PS – LP, 5.06.88

THE TRANSFORMER – COUPLED RESONATOR MODEL OF THE EPA RF CAVITY.

S.Bartalucci,A.Riche.

Cern, PS

PS,LP Note 88 – 47

Introduction.

In a previous note,[1],a rough estimate of the cavity shunt impedance was obtained by comparing the simple one – resonator model with some measurements of the loading angle Φ_L in the presence of the beam. In this note a two – resonator model is presented which is more suitable to describe the EPA RF system than the RLC lumped circuit. The reason for introducing this model here is basically that the rather unconventional design of the system may turn out to be the source of deviations with respect to the standard theory,as far as the beam – cavity interaction is concerned. In particular it has been suggested [2] that the beam loading instability limits,as predicted by Robinson's criterion,would change considerably if a more complicate model of the RF cavity were assumed. In fact some measurement of the instability threshold seems not to agree with Robinson's criterion. To verify this hypothesis,a detailed study of the model is needed,in order to get an accurate description of the RF system,what is essential for the subsequent analysis.

1. The cavity model.

The EPA RF system [3], consists of an accelerating cavity coupling through a magnetic loop to an amplifier cavity ,where the power tetrode is located. the equivalent lumped circuit of the system without beam is shown on Fig. 1. The power tube (tetrode) is represented by a current generator with its plate resistance Rp added in parallel. Using Kirchhoff's laws the following complex quantities can be calculated:

$\tilde{Z}_1 = \frac{\tilde{V}_1}{\tilde{I}_1}$ is the anode impedance, i.e. the impedance seen by the power generator. $\tilde{Z}_{21} = \frac{\tilde{V}_2}{\tilde{I}_1}$ is the anode impedance transformed to the accelerating gap. $\tilde{\tau} = \frac{\tilde{V}_2}{\tilde{V}_1}$ is the voltage step−up (transformation ratio) between the anode and the gap. $\tilde{Z}_2 = \frac{\tilde{V}_2}{\tilde{I}_2}$ is the gap impedance (as seen by the beam).

The analytical expressions are given in Appendix I. With the beam, the equivalent lumped circuit is outlined in Fig.2. The following equations apply:

$$\tilde{V}_1 = \tilde{Z}_{11} * \tilde{I}_1 + \tilde{Z}_{12} * \tilde{I}_2 , \quad \tilde{V}_2 = \tilde{Z}_{21} * \tilde{I}_1 + \tilde{Z}_{22} * \tilde{I}_2 \quad ,with \quad ,$$

$$\tilde{Z}_{11} = \left(\frac{\tilde{V}_1}{\tilde{I}_1}\right)_{\tilde{I}_2=0} \equiv \tilde{Z}_1 , \quad \tilde{Z}_{12} = \left(\frac{\tilde{V}_1}{\tilde{I}_2}\right)_{\tilde{I}_1=0} , \quad \tilde{Z}_{21} = \left(\frac{\tilde{V}_2}{\tilde{I}_1}\right)_{\tilde{I}_B=0} , \quad \tilde{Z}_{22} = \left(\frac{\tilde{V}_2}{\tilde{I}_2}\right)_{\tilde{I}_1=0} \equiv \tilde{Z}_2$$

and $\tilde{I}_2 \equiv \tilde{I}_B$, the fundamental component of the beam current (for short bunches).

Owing to the symmetry of the circuit, the impedance seen by the beam can be calculated by interchanging the indexes 1 and 2 in the expression of $\tilde{Z}_1$. The parameters involved in the calculations of Appendix I are evaluated as follows: $R_{1,2} = Zn_{1,2} * Q_{1,2}$ are the shunt impedances of the 2 isolated cavities; the quality factor $Q_{1,2}$ have been measured several times with the high resolution Network Analyzer HP 3577A, while the characteristic impedances (on the axis) are computed by SUPERFISH for the main cavity (Zn_2), and known since a long time for the amplifier cavity,[4].

Using the following equations (and the measured values of the two resonant frequencies $\omega_{1,2}$, the other parameters are determined : $\omega_{1,2} = \frac{1}{\sqrt{L_{1,2} * C_{1,2}}}$, $Q_{1,2} = \omega_{1,2} * C_{1,2} * R_{1,2}$

$M \approx \left(2 * \frac{\Omega}{\omega_0}\right)\sqrt{L_1 * L_2}$, is the mutual inductance coefficient. The ' big Omega ' Ω is half the distance between the two resonant peaks when both circuits are tuned to the same resonant frequency (19 MHz in our case) and its relationship with the mutual inductance coefficient is derived in the theory of transformer−coupled amplifiers,[5].

The numerical values of the relevant parameters have been used in a FORTRAN program (BMLD) which computes the complex polynomials of Appendix I. The results are compared again with some measurements, and the best setting of input parameters is chosen. An example is shown if Fig.3,where a typical resonance curve of the cavity is displayed,as measured by the Network Analyser. This illustrates the cavity response (as measured by a magnetic pickup in the cavity) when a frequency span of 10 KHz around the central frequency (here ≈ 19080 kHz) was executed at constant excitation voltage on the control grid of the tube. This plot has been taken with about 25 kV on the gap. The loaded Q is 3446 as measured with the 3 dB method. Some points calculated by the program are also shown, for 3 different values of Q_2 (i.e. the Q of isolated cavity when powered). Obviously,this last parameter is not directly measurable. The input and output parameters,as used in the program,are shown in Table I for the case of Fig.3.

The plate resistance Rp can only be estimated from the tetrode characteristics. A program has been written for a HP Desktop computer by A. Susini and R.Giannini to calculate the first three Fourier components of the plate current for a given grid voltage,as a function of the plate voltage, This is just the definition of the plate resistance,as $\left(\frac{\partial I_p}{\partial V_p}\right)^{-1}_{V_G = const.} = R_p$, whose value depends on the plate voltage and on the phase angle of the load. Typical values are between 4.7 kΩ, and 11 kΩ,[4].

In Fig.3, R_p was chosen to be 7.2 $k\Omega$.

The results of BMLD can be compared with some 'cold' measurements taken in the laboratory. The anode impedance(i.e.,the load impedance) was measured with the HP Vector Impedance Meter at 19.1 MHz and found to be $\approx 5\,k\Omega$ at the maximum (i.e. phase $= 0^0$,in agreement with the model (see Table I), also the transformation ratio $\tilde{\tau}$ has been measured by exciting the cavity at the gap with a matched loop and looking at the response of two pick-ups,one located in the main cavity,the other located near the anode. By measuring the attenuation between the two with a Network Analyser, the transformation ratio Vgap / V anode is easily determined. We have found $\left|\frac{V_{gap}}{V_{anode}}\right| = 6.5\ at\ 19.1\ MHz.$

2. Measurements with the beam.

Some measurements of the cavity voltage V_C and the loading phase angle Φ_L as functions of the average beam current I_B^{dc} have been presented in a previous note,[1]. In the case of no control loops they can be used to get an estimate of the parameters involved in our model.Two examples ('a' and 'b') of such measurements are shown in Fig.4 to 7,were the measured $\Phi_L = f(I_B^{dc})$ and $V_C = f(I_B^{dc})$ are represented by solid lines and the theoretical predictions are represented by stars. From the usual phasor diagram (Fig.8),the following equations are derived (with the usual meaning of the symbols):

$$I_G\cos\Phi_L = \frac{V_C}{R_S} + I_B\sin\Phi_S\ , \qquad \tan\Phi_L = \frac{\tan\Phi_Z - \frac{I_B * R_S}{V_C}\cos\Phi_S}{1 + \frac{I_B * R_S}{V_C}\sin\Phi_S}.$$

with $I_0 = \frac{V_C}{R_S}$, $\Phi_S = \arccos\left(-\sqrt{1-\left(\frac{U_0}{eV_C}\right)^2}\right)$, where U_0 is the energy loss per turn per particle in EPA. Since I_G,Φ_Z and R_S are constant, the loading angle Φ_L can be eliminated from the above equations to get a polynomial of the 4th order in V_C, which contains only I_B as a variable. The resolution of this system has been included in a FORTRAN program which computes the shunt impedance R_S as a function of the parameters of the transformer – coupled system and finally performs a stability test of the system applying the Routh – Hurwitz criterion.

The set of parameters used to produce the curves in Fig.4 and 5 is shown in Table II and the one used in Fig. 6 – 7 is shown in Table III. It is clear from Fig. 4 – 5 that the agreement with the model is rather good for the $V_C = f(I_B^{dc})$ curve, but it is very bad for the $\Phi_L = f(I_B^{dc})$ curve. The opposite is true for the data in Fig. 6 – 7. In particular, the values of R_p and Zn_2,as given in tables II and III are quite different between the two cases. The resulting shunt impedances R_S are about 140 $k\Omega$ (from Table II) and 77$k\Omega$ (from table III). The value of Zn_2 in Table II is 41Ω,which is exactly the value predicted by computation, while a value of a few $k\Omega$ for R_p seems reasonable in both cases.

It should be mentioned that the measurement of V_C seems more reliable than that of Φ_L, since the cavity voltage has been checked by measuring both the top energy of the spectrum of the X – rays leaving the accelerating gap [3] and the synchrotron frequency at various voltages. The measurements of Φ_L instead might be affected by non – linearities of the power tetrode, since the phase measurent is made between the cavity voltage V_C and the grid voltage V_G, taken as reference.

Anyhow it is impossible to come to a definitive conclusion at this level also because all the measurements taken in these conditions confirm this discrepancy. A more accurate measurement of Φ_L is certainly needed.

3. Results of the fit.

To further investigate the relative consistency of the V_C and Φ_L measurements we tried a lest square fit of these data to the theoretical model, described in Chapter 2.

The MINUIT package was used. We tried first a fit of $\Phi_L = f(I_B^{dc})$ and *of* $V_C = f(I_B^{dc})$ one at a time. The fitted parameters were the frequency of the isolated cavity f_2 ,its quality factor Q_2 ,the plate resistance R_p and the cavity characteristic impedance Zn_2.

The results are shown in Fig.9 – 10 for the fit on V_C only and in Fig. 11 – 12 for the fit on Φ_L, for the curve 'a'. They produce two sets of best fit parameters, shown in the figures,which are definitely incongruous. Again, the best fit of Φ_L gives numbers which are distant from the expectations. The same holds true for the curve 'b', although the corresponding plots are not included in the text. A fit of both Φ_l and V_C together was also tried, giving unsatisfactory results.

4. Conclusions.

The transformer – coupled resonator model of the EPA RF cavity was presented. The validity of this model is confirmed by ' cold '(i.e. without beam) measurements. Furthermore the measurements of the cavity voltages V_C and of the loading angle Φ_L as functions of the beam current I_B^{dc} , which have been taken during the running – in of EPA, were used to asses the main parameters of the cavity. Since no clear and definite indication comes out from this analysis,a least – square fit of these data to the theoretical model was tried. The results show the need of further measurements and understanding.

5. Acknowledgements.

We thank A. Susini for many discussions and suggestions of invaluable help. Most of the work was possible because R. Garoby took a lot of his time to discuss,participate and share his experience. The constant encouragement and support of K. Huebner is gratefully acknoledged.

References

1. S.Bartalucci, *Beam Loading Measurements in EPA*, PS/LPI Note 87 – 36

2. F.Pedersen, *Private Communication,*

3. S.Bartalucci et al, *A 'monochromatic' RF cavity*. Proceedings of the 1987 IEEE Particle Accelerator Conference (1987)1791.

4. A.Susini , *Private Communication.*

5. S.Seely , *Electron – tube circuits,* Mc Graw – Hill, Chapter 10.

Table 1: Input and output parameters of BLMD

$f_1 = 16857\,kHz \quad Q_1 = 1000 \quad Zn_1 = 45\Omega$

$f_2 = 19025.3\,kHz \quad Q_2 = 6000 \quad Zn_2 = 41\Omega$

$\frac{\Omega}{2\pi} = 335.5\,kHz, \quad f_0 = 19079.33\,kHz$

Results of the calculations for $R_p = 7.2k\Omega$

$|\tilde{Z}_1| = 2710\,Omega, \quad Arg(\tilde{Z}_1) = 0.5^0 \quad |\tilde{\tau}| = 7.15.$

$Arg\left(\frac{\tilde{V}_2}{\tilde{I}_1}\right) = 2.6^0$

Results of the calculations for $R_p = \infty$.

$|\tilde{Z}_1| = 4357\,\Omega, \quad Arg(\tilde{Z}_1) = 2.5^0 \quad |\tilde{\tau}| = 7.14.$

$Arg\left(\frac{\tilde{V}_2}{\tilde{I}_1}\right) = 4.6^0$

Table 2: Input data to produce Fig. 4 – 5

f_1 = 16857 kHz, Q_1 = 1000, Zn_1 = 45Ω
f_2 = 19028.30 kHz for 'a', Q_2 = 6000, Zn_2 = 41Ω
19029.34 kHz for 'b', R_p = 7.2k Ω
$\frac{\Omega}{2\pi}$ = 335.5 kHz, f_0 = 19085.24 kHz bunch frequency in EPA

Table 3: Input data to produce Fig. 6 – 7

f_1 = 16857 kHz, Q_1 = 1000, Zn_1 = 45Ω
F_2 = 19028.30 kHz for 'a', Q_2 = 6000, Zn_2 = 35Ω
19029.54 kHz for 'b', R_p = 3 kΩ
$\frac{\Omega}{2\pi}$ = 335.5 kHz, f_0 = 19085.24 kHz

6. Appendix I

Since the calculation are long and very tedious, only the final results are given : $\tilde{Z}_1 = \frac{\tilde{V}_1}{\tilde{I}_1}$

$$= \frac{s}{C_1}\,\frac{s^2+\alpha_2 s+\dfrac{\omega_2^2}{1-k^2}}{s^4+s^3(\alpha_1+\alpha_2)+s^2\left(\alpha_1\alpha_2+\dfrac{\omega_1^2+\omega_2^2}{1-k^2}\right)+s\left(\dfrac{\alpha_1\omega_2^2+\alpha_2\omega_1^2}{1-k^2}\right)+\dfrac{(\omega_1\omega_2)^2}{1-k^2}}$$

where

$$k = \frac{M}{\sqrt{L1L2}},\quad \alpha_1 = \frac{1}{R_1C_1},\quad \alpha_2 = \frac{1}{R_2C_2},\quad s = j\omega$$

$$\tilde{Z}_{21} = \frac{\tilde{V}_2}{\tilde{I}_1} = \frac{-ks}{C_1C_2\sqrt{L_1L_2}}\,\frac{1}{(s^2+\alpha_2 s+\omega_2^2)(s^2+\alpha_1 s+\omega_1^2)-k^2s^2(\alpha_1+s)(\alpha_2+s)}$$

$\tilde{\tau} = \frac{\tilde{V}_2}{\tilde{V}_1} = \frac{\tilde{V}_2}{\tilde{I}_1}\frac{\tilde{I}_1}{\tilde{V}_1} = \frac{\tilde{Z}_{21}}{\tilde{Z}_1}$, is not explicity given.

$\tilde{Z}_2 = \frac{\tilde{V}_2}{\tilde{I}_2} \equiv \tilde{Z}_1$ when the indexs 1 and 2 are interchanged.

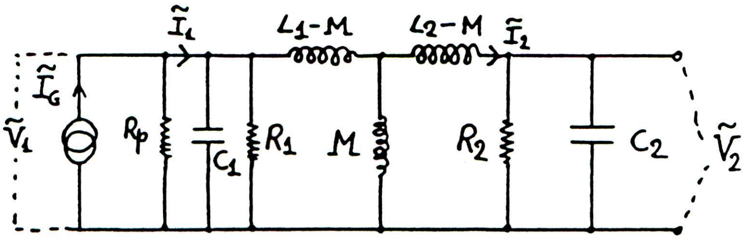

Fig. 1 : Equivalent lumped circuit without beam

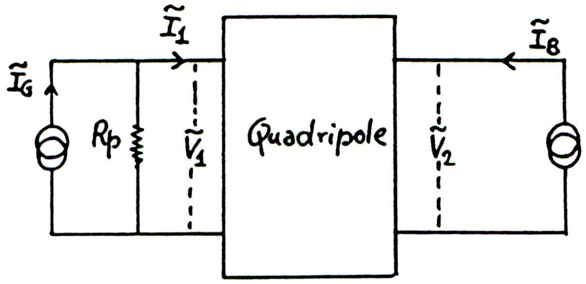

Fig. 2 : Equivalent circuit with beam

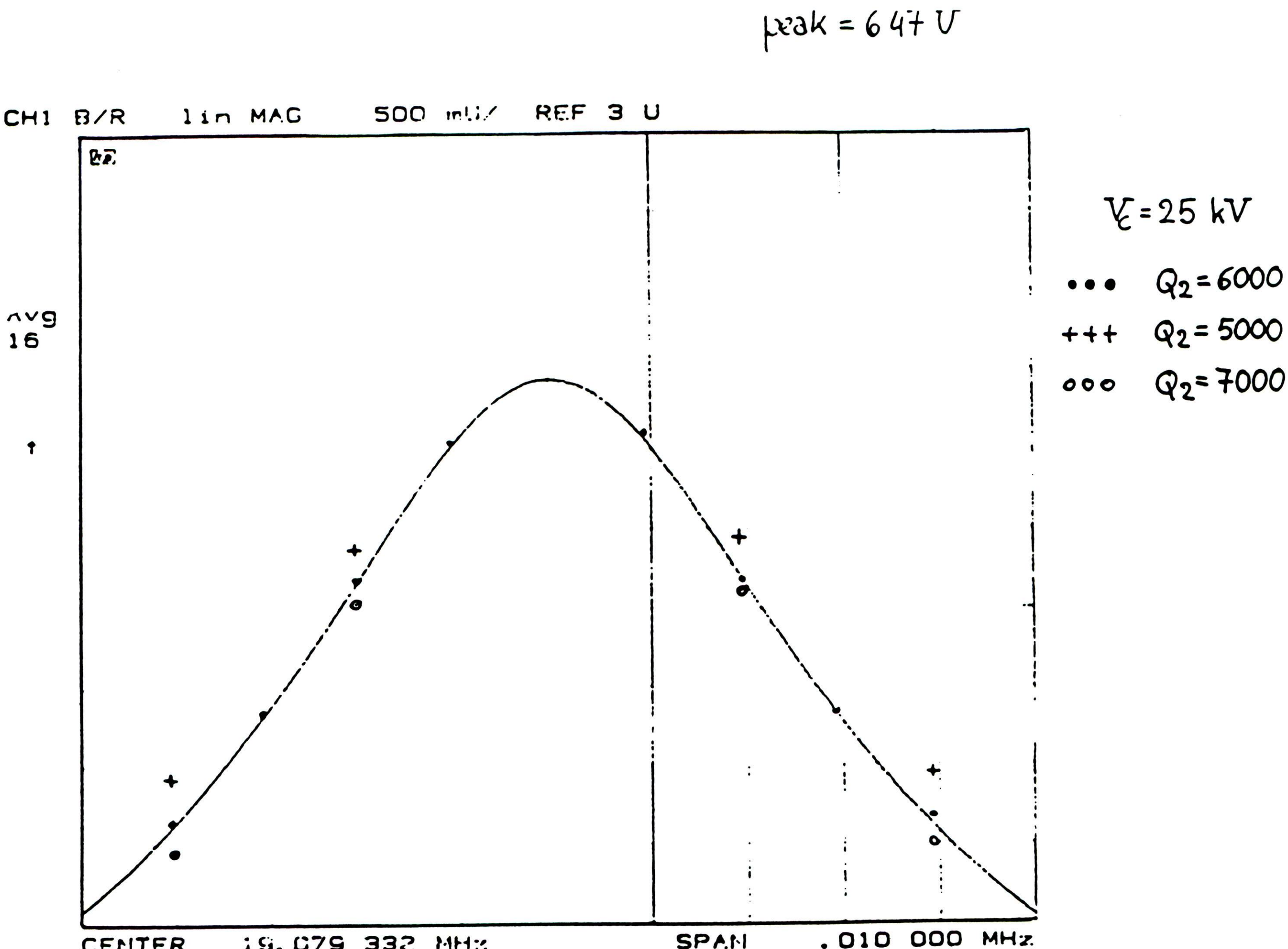

Fig. 3 : Cavity resonance curve (solid line), compared with calculations

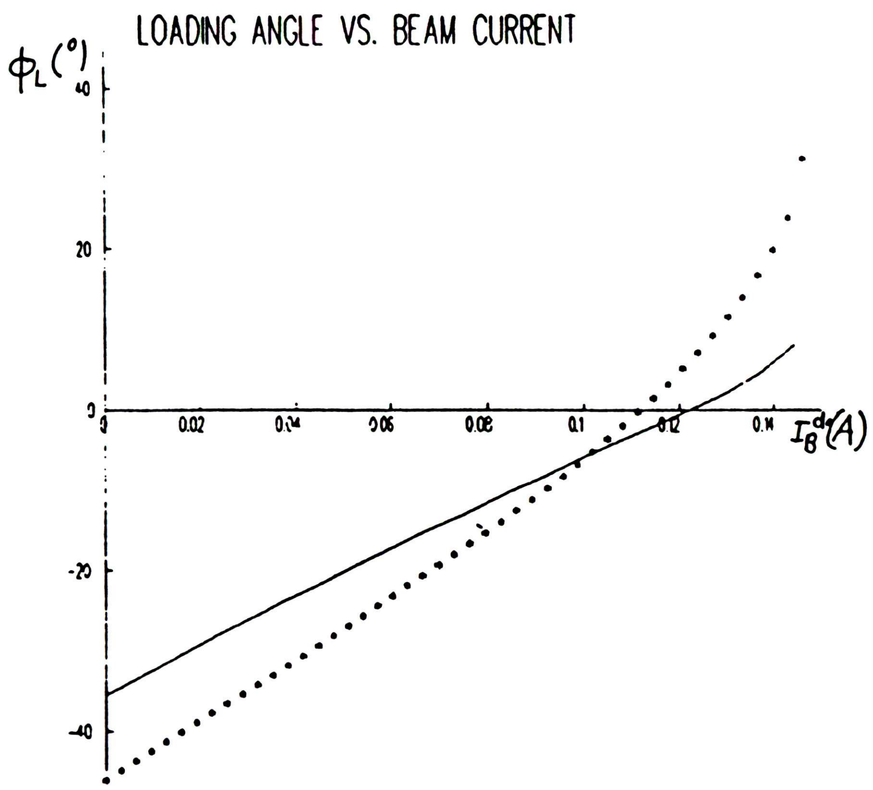
LOADING ANGLE VS. BEAM CURRENT
ϕ_L (°)
40
20
0
-20
-40
0.02
0.06
0.08
0.1
0.12
I_B^{dc} (A)

Fig. 4a

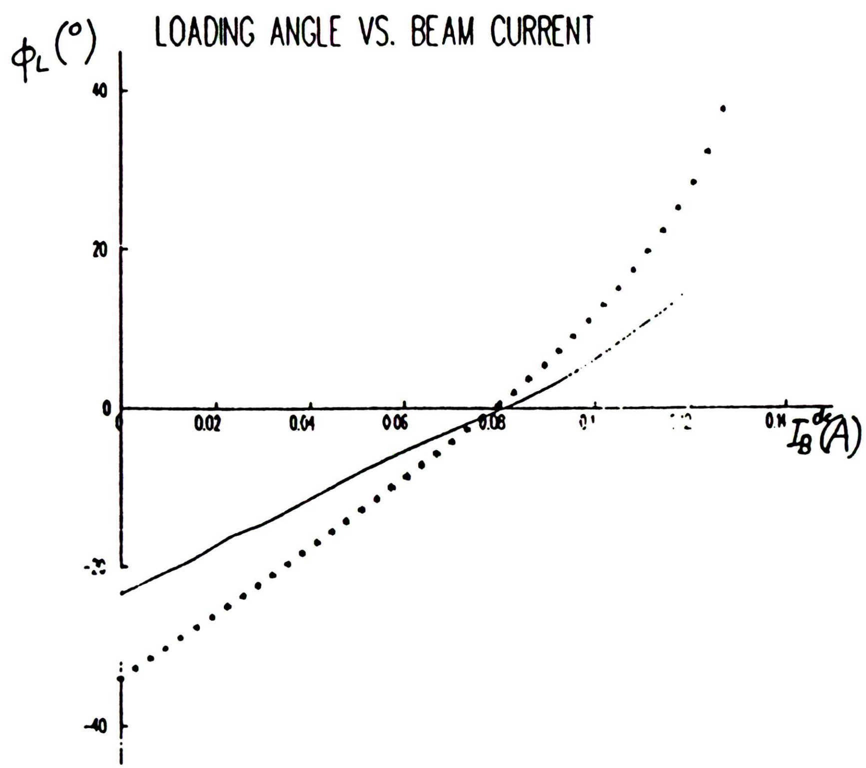
LOADING ANGLE VS. BEAM CURRENT
ϕ_L (°)
40
20
0
-40
0.02
0.04
0.06
0.08
I_B^{dc} (A)

Fig. 4b

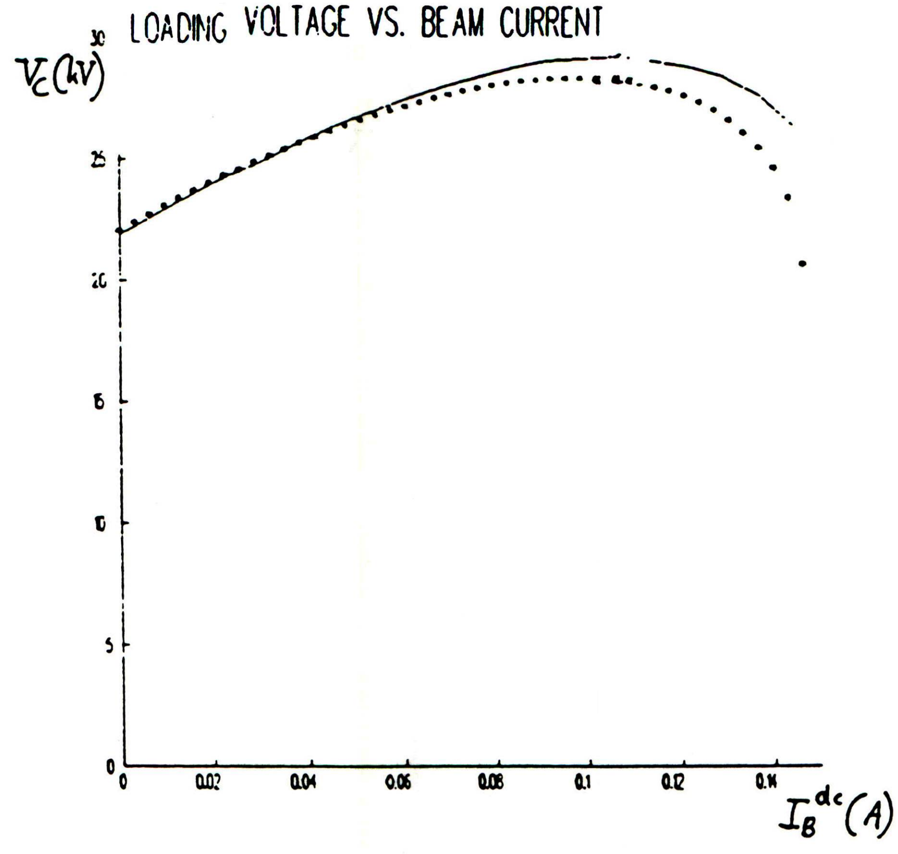

Fig. 5a

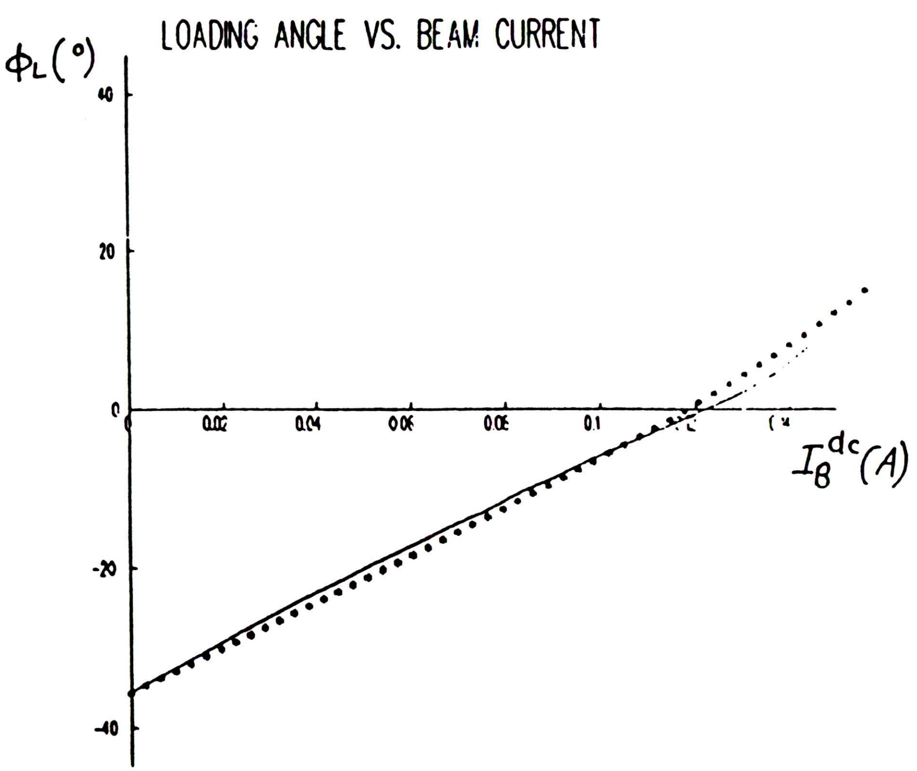
LOADING ANGLE VS. BEAM CURRENT
$\phi_L(°)$
40
20
0
-20
-40
0.02
0.04
0.06
0.1
$I_B^{dc}(A)$

Fig. 6a

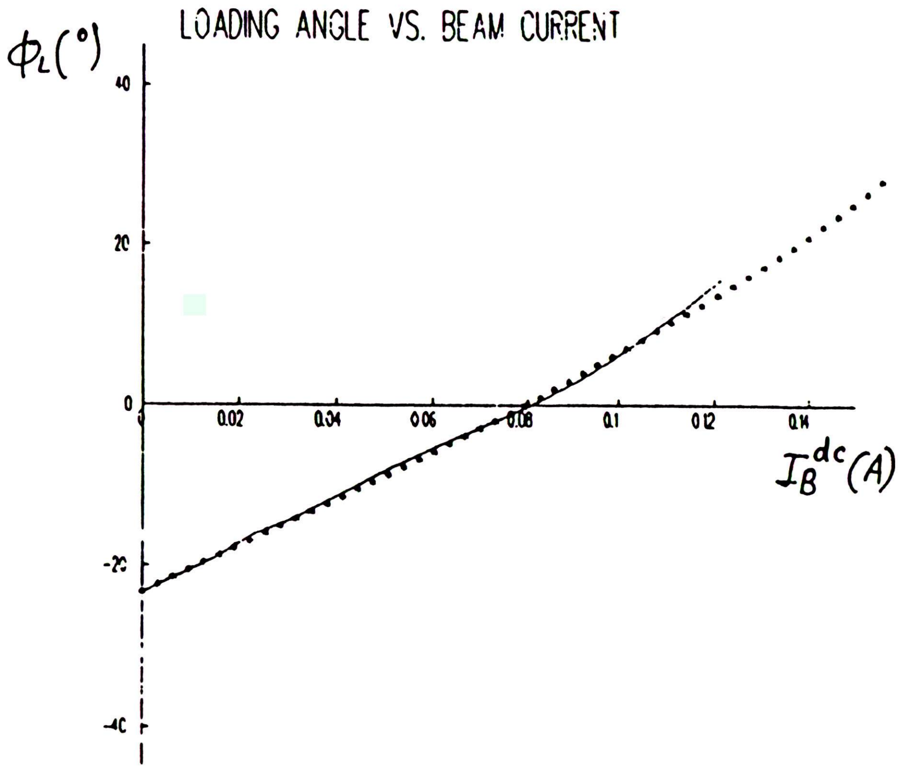
LOADING ANGLE VS. BEAM CURRENT
$\phi_L(°)$
40
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
$I_B^{dc}(A)$

Fig. 6b

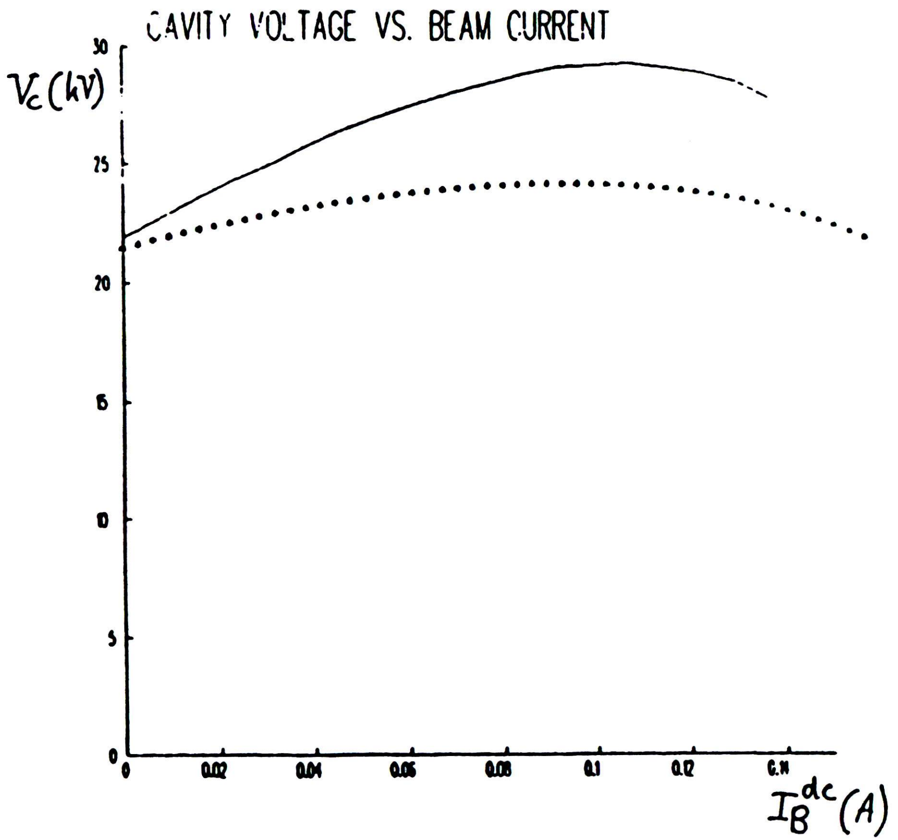
CAVITY VOLTAGE VS. BEAM CURRENT
$V_c(kV)$
30
25
20
15
10
5
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
$I_B^{dc}(A)$

Fig. 7a

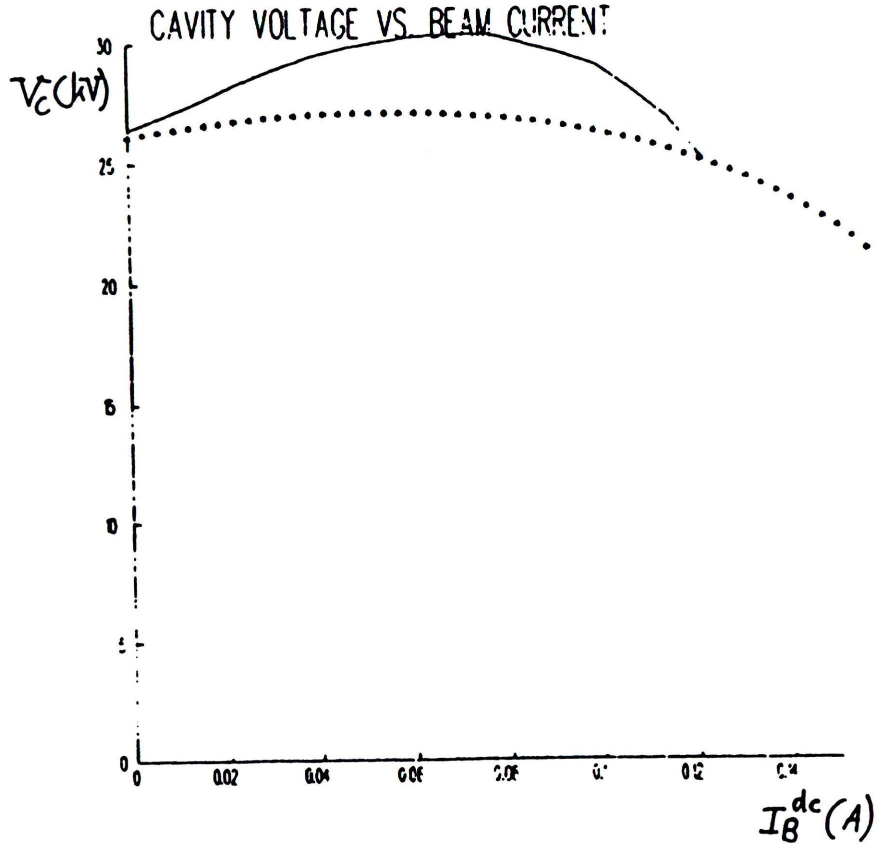
CAVITY VOLTAGE VS. BEAM CURRENT
$V_c(kV)$
30
25
20
15
10
0
0
0.02
0.04
$I_B^{dc}(A)$

Fig 7b

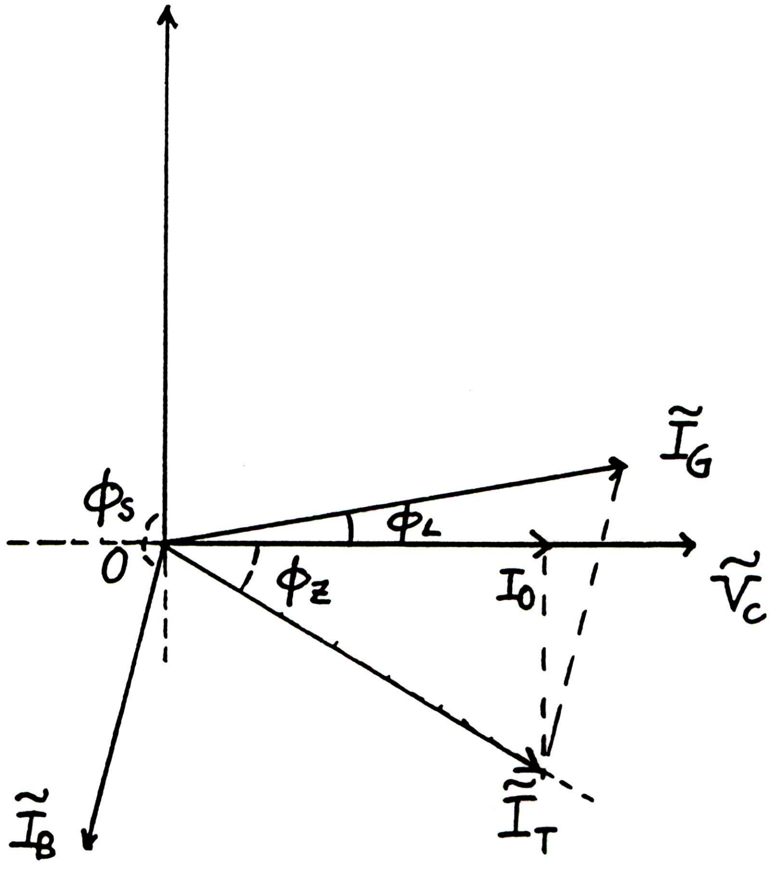

Fig 8 : the phasor diagram

MINUIT FIT #1

$V_c = V_c(I_B)$

$Q_c = 6650$
$Z_n = 41.6\ \Omega$
$R_S = 142\ k\Omega$

} expected

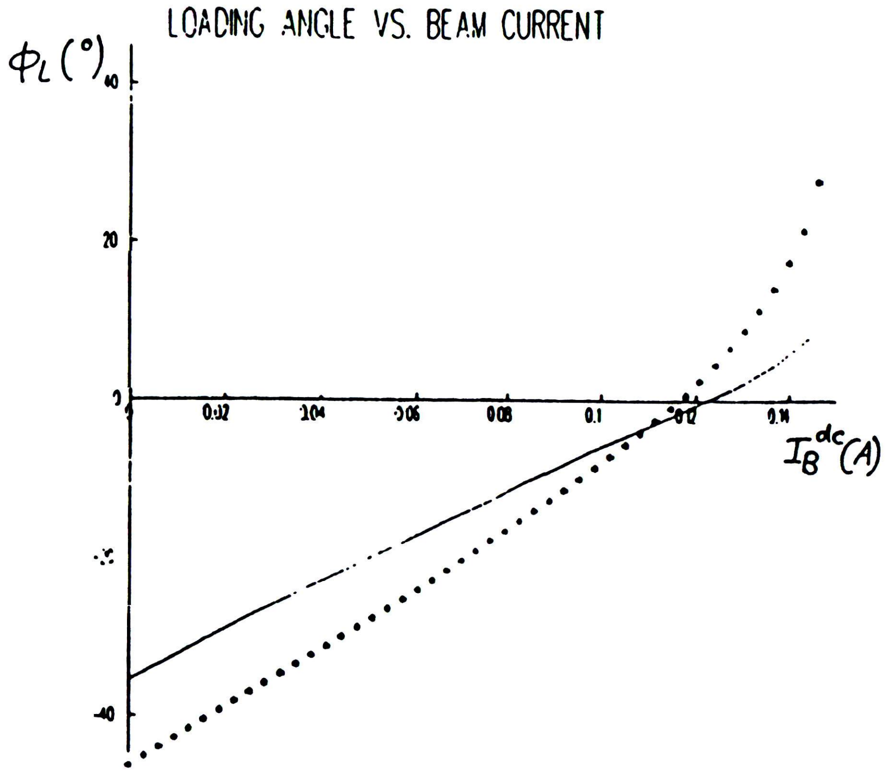

Fig. 9 : Fit on V_c

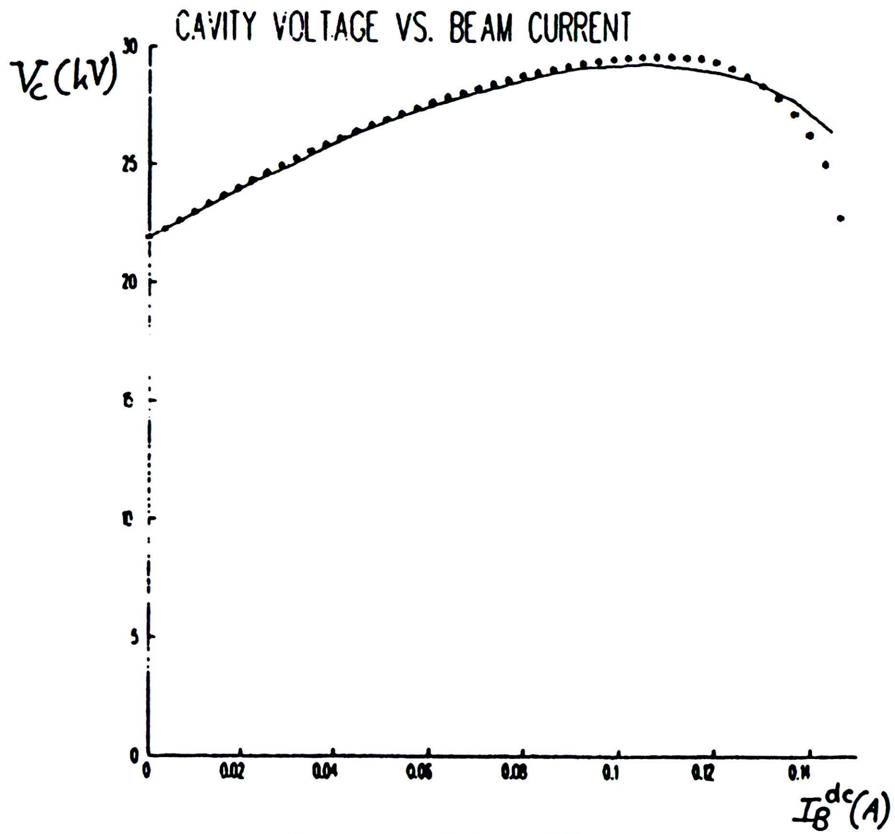

Fig. 10 : Fit on V_c

$\phi_L = \phi_L(I_B)$

$Q_c = 3000$
$Z_n = 36$
$R_S = 75\,k\Omega$
} unexpected

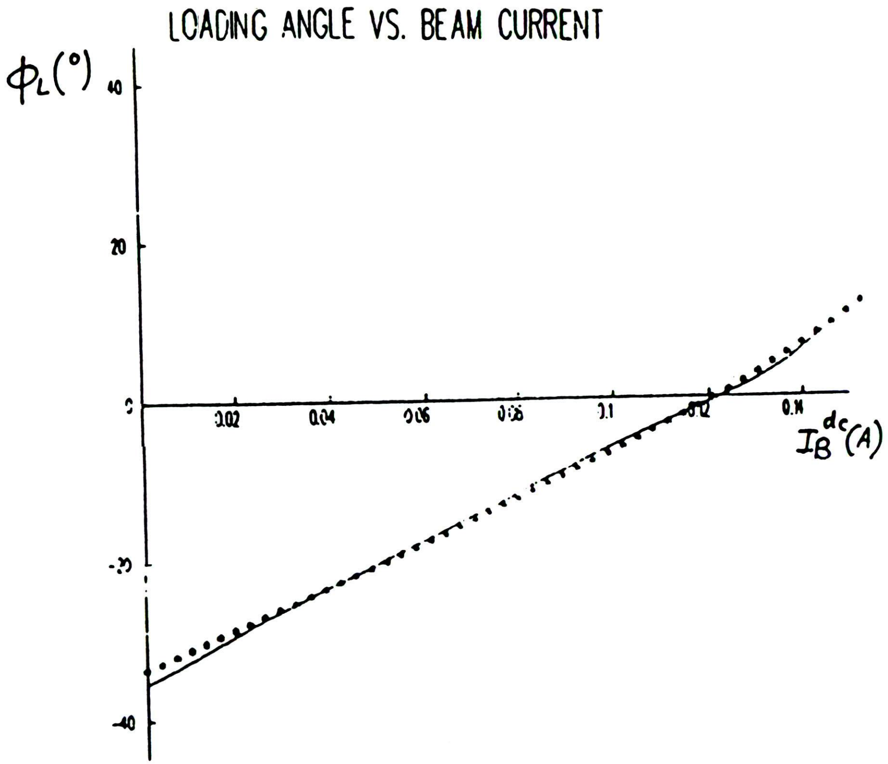

Fig 11: Fit on ϕ_L

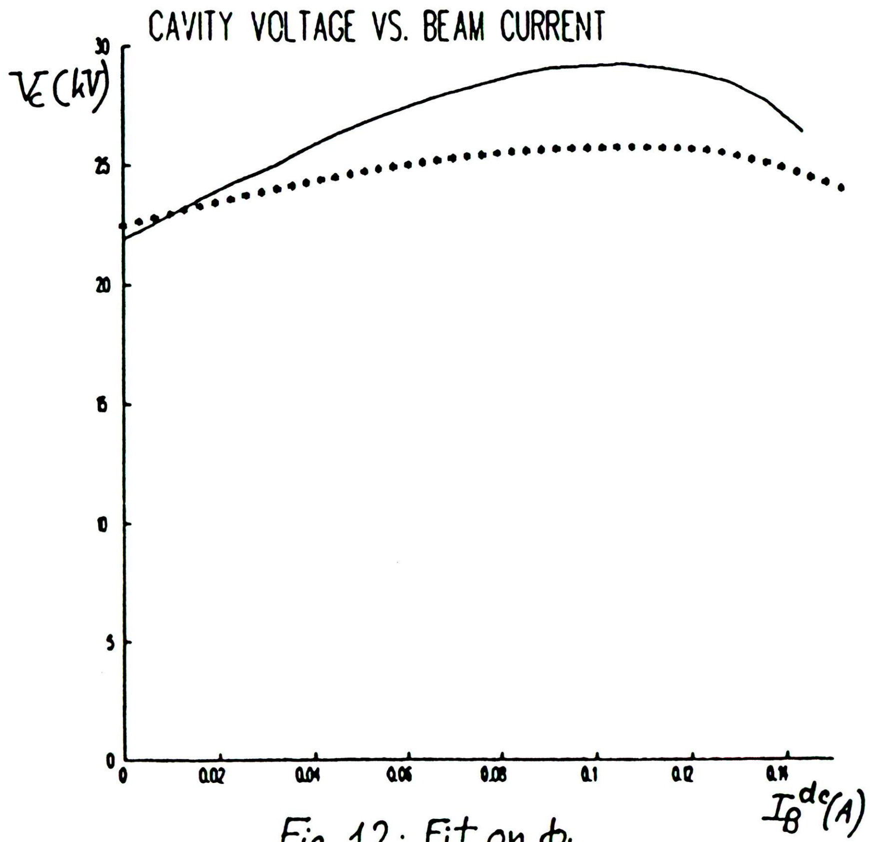

Fig. 12: Fit on ϕ_L