Calculation of some RF dependent parameters for EPA

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A list of computed RF parameters is presented. In these calculations only the case of very low beam current ($I_B \approx 0$) has been considered. The dependence of RF parameters on intensity will be the subject of a subsequent note.

The theoretical formulae are taken from Ref. [1]:

i) for the stable phase angle (synchronous phase) (Fig. 2):

$$\Phi_s = \text{acos}\left(-\left(1 - \left(\frac{U_0}{eV_c}\right)^2\right)^{\frac{1}{2}}\right)$$ where V_c is the cavity total voltage.

For phase stability $\frac{\pi}{2} < \Phi_s < \pi$, U_0 is the synchrotron radiation loss per turn, which is given by:

$$U_0 = \frac{2}{3} r_e \frac{E_0^4}{(mc^2)^3} I_2 \quad \text{and} \quad I_2 = \int \frac{ds}{\rho^2}$$ as from Ref. [2].

The synchrotron integral I_2 has been evaluated from the published values [3] of the vertical damping time constant:

$$\tau_y = \frac{1}{\alpha_y}, \quad \alpha_y = \frac{r_e}{3}\left(\frac{E_0}{mc^2}\right)^3 \frac{c}{L} I_2$$

at the beam energies of 500 and 600 MeV, giving eventually:
$U_0 = 3.481$ keV/turn and $U_0 = 7.235$ keV/turn, respectively.

For the other beam energies considered in the calculations, the 600 MeV value is taken as reference and scaled with the 4th power of E_0.

Physical constants: $L = 125.6636$ m is the circumference of EPA

$r = 2.817938$ fm, $c = 2.997925 \cdot 10^8$ m/s, $m = 0.511003$ MeV/c^2

ii) RF acceptance in the longitudinal phase space (Figs. 3,4):

$$\frac{\Delta E}{E_0} = \left(\frac{2\,U_0}{\pi\, h\, \alpha\, E_0}\left[\left|\cot an\Phi_s\right| - \left(\frac{\pi}{2} - \Phi_s\right)\right]\right)^{\frac{1}{2}}, \text{ where } \pm\frac{\Delta E}{E_0}$$

is the maximum acceptable energy deviation, h is the harmonic number and α is the momentum compaction. Its value has been chosen according to [3]:

$$\alpha = 0.0341 \qquad \text{for } 400 \leq E_0 \leq 500 \text{ MeV}$$

$$\alpha = 0.0337 \qquad \text{for } 550 \leq E_0 \leq 700 \text{ MeV}$$

The angular acceptance is given by:

$\Delta\Phi = \Phi_2 - \Phi_1$, where the maximum elongations Φ_1, Φ_2 are such that:

$$\cos\Phi_i + \Phi_i \sin\Phi_s = \cos(\pi - \Phi_s) + (\pi - \Phi_s)\sin\Phi_s \qquad i = 1,2$$

A trivial solution is $\Phi_1 = \pi - \Phi_s$, the other must be found by solving the above equation numerically or graphically. Finally, the acceptance in the time domain is given by: $\Delta t = \Delta\Phi/\omega_{RF}$

ii) Phase oscillation frequency (Fig. 5)

$$f_s = f_0\left[\frac{-\alpha e\, V_c h \cos\Phi_s}{2\pi E_0}\right]^{\frac{1}{2}}$$ where $f_0 = c/L$ is the revolution frequency.

References:

[1] M. Sands, "The physics of electron storage rings. An introduction," Proceedings of the International School of Physics "Enrico Fermi", Course XLVI, ed. B. Touschek (Academic Press, 1971).

[2] R.H. Helm, M.J. Lee, P.L. Morton and M. Sands. Evaluation of Synchrotron Radiation

[3] M. Bell, H. Kugler, "The EPA Bending Magnet and its Representation in the full description of the Machine; PS/LPI Note 86-01 reviewed on 23.04.86.

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Various RF-dependent parameters for EPA ($I_b \sim 0$)

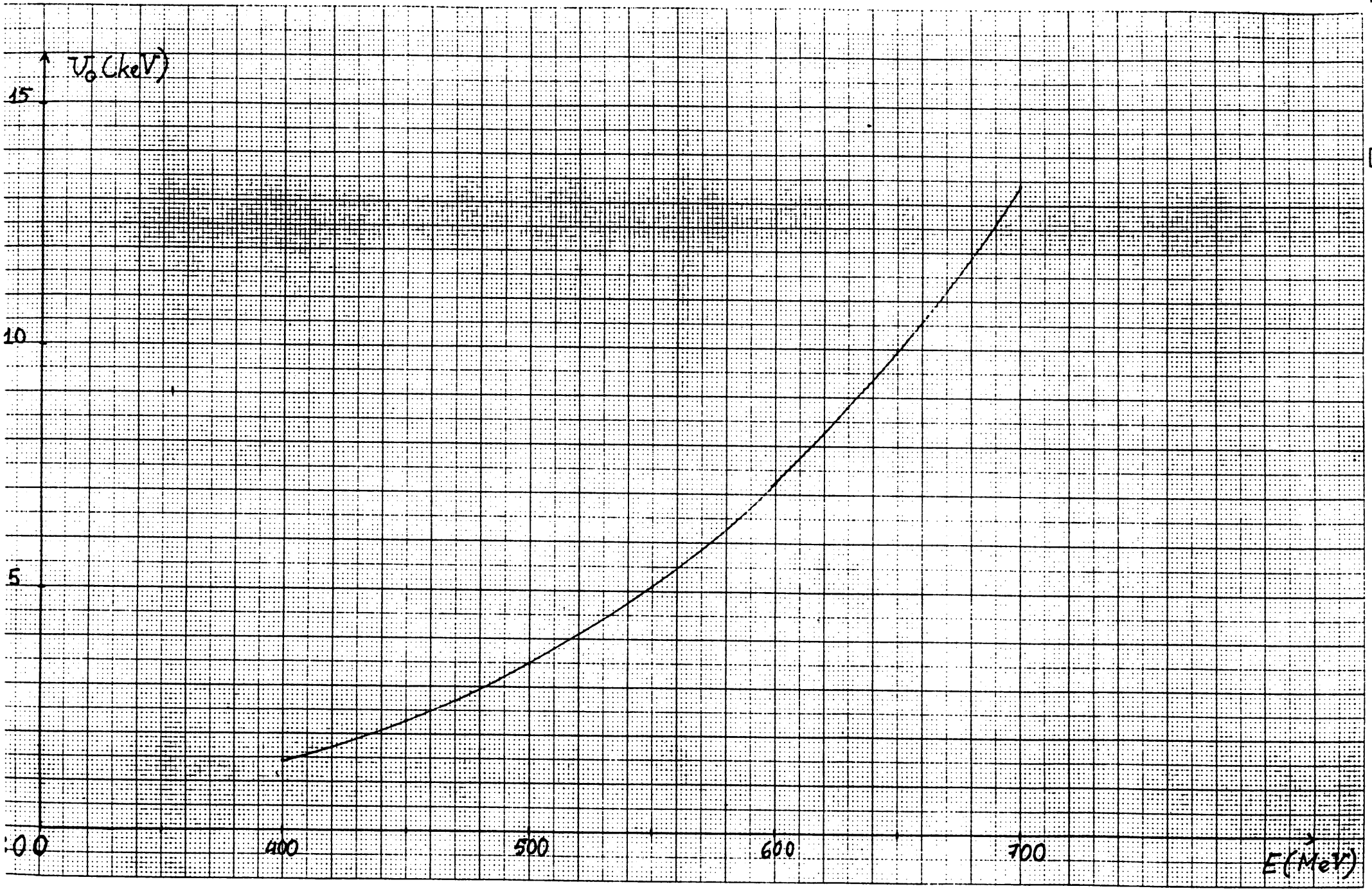
(by S. Bartalucci)

ϕ_s (°)	stable phase angle
$\Delta\phi$ (°)	bucket length
Δt (ns)	bucket length in the Time domain
$\pm \Delta E/E$ (‰)	bucket half-height
f_s (kHz)	synchrotron frequency

E_0 (MeV) / U_0 (keV) \ V_c (kV)	5	10	15	20	25	30
$E_0 = 400$	163.4	171.8	174.5	175.9	176.7	177.3
	239.6	278.9	294.8	304.1	310.2	314.8
	34.9	40.6	42.9	44.3	45.1	45.8
	6.22	8.34	9.98	11.3	12.5	13.7
$U_0 = 1.430$	1.72	2.47	3.04	3.51	3.92	4.30
$E_0 = 450$	152.7	166.8	171.2	173.4	174.7	175.6
	198.5	254.3	275.9	288.1	296.2	302.1
	28.9	37.0	40.2	41.9	43.1	44.0
	6.00	8.15	9.68	11.0	12.1	13.1
$U_0 = 2.289$	1.56	2.31	2.85	3.30	3.70	4.05
$E_0 = 500$	135.7	159.6	166.5	170.0	172.0	173.3
	140.5	224.2	253.3	269.4	279.9	287.5
	20.4	32.6	36.9	39.2	40.7	41.8
	5.46	7.97	9.48	10.7	11.8	12.8
$U_0 = 3.481$	1.33	2.15	2.69	3.12	3.50	3.84
$E_0 = 550$		149.4	160.1	165.2	168.2	170.2
		186.4	226.4	247.5	261.1	270.7
		27.1	33.0	36.0	38.0	39.4
		7.72	9.35	10.6	11.6	12.6
$U_0 = 7.097$		1.95	2.50	2.93	3.30	3.62
$E_0 = 600$		133.7	151.1	158.8	163.2	166.0
		133.9	192.8	221.1	238.8	251.1
		19.5	28.1	32.2	34.8	36.5
		7.00	9.06	10.4	11.4	12.3
$U_0 = 7.235$		1.68	2.31	2.75	3.12	3.44
$E_0 = 650$			138.4	150.1	156.5	160.6
			149.2	189.1	212.2	228.3
			21.7	27.5	30.9	33.2
			8.44	10.0	11.2	12.1
$U_0 = 9.965$			2.05	2.55	2.93	3.26
$E_0 = 700$			116.7	137.9	147.6	153.5
			80.6	147.7	180.2	201.1
			11.7	21.5	26.2	29.3
			6.60	9.37	10.8	11.9
$U_0 = 13.40$			1.53	2.28	2.71	3.06

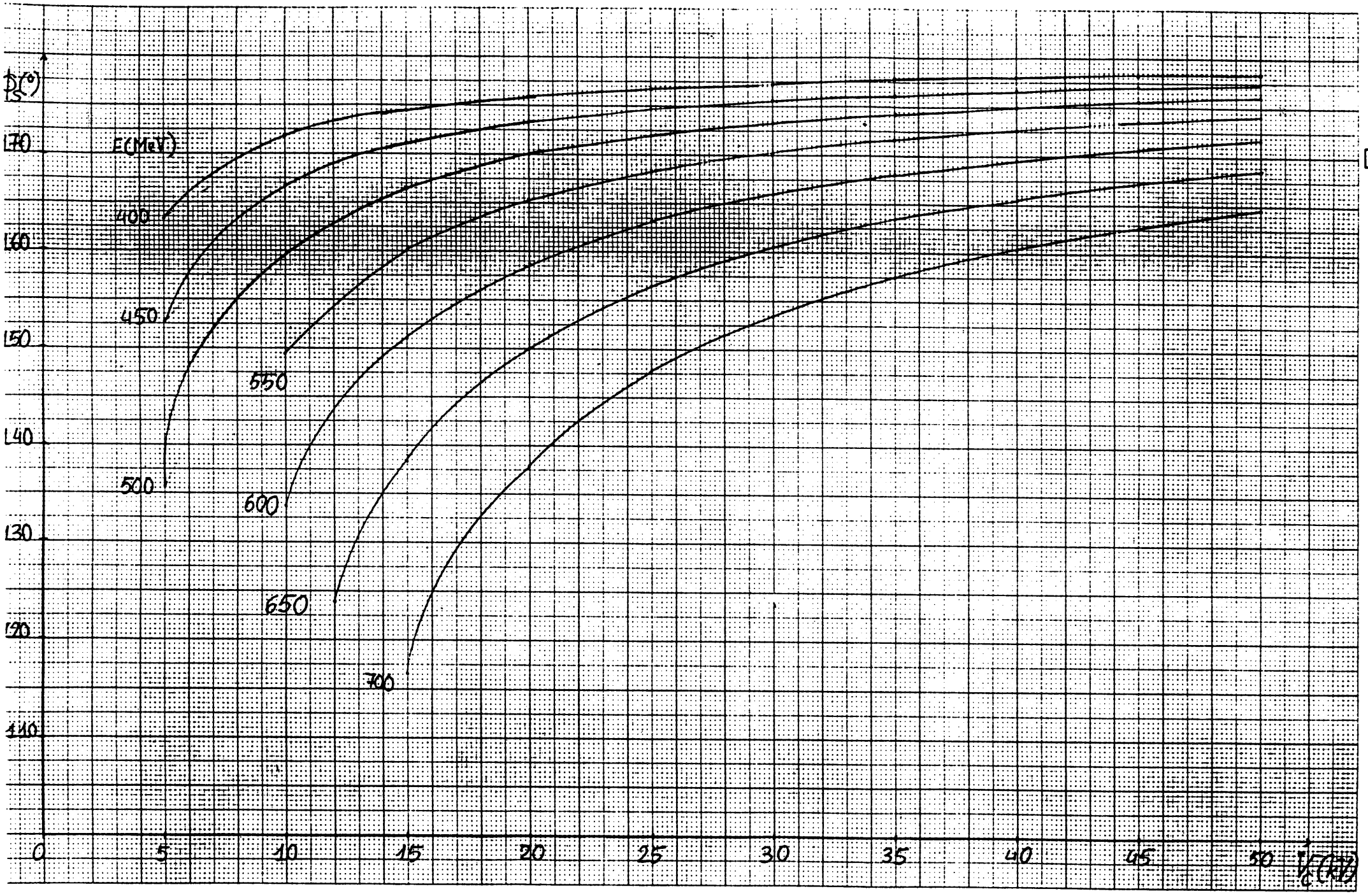
ϕ_s (°)	stable phase angle
$\Delta\phi$ (°)	bucket length
Δt (ns)	bucket length in the Time domain
$\pm \Delta E/E$ (‰)	bucket half-height
f_s (kHz)	& syuchrotron frequency

E_0 (MeV) / U_0 (keV) \ V_c (kV)	35	40	45	50
$E_0 = 400$	177.7	178.0	178.2	178.4
	318.3	321.0	323.3	325.3
	46.3	46.7	47.0	47.3
$U_0 = 1.430$	14.8	15.7	16.6	17.5
	4.65	4.97	5.27	5.56
$E_0 = 450$	176.2	176.7	177.1	177.4
	306.6	310.2	313.2	315.7
	44.6	45.1	45.6	45.9
$U_0 = 2.289$	14.1	15.0	15.8	16.6
	4.38	4.68	4.97	5.24
$E_0 = 500$	174.3	175.0	175.6	176.0
	293.3	297.9	301.6	304.8
	42.7	43.4	43.9	44.4
$U_0 = 3.481$	13.7	14.5	15.3	16.0
	4.15	4.44	4.71	4.96
$E_0 = 550$	171.6	172.7	173.5	174.1
	278.0	283.8	288.5	292.5
	40.5	41.3	42.0	42.6
$U_0 = 5.097$	13.4	14.2	15.0	15.7
	3.92	4.20	4.46	4.70
$E_0 = 600$	168.1	169.6	170.7	171.7
	260.3	267.6	273.4	278.3
	37.9	38.9	39.8	40.5
$U_0 = 7.235$	13.2	13.9	14.7	15.3
	3.73	4.00	4.25	4.49
$E_0 = 650$	163.5	165.6	167.2	168.5
	240.0	249.0	256.3	262.3
	34.9	36.2	37.3	38.2
$U_0 = 9.965$	13.0	13.7	14.4	15.1
	3.55	3.82	4.06	4.29
$E_0 = 700$	157.5	160.4	162.7	164.5
	216.1	227.6	236.7	244.1
	31.5	33.1	34.4	35.5
$U_0 = 13.40$	12.7	13.5	14.2	14.9
	3.36	3.63	3.87	4.10



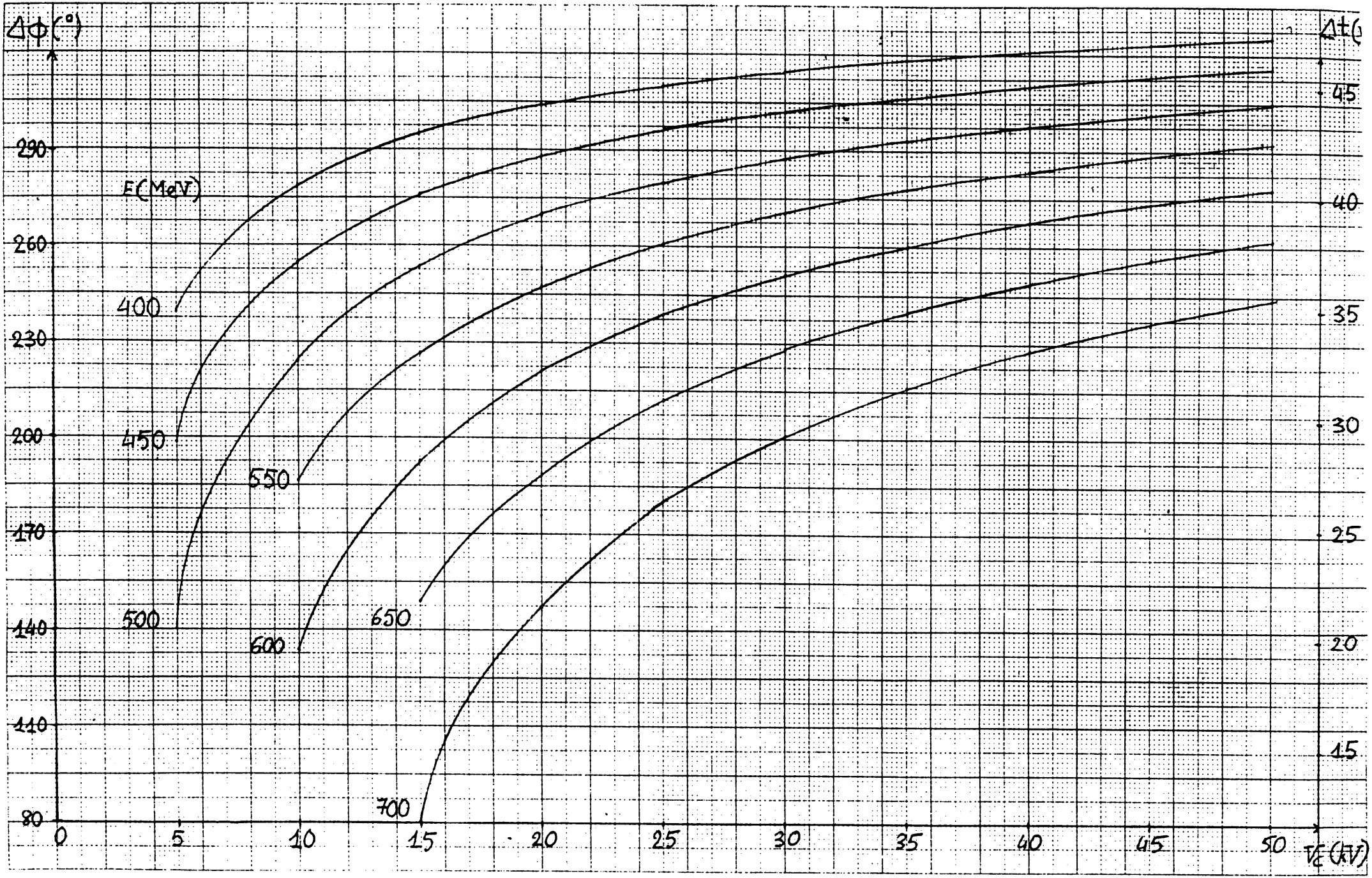
Synchrotron radiation energy loss per Turn vs. beam energy

Fig. 1



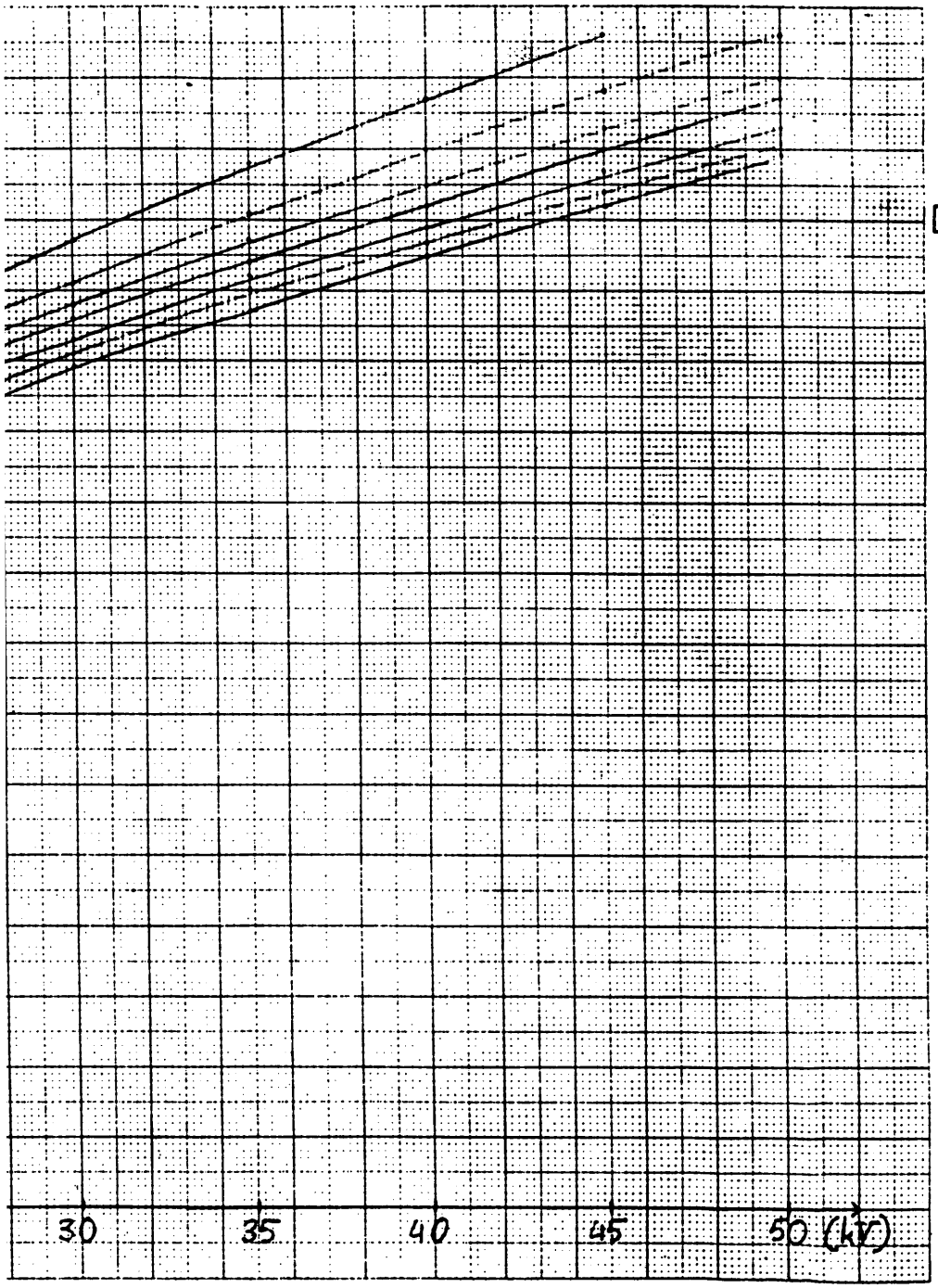
Synchronous phase vs. cavity voltage for various beam energies

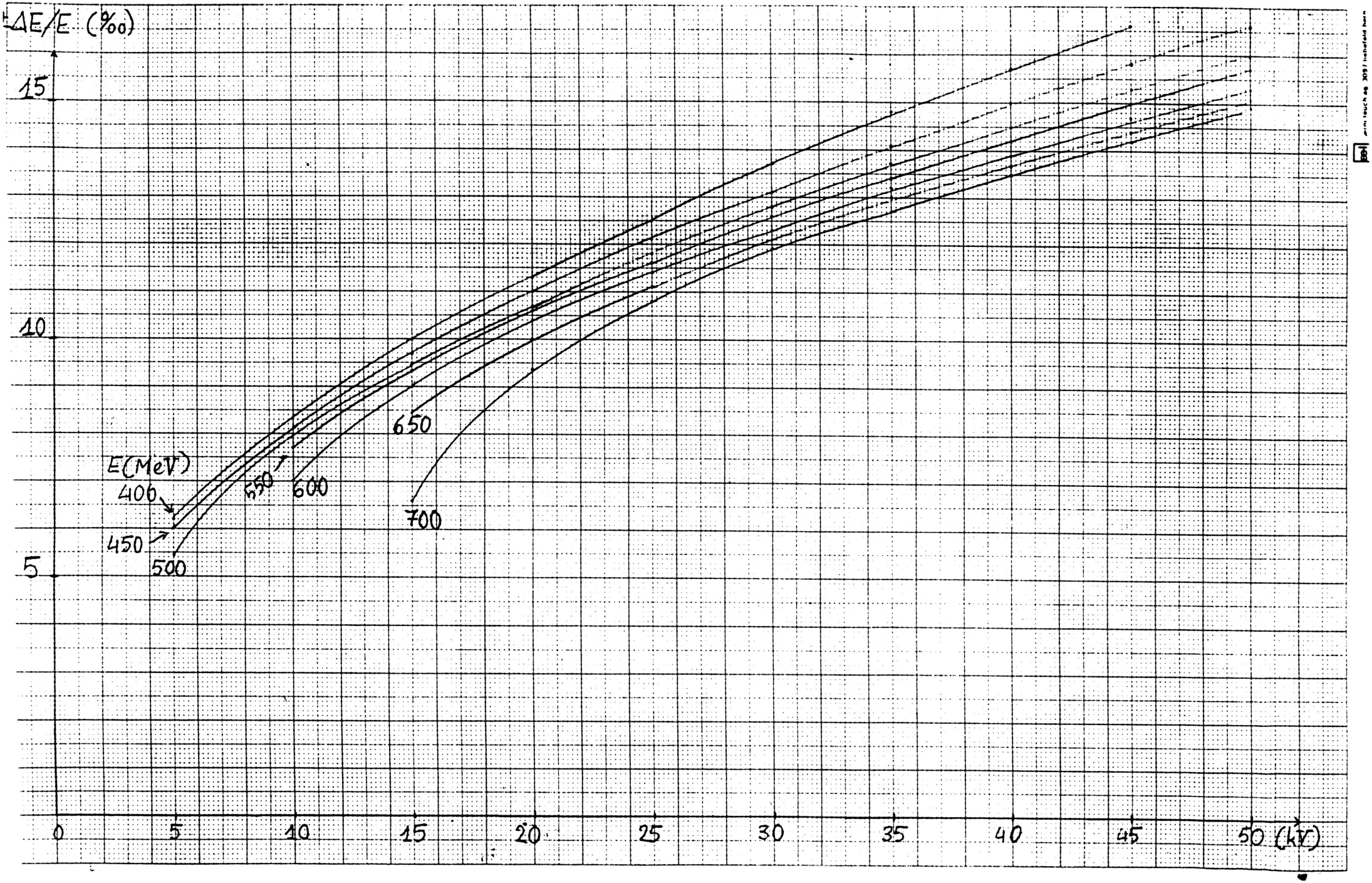
Fig. 2

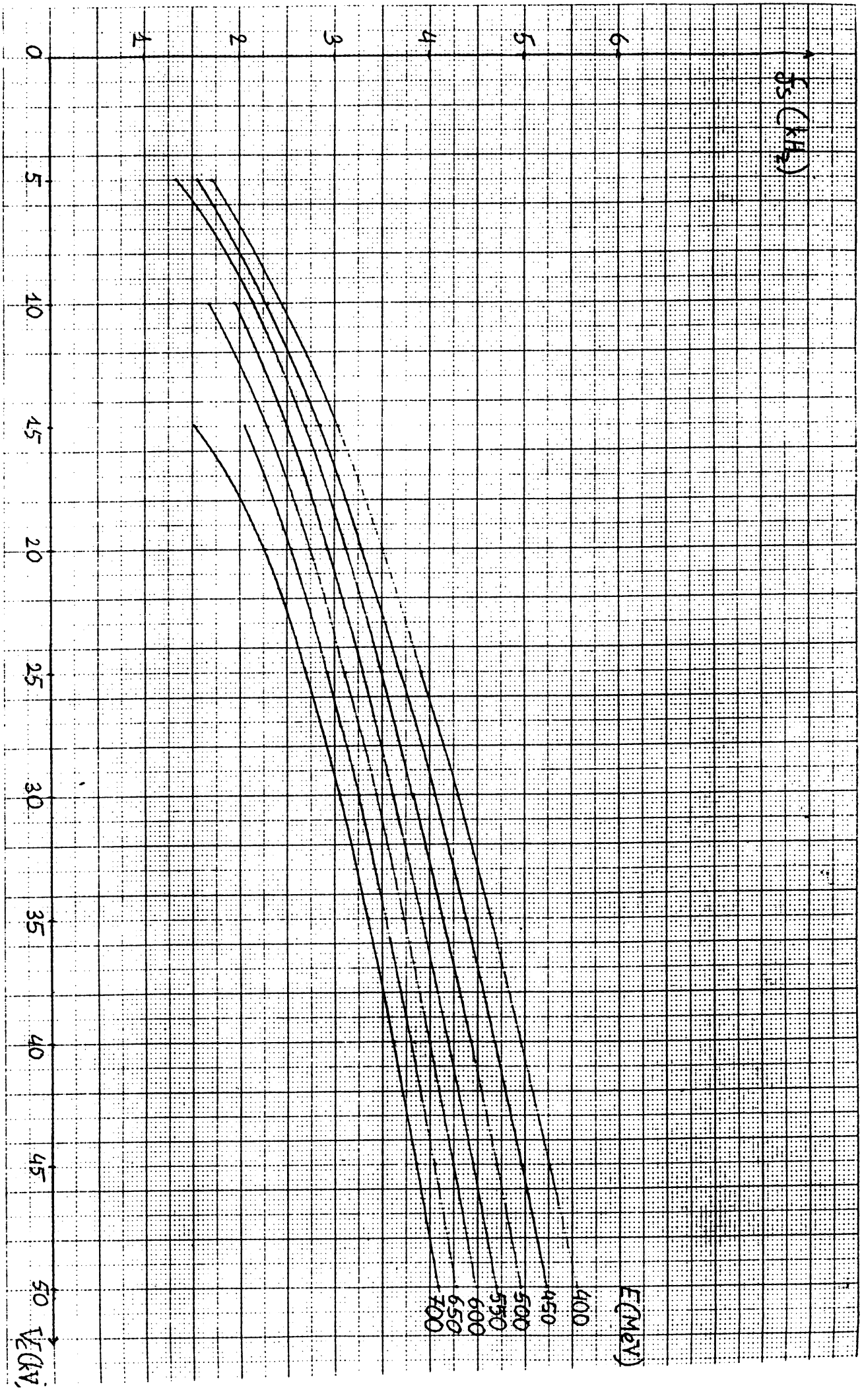


RF (angle / time) acceptance vs. cavity voltage for various beam energies

Fig 3


30
35
40
45
50 (kV)


ΔE/E (‰)
15
10
5
E(MeV)
400
450
500
550
600
650
700
0
5
10
15
20
25
30
35
40
45
50 (kV)



Synchrotron frequency vs. cavity voltage for various beam energies

Fig. 5