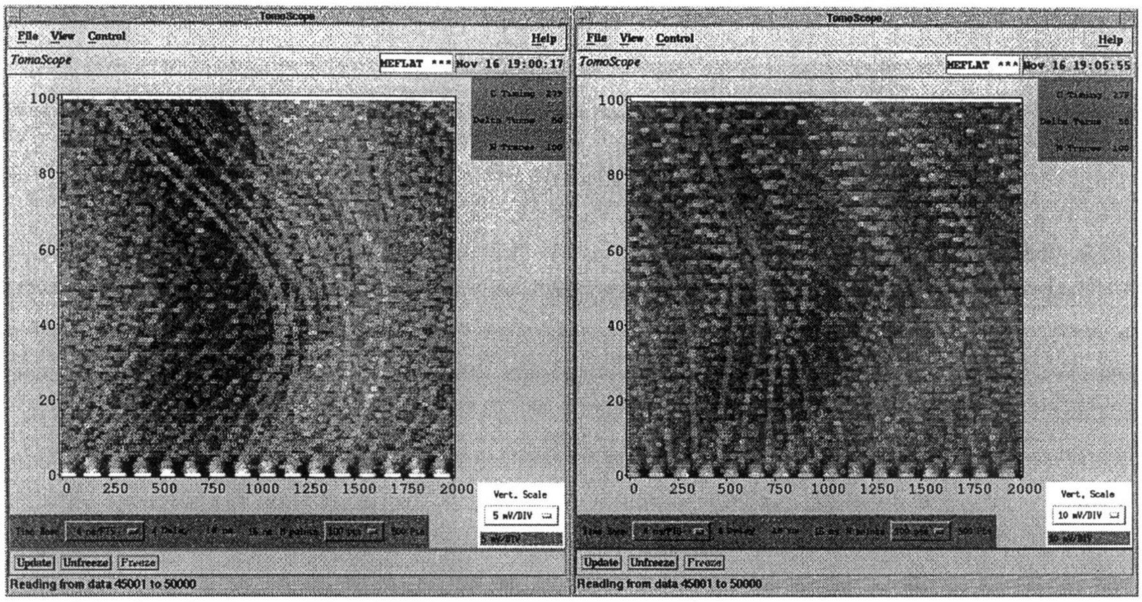

Figure 5: C02 off, 50 turns/trace, 100 profiles, span 8.3 ms, start @ inject + 4 ms; left => 4.4 turns & right => 7.1 turns injected .

The intriguing 4th harmonic periodic structure in right hand of Figure 4 for 7.1 turns, is probably the result of a coasting beam instability that displays current thresholding due to Landau damping. In another MD (22 Nov. 99) the Schottky spectra below and above 5 injected turns were found to differ markedly.

3 C02 cavity on

C02 starts to generate voltage 3 ms after injection or at 278 ms of the C-train.. The rate of phase advance is proportional to momentum deviation, and the change in the momentum is proportional to the square root of the voltage. Consequently, one may expect the effect of the C02 modulation (applied to the beam) to scale linearly with duration ΔT of the modulation and as $\sqrt{V}$. The following cases (Figure 6 through Figure 12) are ordered in increasing $\Delta T \times \sqrt{V}$. For the most part, this scaling seems to fit; but it starts to break down for large voltages because the momentum changes are so large that the distribution starts to change while the modulation is still being applied. This is why the cases C02=0.5 kV and C02=1.0 kV do not appear to fit within the continuum of the other examples. The C16 program was unchanged, so a C16 gap voltage of 3 kV was used.

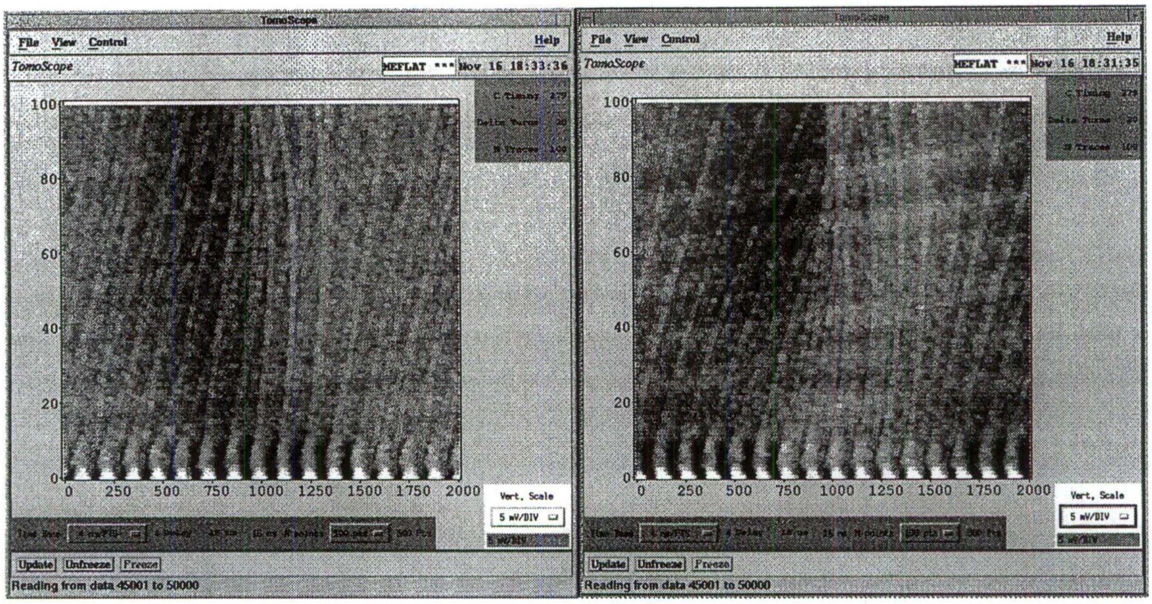

Figure 6: 20 turns/trace, 100 profiles, span 3.3 ms, start @ inject + 4 ms; left => C02 off; right => C02 = 0.1 kV for 0.5 ms, $\Delta T\sqrt{V}$ = 0.50 .

A crude estimate of the energy modulation imposed at h=1 is as follows. Particles receive energy increments between –100eV and +100eV (figure 6-right) each turn of the ring, and they complete (500/1.6)=300 turns, yielding a total modulation peak to peak of approximately ±30keV. Certainly the traces left by the holes tend to converge upon one another (in Figure 6), but the time span is too short to see if they collide and coalesce. However, increasing the modulation will hasten the collisions in real space.

Collision of nearest neighbour holes depends not on the peak to peak modulation, but rather upon the relative energy differences which are much smaller. However, one can estimate how long for collision between two holes separated by ½ RF period of the fundamental.

Now, $d\varphi/dt=2\pi\times\nu_{rf}\times\eta\times(\Delta p/P)$ where φ is the phase in radians, ν_{rf} is the RF in Hz, η=0.8433 is the slip factor, and $\Delta p/P$ is the relative momentum spread. Now, to first order, $\Delta p/P=\gamma/(\gamma+1)\Delta T/T$ where $\gamma=E/(m_0c^2)$, ΔT is the kinetic energy difference and T = 50MeV. Hence, the time taken to slip by π radian is $1/(\nu\eta\Delta T/T)$.

For Figure 7, a crude estimate of the C02 energy modulation is ±60 keV and for this case, with ΔT=120 keV, the time taken to slip by π radian is ≈ 0.82 ms. Bunches which are initially closer together will collide after a smaller phase slip, but because their relative energy differences are also smaller, they take the same time (0.82 ms) to collide (in the approximation of a linearized RF waveform). The situation in figure 7 is as follows. The horizontal span is 1.2 RF periods; the leftmost 13 bunches are focused toward a common centre, and the rightmost 4 bunches are virtually a replication of the first 4 drawn from the leftmost 13. If one recalls that 1 ms is spent building up the energy modulation, and the next 0.82 ms converging, then bunches should collide after about 2 ms, or • the way up the waterfall plot – and this is just what is seen in Figure 7 left. In Figure 7 right, there are collisions of two groups each of two holes in the upper central portion of the waterfall plot.

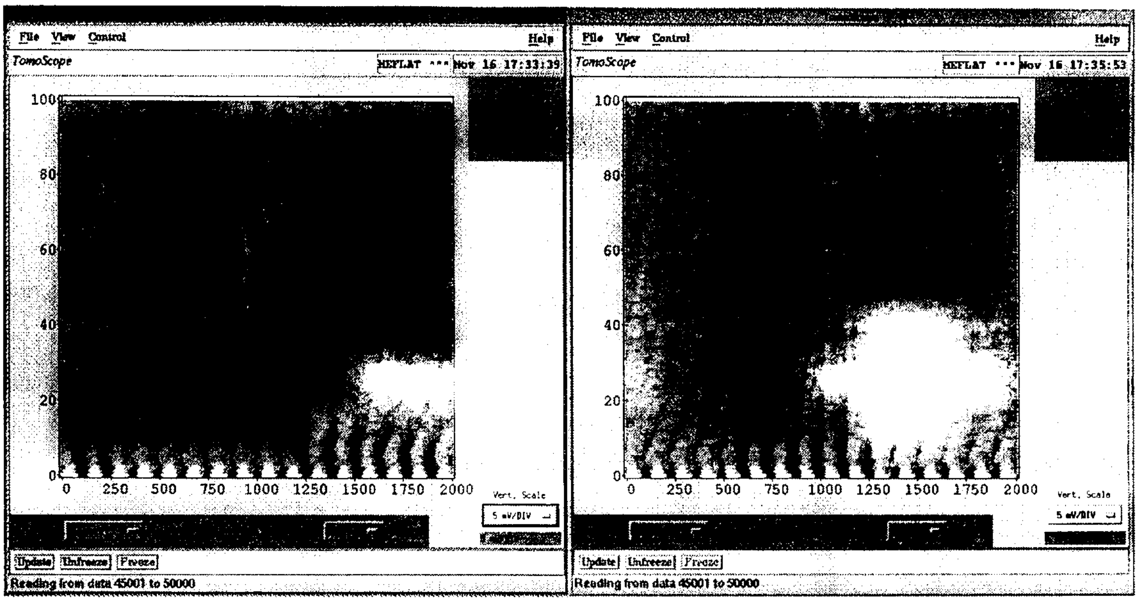

Figure 7: 20 turns/trace, 100 profiles, span 3.3 ms, C02 = 0.1 kV for 1 ms, $\Delta T\sqrt{V}$ =1.00 .

For the case in Figure 8, with ΔT=240 keV, the time taken (after the modulation is completed) to slip by π radian is $\approx$ 0.41 ms. Because the energy modulation is greater than the height of the h=13 empty buckets (198 keV), one would expect holes initially separated by ½ wavelength of h=1 not to collide in phase space. Bunches, initially closer, however are anticipated to make collisions. In figure 8, below, the h=13 holes do converge more quickly. In figure 8-right, there is evidence of coalescing between holes initially separated by up to 3/13 of the fundamental wavelength, but not for holes with greater separations.

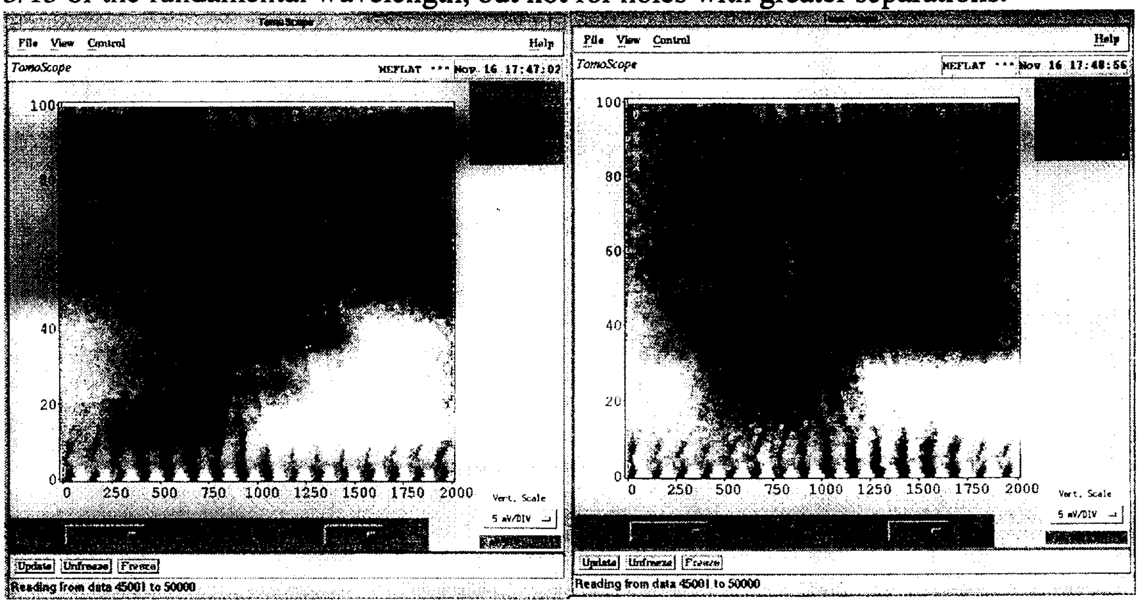

Figure 8: 20 turns/trace, 100 profiles, span 3.3 ms, C02 = 0.2 kV for 1 ms, $\Delta T\sqrt{V}$=1.41 .

For figure 8, a crude estimate of the C02 energy modulation is ±120 keV.

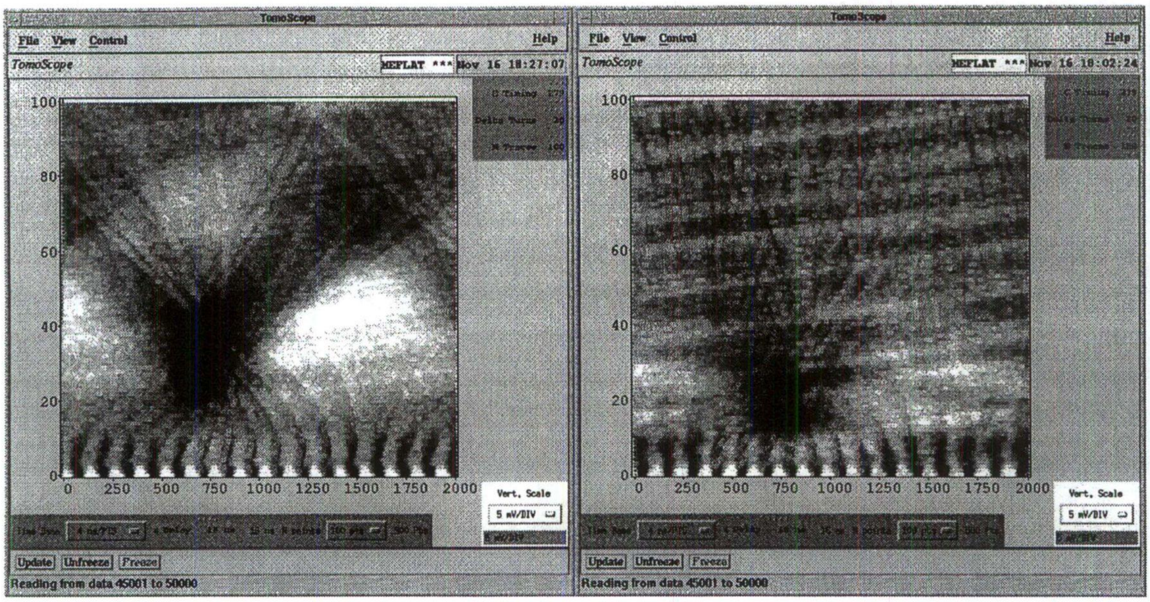

Figure 9: 20 turns/trace, 100 profiles, span 3.3 ms; left => C02=0.1 kV for 2 ms, $\Delta T\sqrt{V} = 2.00$ & right =>C02 = 0.5 kV for 1ms, $\Delta T\sqrt{V} = 2.23$.

For Figure 9 (left/right) a crude estimate of the C02 energy modulation is ±120 keV left and ±300 keV right. This latter modulation is large enough that not all holes can collide.

The energy modulation which causes the holes to converge also focuses the occupied phase space toward a common centre. This has the unfortunate consequence that the waterfall display becomes dominated by a patch of high density (black in the plot) that "eats into" the dynamic range of the greyscale leaving less room to resolve the evolution of the traces left by the phase space holes. Ideally those profiles with clear h=1 bunching should be omitted from the plot.

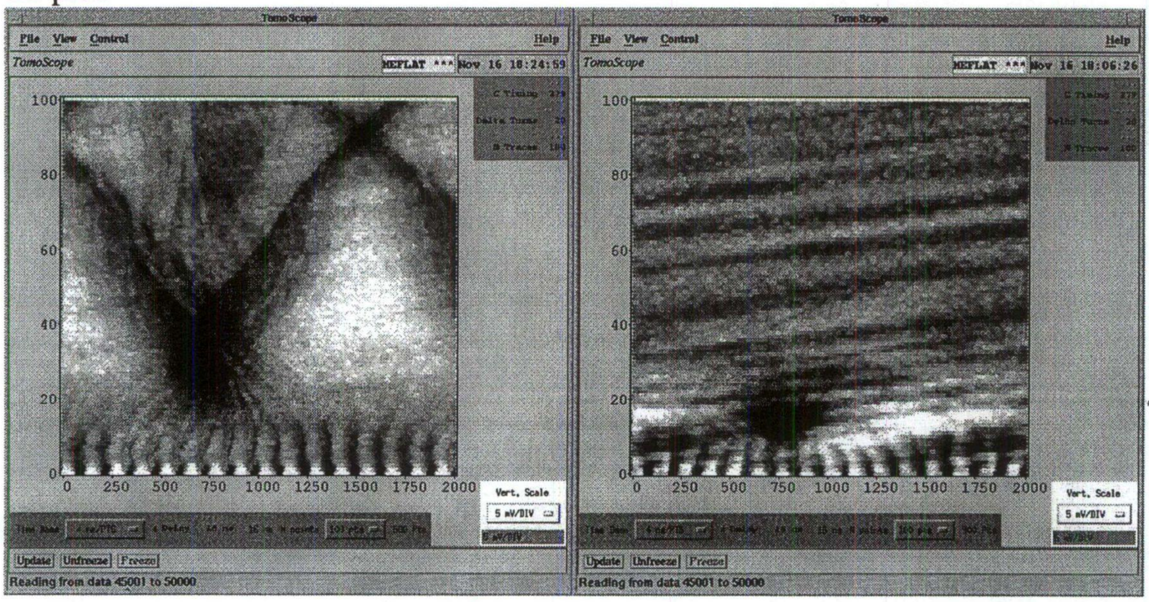

Figure 10: 20 turns/trace, 100 profiles, span 3.3 ms; left => C02 = 0.1 kV for 3 ms, $\Delta T\sqrt{V} = 3$; right => C02=1.0 kV for 1 ms, $\Delta T\sqrt{V} = 3.16$.

For Figure 10 (left/right) a crude estimate of the C02 energy modulation is ±180 keV left and ±600 keV right. This latter modulation is large enough that not all holes can collide. There is some evidence of phase space collisions in the upper left portion of figure 10-left, but little evidence in 10-right.

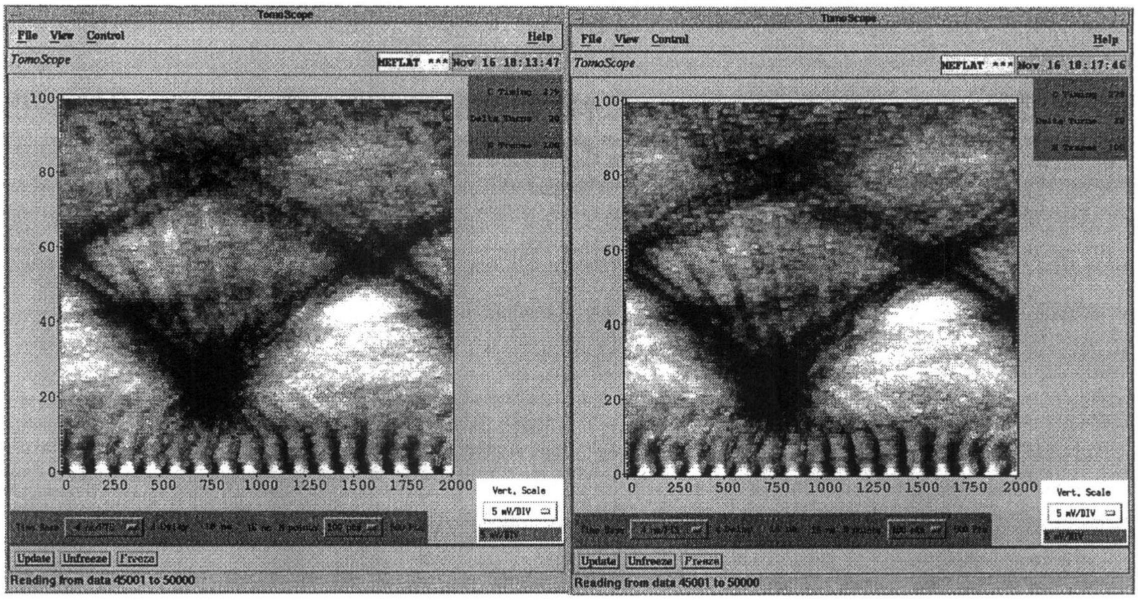

Figure 11: 20 turns/trace, 100 profiles, span 3.3 ms, C02 = 0.3 kV for 2 ms, $\Delta T\sqrt{V}$ = 3.46 .

For figures 11 and 12, a crude estimate of the C02 energy modulation is ±360 keV. This modulation is so large that the beam redistributes while the h=1 RF is applied, and a better estimate of the convergence time comes from the synchrotron oscillation period of the temporary h=1 RF bucket. For 3 kV applied at h=1, the synchrotron period is 1.2 ms and a 90° rotation in phase space takes 0.3 ms for small amplitude oscillations, but rather longer for large amplitude motions. Thus in Figure 11 we see a kind of aberration: the h=13 holes all converge upon a common centre-line but their arrival time on that line is progressively longer the farther their starting phase is from that line. There is some evidence of coalescing in as much as there are fewer hole traces emerging from the convergence area (dark patch) than enter, but it is hard to make a strong case for this interpretation of the waterfall plot.

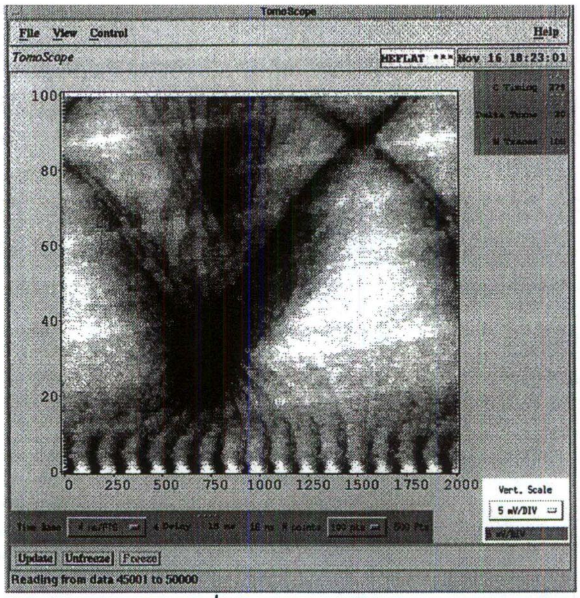

Figure 12: C02 = 0.2 kV for 3 ms, $\Delta T\sqrt{V}$ =4.24; 20 turns/trace, 100 profiles, span 3.3 ms .

4 Conclusion

The longevity of the h=13 holes, far in excess of the debunching time in a naïve calculation, indicates internal focusing forces must be at work. We have clearly demonstrated that the relative motion between high harmonic holes is governed by the momentum differences between them, and as is consistent with the "derivative of line density" model for longitudinal space-charge there are no forces of attraction or repulsion between the holes until the time they collide. If one compares figures 2, 4 and 5 (with 8.3 ms span) of section 2 with figures 6, 7 and 8 (with 3.3 ms span) of section 3 it is clear that collision of holes can be hastened by an applied h=1 energy modulation of the coasting beam.

With hindsight, we should have studied the small modulation parameter range (e.g. 0.1 kV for 1 ms) more carefully; as the large modulations (e.g. 0.3 kV for 2 ms) lead to cases where holes fail to collide in phase space but rather pass over one another. To study hole collisions resulting from the small modulations, we need to observe for time spans greater than 8.3 ms. Unfortunately, during the MD it was not realized that the acquisition frequency of the "tomoscope" has to be adjusted to follow the revolution frequency of the holes and so we were prevented from acquiring long spans by the fact that the wrapped-around motion presented in the plots quickly becomes confusing and less meaningful.